

An investigation of the hierarchy of simplifications for risk margins in the Solvency Assessment and Management framework

by PRF Carswell, RT Farnaby and N Nirghin

Presented at the Actuarial Society of South Africa's 2018 Convention
24–25 October 2018, Cape Town International Convention Centre

ABSTRACT

Regulation requires South African insurers to follow the Solvency Assessment and Management (SAM) framework and requires insurers to calculate a risk margin as part of the technical provisions that they must hold. The full calculation of the risk margin can be computationally intensive and consequently many insurers may make use of simplifications. This paper aims to assess the appropriateness of the simplifications specifically for a life insurer offering annuity, credit life and whole-of-life assurance products to assess whether there are biases within the simplifications. It was found that there are systemic biases resulting in understatements of the risk margin for annuities and the factors affecting the variability of the bias are explored. Lower escalation rates and younger lives have larger biases. The Level 1 simplification for Credit Life products are shown to be good. Whole-of-life products are shown to be underestimated on the Level 1 and Level 2 simplification. Where the simplifications are found to introduce bias for annuities and whole-of-life assurances, some preliminary analysis of that bias is provided which leads to improved estimators of the risk margin being proposed for both products.

KEYWORDS

Risk margin; solvency assessment management; solvency capital requirements; hierarchy of simplifications; cost-of-capital; annuity; credit life; whole of life

CONTACT DETAILS

Mr Peter Carswell; Email: pete.carswell@gmail.com

Mr Ross Thomas Farnaby; Cell: +27(0)74 830 3465; Email: rtfarnaby@gmail.com

Ms Nikhara Nirghin; Cell: +27(0)83 520 7040; Email: nikhara.nirghin@gmail.com

1. INTRODUCTION

1.1 South African insurance companies determine solvency using a framework known as the Solvency Assessment Management (SAM) framework. The primary legislation for SAM is the Insurance Act, 2017¹ (which came into force on 1 July 2018), followed by the secondary legislation known as the Financial Soundness Standards for Insurers (FSI).² Guidance notes have been issued to assist insurers in interpreting and complying with requirements of certain FSIs although the guidance notes do not have the same level of enforceability as the primary or secondary legislation.³ The objective of the Act is to promote a fair, safe and stable insurance market for the benefit and protection of policyholders for both long- and short-term insurance contracts.⁴

1.2 SAM requires insurers to calculate the level of Own Funds and compare these against the Solvency Capital Requirement (SCR). Own Funds are calculated as the value of assets less the value of liabilities. The most complex and typically material liability calculation is that for the technical provisions. The technical provisions are defined in Prudential Standard FSI 2.2⁵ as the value of their insurance obligations in the event that such obligations were transferred immediately to another insurer.⁶ SAM requires technical provisions to be:

- calculated as a whole; or
- equal to the sum of the best estimate liability (BEL) and a risk margin.

1.3 The risk margin is the part of the technical provisions that ensures that the value of the technical provisions is equivalent to the amount another insurer would be expected to pay to take over and meet the insurance obligations of the insurer.⁷ The risk margin is calculated using a cost-of-capital approach where the capital involved is the SCR for non-hedgeable risks.⁸

1.4 The SCR is the risk responsive capital that an insurer needs to hold in order to ensure that the probability of ruin is no more than 0.5% in the next 12 months. The

1 Insurance Act (Act 18 of 2017, Republic of South Africa)

2 Financial Services Board, Prudential Standards Framework for Financial Soundness of Insurers, 2018

3 Financial Services Board, Guidance Note (GN) FSI 2.2, 2016, <https://www.fsb.co.za/Departments/insurance/Documents/Guidance%20Note%20on%20FSI%202.2%20Valuation%20of%20Technical%20Provisions.pdf>, 28/05/2018

4 Long Term Insurance Act 2017, Section 3.

5 Financial Services Board, Prudential Standard Financial Soundness of Insurers (FSI) 2.2, 2016, <https://www.fsb.co.za/Departments/insurance/Documents/Prudential%20Standard%20FSI%202.2%20Valuation%20of%20Technical%20Provisions.pdf>, 28/05/2018

6 Prudential Standards FSI 2.2, *supra* at 4.1

7 Prudential Standards FSI 2.2, *supra* at 14.1

8 Prudential Standards FSI 2.2, *supra* at 14.2–14.3

SCR is calculated so as to ensure that all non-hedgeable risks are taken into account. (Floreani, 2011).

1.5 The Act permits⁹ the use of simplified methods and techniques when calculating technical provisions where those methods and techniques are proportionate to the nature, scale and complexity of the risks supported. The explicit calculation of the risk margin may be disproportionately intensive to calculate. This is reiterated in FSI 2.2¹⁰ and a hierarchy of some possible simplifications are provided in a Guidance Note.¹¹ The highest level of the hierarchy estimates the projected SCR components for each risk separately, the second highest level estimates the projected total SCR while the third level estimates the value of all future SCRs in a single step. Moving down the hierarchy simplifies the calculation with new assumptions explicitly or implicitly introduced at every level. Where these assumptions are incorrect they may result in misstatement of the future SCR and ultimately the risk margin. Where the misstatements are systematically over- or under-statements this would result in a systemic over- or under-statement of the risk margin respectively.

1.6 Objectives and Research Questions

1.6.1 The overall objective of the research is to establish the effectiveness of the first two levels of the hierarchy of simplifications in estimating the risk margin of a life insurance company.

1.6.2 The research questions of this paper are:

- Do any of the simplifications introduced under FSI GN 2.2 accurately predict the value of the future SCR?
- Is there a systemic bias in the prediction?
- Where there is a systemic bias, can we explain the bias and correct for it through an alternative approach?
- Can specific combination of products and simplification be identified as sufficiently biased that they should be avoided?

1.7 Importance of Research

1.7.1 The research questions are relevant in practical application, as insurers might be incorrectly assuming that using a certain level of the hierarchy is reasonable; however, the approaches may be producing severe biases when using it on a life insurer's total liability profile. SCRs could be far lower or higher than it should be, due to these biases. This in turn affects the calculation of the risk margin.

9 Insurance Act, 36. (6)(c)(ii)

10 Prudential Standards FSI 2.2, 14.18

11 Guidance Note FSI 2.2, *supra* at chapter 3 section A part 1

1.7.2 A bias that exists for a product class would result in all insurers misstating their liabilities and could pose a systemic risk in the event of an industry-wide shock. This would pose a risk to a regulator's objective of financial stability. According to the SAM QIS3 Report,¹² only ten insurers out of a total of 197 use a full calculation approach which is a full, comprehensive approach in calculating future SCRs without simplification.¹³ This implies that there were 187 insurers in South Africa that did not make use of the full calculation and therefore were at risk to the biases that may be a by-product of the simplifications. Table A.1 in Appendix A, lists the full breakdown of what simplification insurers are using in the South African insurance market.

1.7.3 Simplifications are often used at the time of pricing products and overstatements of capital could result in customers being unnecessarily overcharged for the cost of the capital. If there is a bias for a product class then all consumers purchasing that product could be adversely affected. If the simplification understates the future capital requirements, the pricing could be insufficient to meet the insurer's actual cost of capital.

1.7.4 The risk margin's value is of importance as it affects the insurer's ability to meet its current and future obligations and to make efficient use of their capital (Doff, 2008). This follows because if the risk margin is too high the insurer will be unnecessarily holding funds that could be better utilised elsewhere to provide better returns to the company and its shareholders or returned as dividends.

1.7.5 Simplifications may be used in transaction work (e.g. liability take-over, company mergers) when the value of liabilities are being assessed by the buyer's actuarial team. An misstatement of the risk margin could provide incorrect information to the team who are negotiating the transaction price, or leave an expectation gap between the expected solvency of the company (determined using simplifications during the transaction) and the actual solvency of the company (determined using more explicit calculations at the time of reporting).

1.7.6 An inaccurate calculation of future SCRs would impact the embedded value of an insurer by misstating the cost of capital of the insurer and the cost of residual non-hedgeable risks for insurers calculating their embedded value using the Market Consistent Embedded Value method. The embedded value calculation is important due to two reasons: the first is to accurately value a company for use in capital transactions and the second is to measure the value added by the management of an insurer (O'Keeffe et al., 2005) which can directly affect the remuneration and incentives. It is important to note that this research does not examine the impact of the inaccurate

12 Financial Services Board (FSB) Quantitative Impact Study (QIS) 3 Report, 2014, <https://www.fsb.co.za/Departments/insurance/Documents/SAM%20SA%20QIS3%20Report.pdf>, 26/1/2017

13 Ibid.

calculation of cost of capital and cost of residual non-hedgeable risks components of embedded value, but the findings can be applied directly to these calculations.

1.7.7 This research also has implications in the new International Financial Reporting Standards 17 (IFRS17) where insurers choose to use a projected SCR in determining the ‘unit of measure’ when deciding the pattern of future revenue recognition. As with embedded value, IFRS17 is not a focus of this paper, however the findings would be directly applicable.

1.7.8 Consequently it is important to ensure that there are no biases present in the future SCR estimations and where there are biases present, we need to understand why they occur. This will enable actuaries to take preventative measures to avoid the occurrence and impact of such biases.

1.7.9 There is no research in any published literature on the effects of the hierarchy of simplifications and the potential negative impact of them, under SAM or Solvency II regulations. This research paper aims to fill this gap. It would provide a useful practical tool for insurers, in order for them to avoid these biases and to be more accurate in their estimations of risk margins. Therefore, this ultimately enables the insurer to treat all stakeholders fairly.

1.8 Structure of the Research Paper

This research paper firstly collates and contrasts the relevant research on this topic in section 2. The SCR and the risk margin are further analysed in the literature review under sections 2.1 and 2.2 respectively. The methods that were utilised to perform the objectives of the research are examined under section 3. In section 4 the results of the paper are presented and there is a discussion of these key results in order to identify if there are any biases, and if so why they occurred. Lastly, a more accurate method of estimating an insurer’s SCR is introduced in this section. This is followed by a summary and conclusions in section 5 and 6. The paper ends off with areas for further research, noted in section 7.

2. LITERATURE REVIEW

2.1 Solvency Capital Requirement

2.1.1 The SCR covers all quantifiable risks for existing business and new business expected to be written in the following 12 months.¹⁴ The SCR for the risk margin is quantified on the basis of all non-hedgeable risks. In addition, the SCRs are calculated annually.¹⁵

¹⁴ Financial Services Board, Prudential Standard Financial Soundness of Insurers (FSI) 4, 2016, <https://www.fsb.co.za/Departments/insurance/Documents/Prudential%20Standard%20FSI%204%20Calculation%20of%20the%20SCR%20Using%20the%20Standardised%20Formula.pdf>, 28/05/2018

¹⁵ Prudential Standards FSI 4, supra at 4.6

2.1.2 A level hierarchy involves characterising various levels according to the complexity of calculations involved in that level. Therefore, the higher the level the more extensive the level of calculation involved (Lane, 2006).

2.1.3 FSI 2.2 introduces a full, comprehensive calculation of an insurer's SCR. This requires the calculation of the SCR for each insurer, at each future time and for each module of risk. This is computationally extensive and the level of complexity involved in the calculations might not be proportionate to the nature, scale and complexity of the risks of the insurer's business.¹⁶ Therefore, simplifications are permitted in FSI 2.2 section 14.18. Possible simplifications are introduced in FSI GN 2.2, chapter 3 section A. The Guidance Note lays out three levels of hierarchies for life insurers as well as a fourth for non-life insurers which is not considered as part of this research. The hierarchy is characterised by complexity of calculation where the lower the level the more extensive the calculations involved. According to the FSI GN 2.2, chapter 3 section A the following levels of simplifications can be used to calculate SCRs for life insurers:

- (1) Approximate the individual risk categories or components within some or all of the modules used for the calculation of future SCRs;
- (2) Approximate the whole SCR for each future year by, for example, using a proportional approach; and
- (3) Estimate all future SCRs "at once" by, for example, using an approximation based on the duration approach.

2.1.4 The BEL involves considering uncertainty in the future cashflows. This is done by assigning probabilities to each possible cashflow for all decrements, so that it represents the mean of the distribution of cashflow values. Additional margins are not required in the best estimate calculation and it needs to be calculated for all future durations of the policy. The FSI 2.2 requires that we use this method to calculate an initial value of liabilities.¹⁷ The discounted cashflow approach to modelling the liabilities has been expressed as a method most consistent with the long-term value creation (Yao, Chen & Lin, 2004). In addition, it is a method that can provide a more realistic picture of a company's value than many other approaches (including the accounting approaches) (Copeland, Kollen & Murrin, 1994).

2.2 Risk Margin

2.2.1 The risk margin is calculated by discounting the capital required by another insurer – at each duration – in order to finance the insurance obligations of the insurer – if the need arose.¹⁸ The value calculated is then subject to a 6% cost-of-capital

¹⁶ Prudential Standards FSI 2.2, *supra* at 14

¹⁷ Prudential Standards FSI 2.2, *supra* at 6

¹⁸ Prudential Standards FSI 2.2, *supra* at 14.1

rate, which is standardised. It therefore follows that the risk margin is calculated by discounting the SCR values, subject to a 6% cost-of-capital rate.¹⁹ The SCR values, could be calculated in full or by the simplifications introduced above.²⁰ The formulae required for the calculations of the risk margin are presented in section 3.7.

2.3 Consequences of Miscalculation of the Risk Margin

In the event that the risk margin is underestimated, the insurer will hold less capital than required to be at an acceptable solvency level (Cummins, Harrington & Klein, 1995). This might lead to risky behaviour by the board of directors due to implications on financial plans of underestimated liabilities, putting policyholders' benefits in danger. If the risk margin is overestimated, it will lead to excessive shareholders' funds being tied up in regulatory capital, reducing shareholders return (Färe, Grosskopf & Weber, 2004).

2.4 Potential Issues presented in the Hierarchy of Simplifications

2.4.1 Under the FSI, the SCR values calculated can never be negative.²¹

2.4.2 Certain levels of the hierarchy of simplifications make use of the BEL, in order to run off the future SCR values. With this in mind, several types of insurance policies give rise to negative best estimate liabilities (Warren, 1968). In addition all profit making business should produce a negative BEL (or in the case of investment contracts a negative rand reserve), at least in the early years. What is important to note is that SAM explicitly states that negative best estimate liabilities are allowed, and zeroisation of negative best estimate liabilities is not allowed.²² The issue of negative best estimate liabilities may be exacerbated if the reserves are calculated on all the insurer's policies rather than on a policy-by-policy basis.

2.4.3 There is no significant research either in support or against the use of this hierarchy. More importantly, there is no research on the hierarchy of simplifications.

2.5 Statistical Methods

The results will be analysed using score functions for the estimations of the risk margin values compared to their actual value. Christoffersen & Jacobs (2003) recommended the use of score functions for judging the different estimates under different pricing models for options. This was done by assigning a score to each possible model by using percentage errors, that is, the percentage by which the price of the option determined using an estimation is different to the actual option price. This percentage error reflects the effectiveness of any given model (Christoffersen & Jacobs, op. cit.).

19 Prudential Standards FSI 2.2, supra at 14.14

20 Guidance Note FSI 2.2, supra at chapter 3 section A part 1

21 Prudential Standards FSI 4, supra at 5.2

22 Prudential Standards FSI 2.2, supra at 6.11

3. METHOD

3.1 Overview

3.1.1 The expected cashflows for the annuities, credit life and whole-of-life products were modelled and the risk margin and other key metrics such as the BEL were explicitly calculated. The risk margin was then recalculated under each level of the hierarchy of simplifications. Where a simplification required the user of a driver, a number of different drivers were considered.

3.1.2 Sensitivity testing was performed on key policy inputs and key assumptions to identify the level of stability of inaccuracies of the calculations for each level of the hierarchy.

3.2 Driver Projections

3.2.1 Some simplifications require use a driver approach to project a future capital component. In a driver approach the ratio of a known metric (called the driver) over the metric to be estimated (called the target) is assumed to be constant over time. In formulae this can be expressed as:

$$DRIVER(t) / TARGET(t) = DRIVER(f) / TARGET(f) \quad (1)$$

where:

Driver(x) is the value of the driver metric at time *x*

Target(x) is the value of the target metric at time *x*

3.2.2 When a driver metric approach is used in these projections we typically know the value of the target at time *t*=0 and the value of the driver for all time periods allowing the estimation of the target at all future periods.

3.3 Policyholder Portfolio Construction

3.3.1 Appendix B summarises the portfolios that were constructed.

3.3.2 A range of annuity benefit escalation rates were considered with the lowest escalation rate being zero (i.e. a level annuity) and the highest being an eight percent per annum increase. Only annually compound escalation rates were considered.

3.3.3 A range of ages on the balance sheet date were considered with the lowest age being 60 years old and the highest 95 years old.

3.3.4 A range of outstanding guarantee periods were considered with the lowest period being zero months and the highest 120 months. During the guarantee period the annuity benefit is payable irrespective of whether the policyholder has survived or not.

3.3.5 The annuitant was assumed to receive R8 000 per month in the month following the valuation date.

3.3.6 For credit life policies the policy was assumed to have a level premium throughout the term of the policy, with a sum assured that reflects the outstanding balance on the loan. The policy was assumed to only provide a benefit on death.

3.3.7 A range of ages on the balance sheet date were considered with the lowest age being 25 years old and the highest 75 years old.

3.3.8 Three different initial sum assured levels were considered: R10 000; R100 000; R2 000 000.

3.3.9 The original loan term was varied with a lowest value of two years and the highest value of seven years.

3.3.10 The whole life assurance product pays a benefit out on death only. The premium and sum assured were assumed to increase at the same rate.

3.3.11 A range of ages on the balance sheet date were considered with the lowest age being 25 years old and the highest 75 years old.

3.3.12 Three different initial sum assured levels were considered: R10 000; R100 000; R2 000 000.

3.3.13 A range of premium and sum assured escalation rates were considered with the lowest being 0% and the highest 10%. Only annually compounding escalation rates were considered.

3.3.14 Only single life products were considered and consideration of joint life products is left to future research.

3.4 Risks analysed

3.4.1 The risks analysed were life insurance underwriting risk (as defined in FSI 4.2) and operational risk. The sub-modules analysed under life insurance underwriting risk include: longevity and expenses for annuity products; mortality, expenses, lapses and catastrophe for risk products.

3.4.2 Market risk was not analysed. One could argue that the long-dated interest rate risk on the annuities is non-hedgeable and so should be included in the reference undertaking SCR calculation. It is excluded in this paper for the sake of simplicity.

3.5 Assumptions

3.5.1 Assumptions about mortality were carried out in accordance with the South African Life Tables 85–90.²³ It would be more appropriate to model annuities using an annuitant mortality table, however we demonstrate that the sensitivity to mortality rates does not affect the overall conclusion.

3.5.2 A constant longevity improvement rate of 1% a year was assumed for annuity products. This was done in accordance with Cooper-Williams, Albertyn & Lewis (unpublished).

3.5.3 A constant force of mortality was assumed throughout the year.

3.5.4 Expenses, expense inflation and interest rate assumptions were taken from the Financial Services Board (FSB) report on long term insurance.²⁴ The average value of all the long-term insurers were used. These assumptions are as in Table 1.

TABLE 1 Assumptions provided by the FSB

Risk	Value
Renewal expenses	R4,50 p.m.
Expense inflation	5,3%
Interest rate (N.A.C.A.)	8,2%

3.5.5 Interest rates and expense inflation rates have been modelled as flat rates rather than on a curve for the sake of simplicity.

3.5.6 Survival benefits are assumed to be paid at the end of the month of occurrence based on survival to the start of the month.

3.5.7 All expenses were assumed to be paid at the end of the month and are incurred based on survival to the start of the month.

3.5.8 For credit life as well as life policies, the assumed lapse rates are detailed in Appendix C.

3.5.9 Death benefits are assumed paid at the end of the month.

23 Actuarial Society of South Africa, South Africa 85–90 Assured Lives Mortality Table, 1996, http://www.actuarialsociety.org.za/Portals/2/Documents/Graduation1985_90AssuredLifeMortalityExperience.pdf, 05/02/2017

24 Financial Services Board (FSB), 17th Annual report of the Registrar of Long-term Insurance, 2014, <https://www.fsb.co.za/Departments/insurance/Documents/17th%20Annual%20Report%20of%20the%20Long%20Term%20Insurance%202014.pdf>, 9/4/2017

3.6 Model BEL

The best estimate cashflows were calculated using the expectations of all decrements. The BEL was calculated for every anniversary.

3.7 Calculation of the Risk Margin

The explicit risk margin was calculated using the following formulae:

$$RM = CoC * \sum_{t=0}^{\infty} \frac{SCR(t)}{(1 + r_{t+1})^{t+1}}; \quad (2)$$

where:

RM is the risk margin;

CoC is the cost-of-capital rate and is set at 6%;²⁵

t is the time in years from the current time 0;

$SCR(t)$ is the aggregated SCR value for all modules at time t ; and

$r(t)$ is the risk-free rate of return at time t .

3.8 Calculation of the SCR using no Simplifications

3.8.1 Capital requirements for each risk were calculated annually.

3.8.2 The capital requirement for each risk was calculated in a full, comprehensive calculation with no simplifications by recalculating the BEL under the specified stress scenario and using the revised BEL to calculate the change in basic own funds. The risk margin was not recalculated in the stress scenario.

3.8.3 The capital requirements for each risk were then aggregated using correlation matrices (prescribed in FSI 4) and formulae to calculate the basic solvency capital requirement (BSCR). The aggregation was recalculated annually.

3.8.4 The operational risk capital requirement was recalculated as a fixed percentage (30%) of BSCR.

3.8.5 No allowance has been made for diversification of risks with an existing portfolio of business. The SCR calculation has been performed on a 'standalone' basis rather than a 'marginal' basis.

3.9 Calculation of the SCR with Simplifications

3.9.1 For Level 1 of the hierarchy of simplifications, individual sub-module risks were calculated on a driver approach as seen in equation (3) below. A driver was chosen for each sub-module based on the following considerations: smaller errors are preferable;

²⁵ Prudential Standards FSI 2.2, supra at 14.14

a low variance of relative errors is preferable; smaller absolute errors for large values of the risk margin are better. The reason the drivers with the lowest variances were considered superior was because of the following: these drivers are easier to correct as there is some tight stability in its misestimation so one can apply a relatively known factor to these stable misestimations to bring it to the explicit risk margin value. The capital requirement for individual risks were then aggregated using the method and correlation factors provided for by FSI 4.

$$Capital_{Risk}(t) = \frac{Capital_{Risk}(0)}{Driver_{Risk}(0)} (Driver_{Risk}(t)); \quad (3)$$

where:

$Capital_{Risk}(t)$ is the capital requirement for the *Risk* module; and
 $Driver_{Risk}(t)$ is the driver associated with the *Risk* module

3.9.2 The driver with the lowest relative error was determined by considering the sum of absolute values of the relative error and the sum of squares of the relative errors. For this calculation a subset of data points were chosen that are not necessarily representative of a realistic portfolio. We recognise this as a weakness of the method. A subset of data points were used to reduce the time taken to calculate the results.

3.9.3 The driver used under Level 2 of the simplifications was worked out in the same way as Level 1, except now the variance and the smallest value of errors were considered by analysing each driver's accuracy in estimating the whole risk margin (under the explicit calculation) rather than each sub-modular risk.

3.9.4 For Level 2, the SCRs were calculated on a proportional approach to the driver using the formula:

$$SCR(t) = \frac{SCR(0)}{DRV(0)} (DRV(t)); \quad (4)$$

where:

$SCR(t)$ is the aggregated SCR at time t ; and
 $DRV(t)$ is the driver value at time t .

3.9.5 The authors were unaware of any resource available to establish what drivers should be used. Four drivers were considered:

- As per the FSI 2.2, the BEL was used for all policies.²⁶
- Expense reserve (the present value of all future expenses),

²⁶ Guidance Note FSI 2.2, *supra* at chapter 3 section A part 1

- BEL less the expenses reserve (i.e. benefit reserve).
- The BEL excluding any payments made in the guarantee period.

3.9.6 The Level 3 simplification exists but we have excluded it from the scope of this paper.

3.10 Sensitivity Testing

The purpose of sensitivity testing is to assess the importance of certain assumptions on the results calculated. In addition, the accuracy of the estimation changes under certain assumptions is established with sensitivity testing. Therefore, once the results were calculated, sensitivity testing was carried out on a subset of policies. The sensitivity testing was done based on the assumptions used in the calculation of the BEL. The variables that are deterministically changed include: mortality factors, longevity improvements, expenses and the interest rate. The variables tested are tabulated in each section of the results.

3.11 Analysis and Statistical Tests

3.11.1 In §2.5, the use of a score function to assess the fit of a model was recommended (Christoffersen & Jacobs, op. cit.). As companies are only ultimately concerned with the end risk margin value that they must hold, it was decided to use a score function based on the differences of the calculated risk margin from its true value, that is, the value produced under the explicit calculation.

3.11.2 Thus, it was decided to use the percentage difference between the actual risk margin calculated with no simplification and the risk margin value calculated using a simplification. This follows what was described by Christoffersen and Jacobs (op. cit.). However, bounds for the acceptance of a given simplification are not imposed as the required bound will differ under different applications of this method. For example, the level of acceptance depends on the company, the size of the portfolio amongst various other factors.

4. RESULTS AND DISCUSSIONS: ANNUITIES

The results of the modelling are discussed in this section. The results are dealt with for annuities Level 1 simplifications (in section 4.1) followed by annuity Level 2 simplifications (in section 4.2). Section 4.3 describes the causes of the bias in the estimation of the future SCR and a potential alternative solution is presented for annuities in section 4.4. Section 4 concludes with a sensitivity test of the results derived in section 4.1, 4.2 and 4.4.

4.1 Annuity Level 1 Simplifications

4.1.1 The optimal Level 1 simplification requires the determination of optimal capital drivers for each of the two relevant capital requirements: longevity risk and

expense risk. The drivers considered are given in Table 2. Figures 1 and 2 show box plots of errors in the calculation of the simplification compared to the comprehensive calculation risk margin for both risk capital components. A driver that has a tighter variance, that is closer to 0% error and that acts more or less consistently as the size of the risk margin increases, was the driver that was chosen.

TABLE 2 A key for drivers used

Driver name	Driver description
BEL	BEL including the following cashflows: future annuity and maintenance expenses
BELNoExp	BEL less expense cashflows
exGuarPeriod	BEL less annuity payments made in the guarantee period
Expense Reserve	Expense reserve

4.1.2 From Figure 1, it can be seen that the expense reserve has the lowest error and variance for the longevity risk margin. However, it can be seen from Figure 2, that the expense reserve is a good fit for the explicit calculation of the risk margin for low values, but as the explicit risk margin increases it can be seen that the expense reserve diverges from it. The expense reserve is thus not an appropriate driver given the

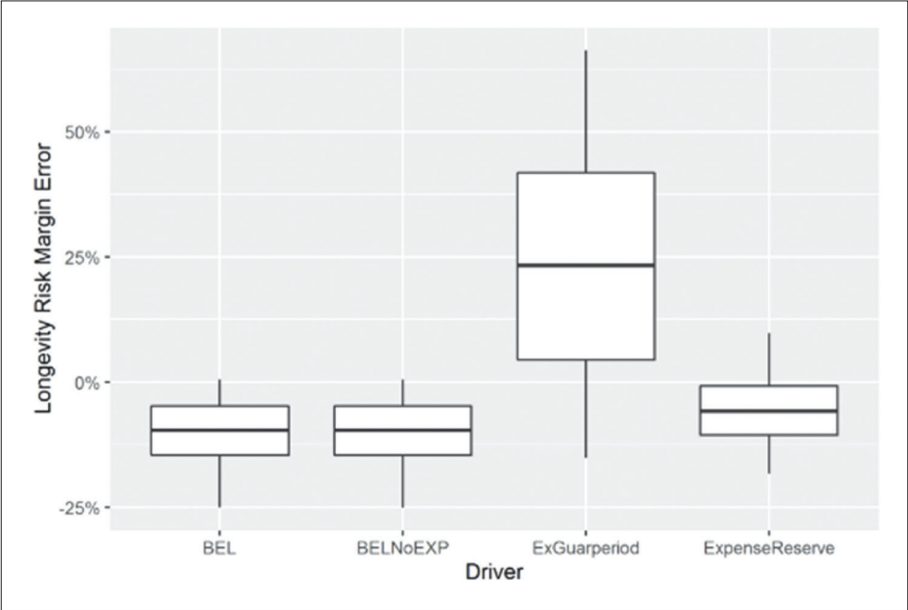


FIGURE 1 Box plots for longevity risk margin errors for the annuity products considered, under each driver

criteria in §3.9.1. BEL was chosen as the optimal longevity capital driver due to its low variance, low error and consistency over time to the explicit risk margin calculation.

4.1.3 It can be seen from Figure 3, that the expense reserve is the best driver for the expense risk margin. This is because the expense reserve driver displays: lower variance, lower errors and it is consistent over time to the explicit risk margin calculation. Therefore, this driver was chosen as the optimal expense capital driver.

4.1.4 The two drivers BEL and expense reserve have been chosen as the base for the Level 1 comparisons.

4.1.5 Figure 4 shows a comparison of the Level 1 simplification risk margin on the y -axis against the explicit calculation risk margin (on the x -axis). The figure shows that in the majority of cases the simplification understates the true value.

4.1.6 An analysis of the errors shows that the error spans -25.2% to $+4.8\%$. It can be seen that 96.67% of the errors fall below 0. The remaining 3.32% are above zero and

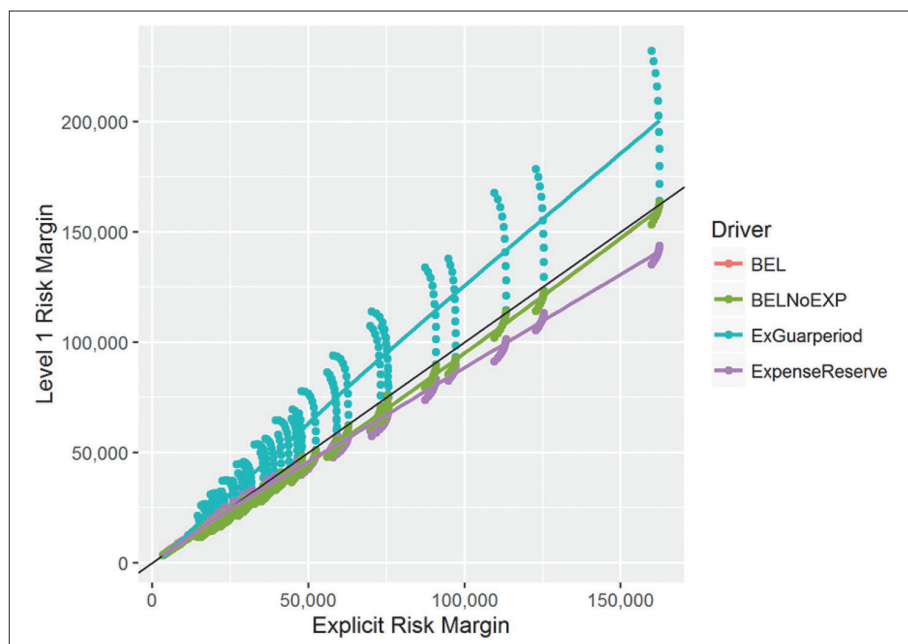


FIGURE 2 Scatter plot (trended) of the longevity risk margin, using different drivers under the Level 1 simplification, plotted against the explicit calculation of the risk margin. BEL is plotted but the results are so similar to BELNoExp that the BEL points are hidden on the graph.

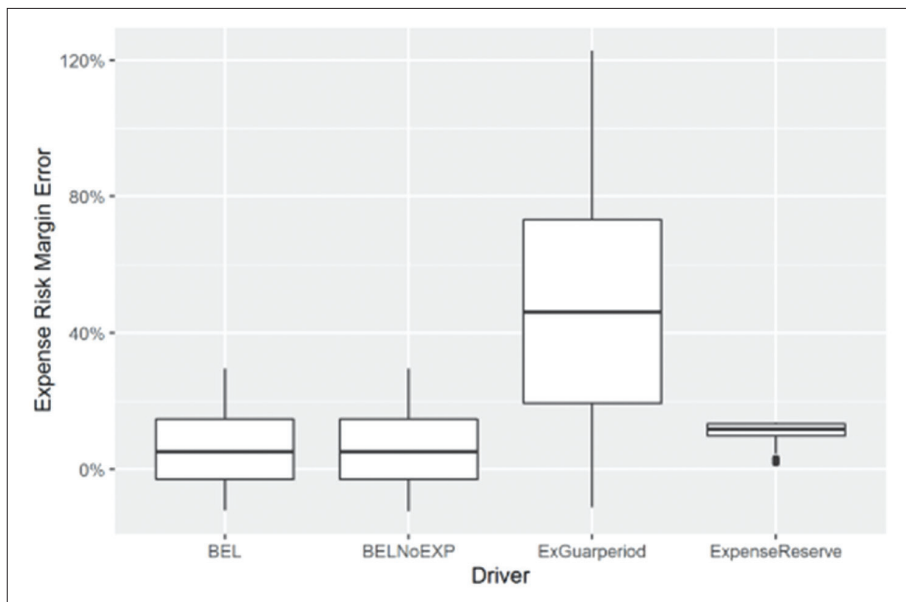


FIGURE 3 Box plots for expense risk margin errors for the annuity products considered, under each driver

below 4.8%. From this we can conclude that there is a systemic bias in the Level 1 simplification that leads to the understatement of the risk margin.

4.1.7 The mean, standard deviation of the errors (or other descriptive statistics) are not presented because the data points being used are artificially created and give equal weights to each combination of model points. This is unrealistic as an annuity writer is unlikely to have the same number of 60-year-olds with a 0-year outstanding guarantee term, as they have 95-year-olds with a 10-year outstanding guarantee term.

4.1.8 The bias is sensitive to a number of different factors and these are shown in the next six figures.

4.1.9 In Figure 5 the relationship between the relative estimation error of the Level 1 simplification and the outstanding guarantee term of the annuity is considered by plotting the estimation error (y -axis) against the outstanding guarantee term (x -axis) for three distinct annual benefit escalation rates: 0%, 3% and 6% for a policyholder aged 65.

4.1.10 For policyholders aged 65, the relationship between outstanding guaranteed term and estimation error is a clear one: as the guarantee period increases, the

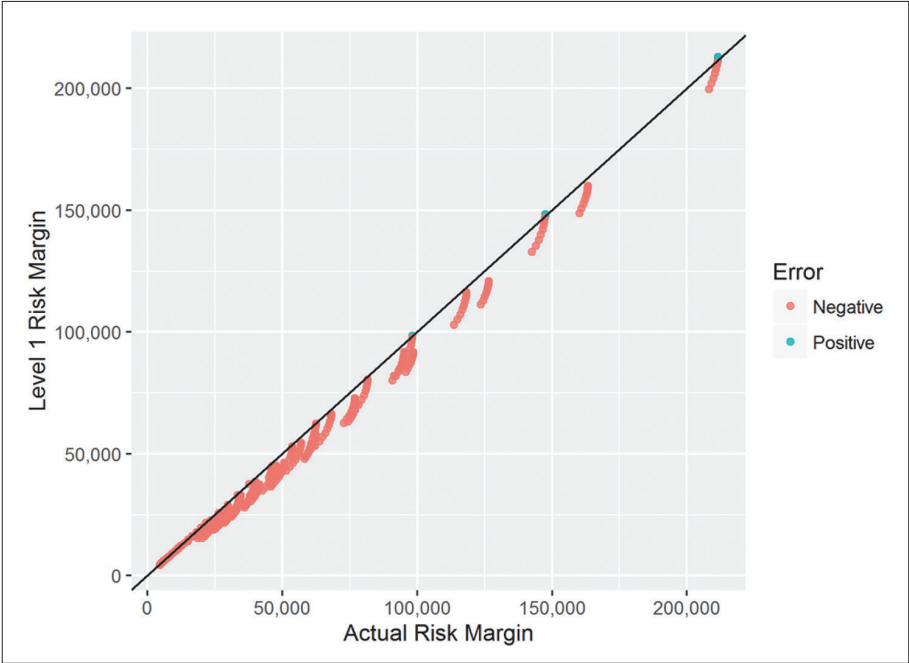


FIGURE 4 Scatter plot of the Level 1 simplification against the explicit calculation of the risk margin

Level 1 relative error increases in absolute terms. This behaviour is consistent across all escalation rates with higher escalation rates demonstrating less bias than lower escalation rates. Similarly, as the escalation rate decreases, the Level 1 error increases in absolute terms.

4.1.11 Lastly, in Figure 6 the relationship between the relative estimation error of the Level 1 simplification and the age of the policyholder is considered by plotting the relative error (y-axis) against the age on the balance sheet date (x-axis) for an escalation rate of 0% and with three distinct outstanding guaranteed periods.

4.1.12 The relationship between age and error is less clear and there is a clear interaction effect between age and outstanding guarantee term. This interaction should be theoretical because annuity writers are unlikely to have many policies with old ages and high guaranteed outstanding terms. For policies outside of guarantee there is a clear relationship between age and relative error: as the policyholder gets older the relative error becomes smaller. The results were very similar for an escalation rate of 5%.

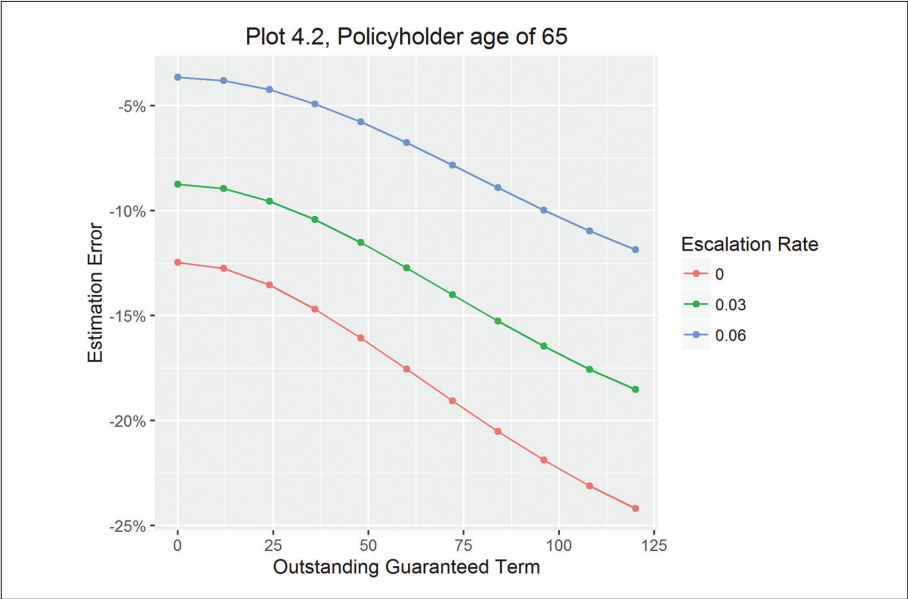


FIGURE 5 Plot of relative errors for the Level 1 simplification under different escalation rates, for a policyholder aged 65 on the balance sheet date

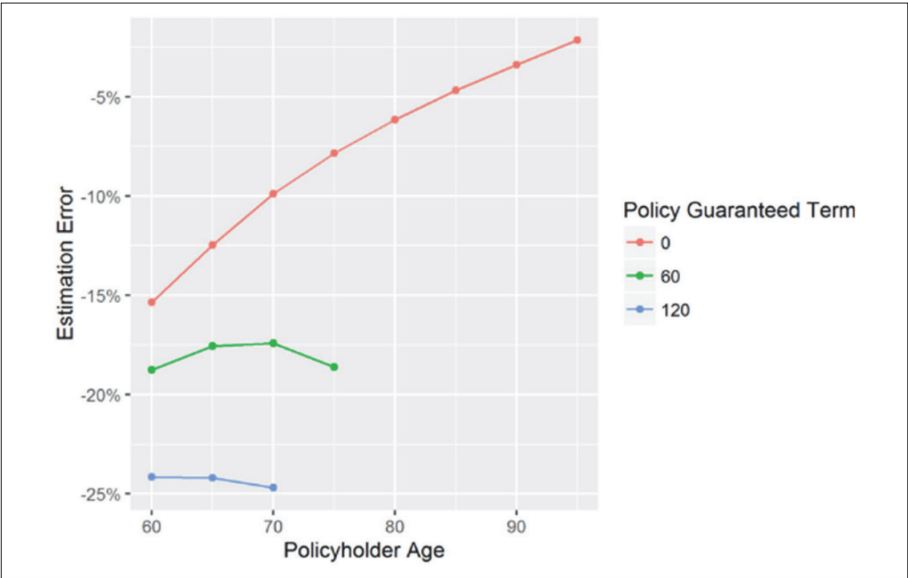


FIGURE 6 Plot of relative errors for the Level 1 risk margin estimate, against the age of the policyholder, on the date of the calculation for escalation rate 0%

4.1.13 Figures 5 and 6 suggest that the Level 1 simplification will be most error-prone for an annuity portfolio that has high outstanding guarantee periods and low escalation rates. This would have particular impact when estimating the capital resources required to support writing future new annuity business in business planning exercises or when pricing annuity business.

4.2 Annuity Level 2 Simplifications

4.2.1 The analysis of the Level 2 simplification requires the determination of optimal drivers for estimating the overall SCR. The estimation error for four drivers considered are shown in Figure 7. As stated in §3.9.1, a lower variance, a lower error and the driver's consistency over a range of values of the explicit risk margin value indicates a good driver.

4.2.2 From Figure 7, it appears that the expense driver is the best driver due to it having the lowest error and lowest variance. However, as for Level 1, the expense driver worsens as the explicit risk margin increases as shown in Figure 8. Therefore the BEL is chosen as the optimal driver for the Level 2 simplifications.

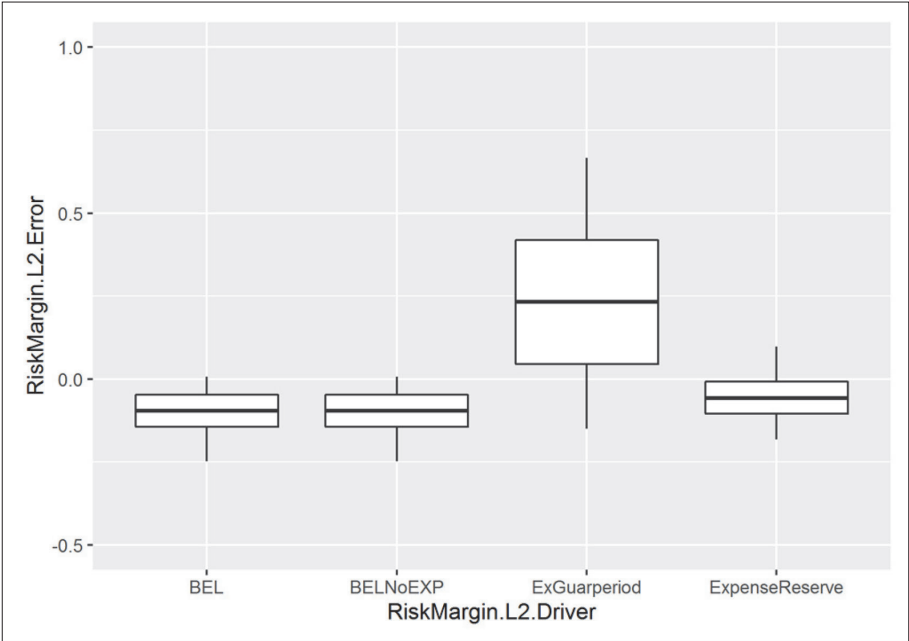


FIGURE 7 Box plots for risk margin errors using each driver to estimate the full risk margin (i.e. Level 2 approach)

4.2.3 It also can be seen from Figure 8, that all the drivers (except the BEL excluding guarantee period) underestimate the explicit value of the risk margin. In the case of the BEL excluding guarantee period, the risk margin is severely overestimated. The same results were established for Level 1.

4.2.4 Further analysis of the estimation error show that the relationship to key variables (age, outstanding guarantee term, escalation rate) is similar to those identified in the analysis of the Level 1 simplification. Hence, the annuity portfolio most prone to errors would be one with high outstanding guarantee periods and low escalation rates as it was for Level 1.

4.3 Understanding the Cause of the Bias for the BEL Driver

4.3.1 The intuitive driver for projecting annuity risk capital is the BEL but the analysis done above shows that the BEL is a biased estimator that results in the underestimation of the risk margin in almost all cases. The reason for this is considered in this section.

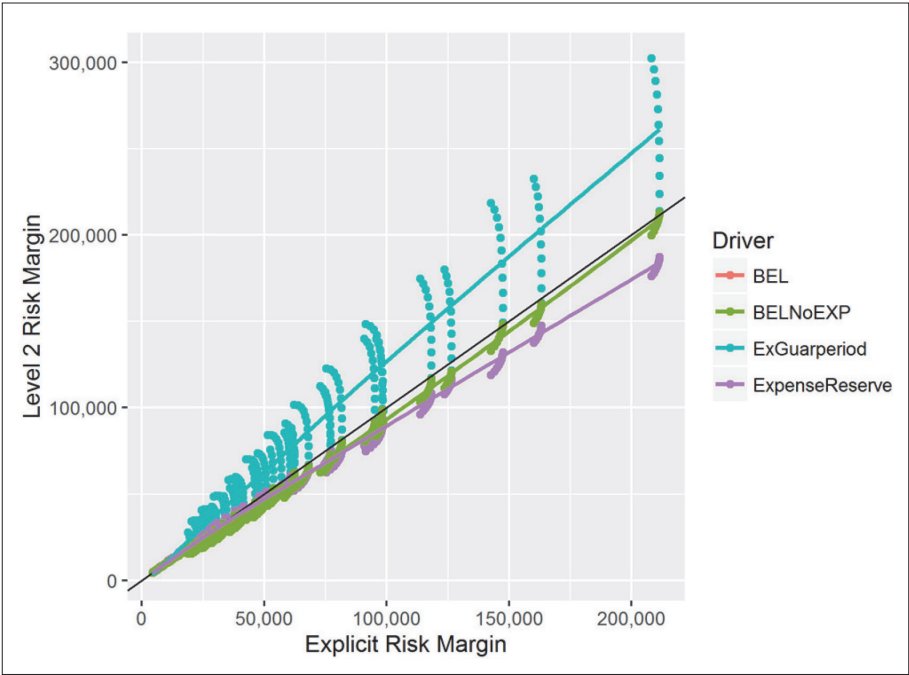


FIGURE 8 Scatter plot (trended) of the longevity risk margin, using different drivers under the Level 1 simplification, plotted against the explicit calculation of the risk margin. BEL is plotted but hidden by the similar values of BELNoExp.

4.3.2 The relative run-off pattern of the expected future BEL is plotted in Figure 9 along with the run-off pattern of the explicitly calculated future SCR. Both patterns are plotted on a relative scale where 1.0 represents the initial value. The policy considered is for a 65-year-old policyholder with no outstanding guaranteed term and a 0% benefit escalation rate.

4.3.3 Figure 9 shows that the SCR runs off far slower than the BEL which results in the understatement of the Level 2 risk margin simplification.

4.3.4 A more intuitive explanation of the bias in the Level 1 simplification presents itself when the actual relationship between the SCR and the BEL is considered. Figure 10 plots the ratio of SCR/BEL for the same policy used in Figure 9, along different guarantee periods and different escalations rates. A horizontal line is plotted representing the assumed relationship between SCR and BEL in the Level 2 simplification.

4.3.5 Figure 10 shows that SCR/BEL is increasing with respect to time while the assumed relationship is constant for the Level 2 simplification. From this we can see

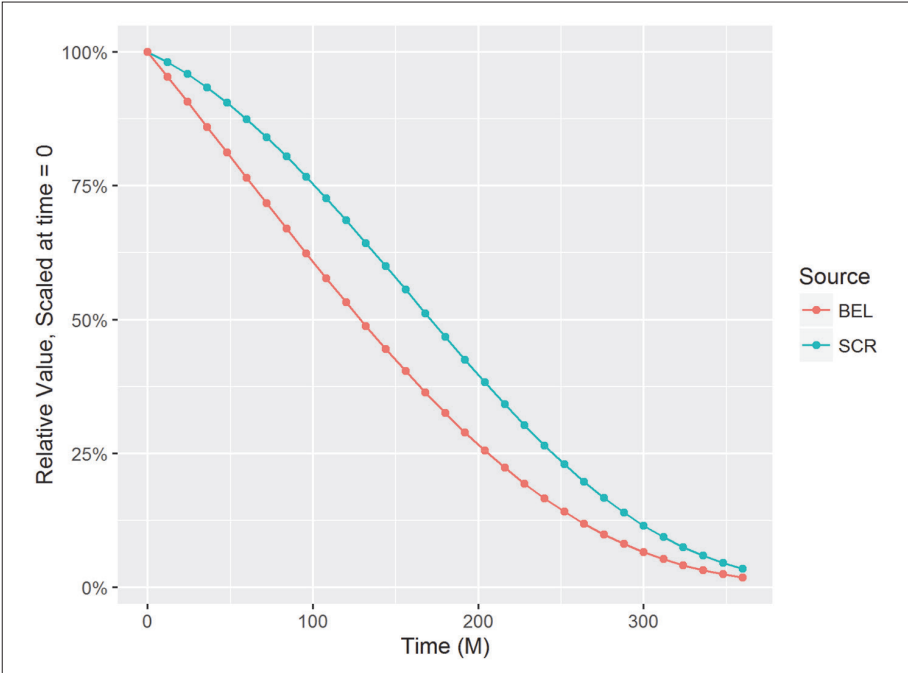


FIGURE 9 The BEL and the SCR run-off

that the Level 2 simplification will consistently underestimate the future SCR over time, consequently underestimating the risk margin.

4.3.6 It also can be seen that the ratio of SCR over BEL is approximately linear after the end of the annuity guarantee period and the gradient differs for different escalation rates.

4.4 A Potential Solution

4.4.1 The near linear relationship between SCR and BEL after the expiry of the guarantee period shown in Figure 10 lends itself to a potential solution to improve the estimation of the future SCR. This approach relies on the calculation of one additional SCR, $SCR(f)$ at some point in the future, f . $SCR(f)/BEL(f)$ and $SCR(0)/BEL(0)$ are used to fit a straight line and then predict future $SCR(t)/BEL(t)$ values at any time t using this fitted line. The value of $BEL(t)$ can be easily projected and so the value of $SCR(t)$ can be calculated.

4.4.2 For annuity products, a value of 15 years was used for f because it was longer than the longest guarantee term.

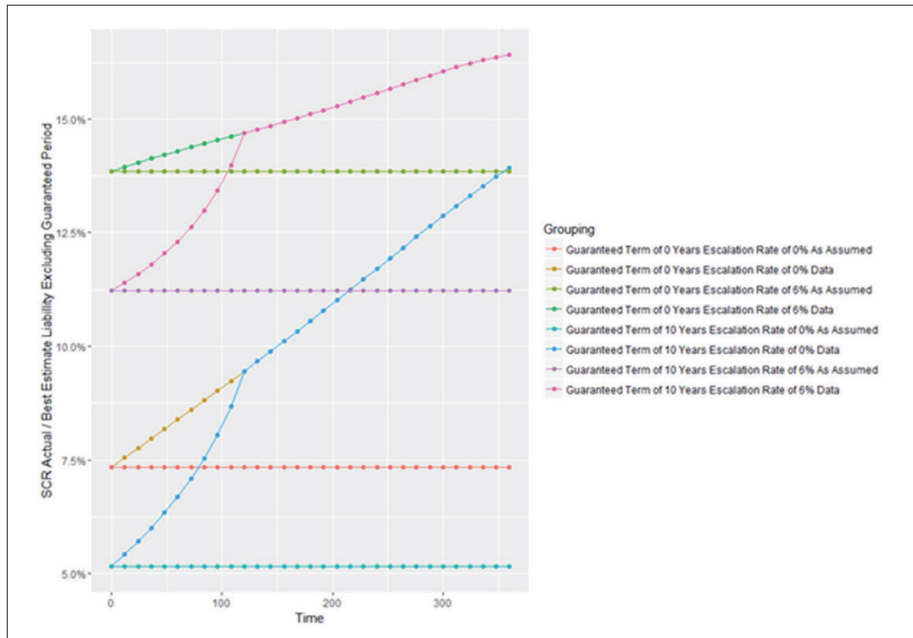


FIGURE 10 The ratio of SCR/BEL for different combinations of guarantee periods (0 and 10 years) and at different benefit escalation rates (0% and 6% p.a.), specifically for a policyholder aged 65

4.4.3 Figure 11 considers the risk margin error present under Level 1, 2 and the new proposed solution. From this it can be seen that the proposed solution has lower errors and a lower variance. Figure 12 then considers the behaviour of the proposed solution as the explicit risk margin increases. From this, it can be seen that the proposed solution maintains a tight track with the explicit risk margin. The proposed solution is far from perfect. This can be seen when one considers policies with non-zero outstanding guarantee terms. Even with its lack of perfection, it does give a better estimate of the explicit risk margin than when the BEL as a driver is used.

4.5 Sensitivity Testing

4.5.1 The initial finding described in sections 4.1 and 4.2 is that there is a systemic bias in the Level 1 and Level 2 simplifications of the risk margin for annuity products.

4.5.2 As discussed in ¶3.10, a key part of the analysis is to test the sensitivity to changes in key assumptions, namely, longevity, mortality, interest rates and expenses in order to see how key results respond to these changes. Table 3 contains the abbreviations used for the sensitivity tests carried out on the calculations. In addition Table 3 contains the scenarios considered under each of the sensitivity changes.

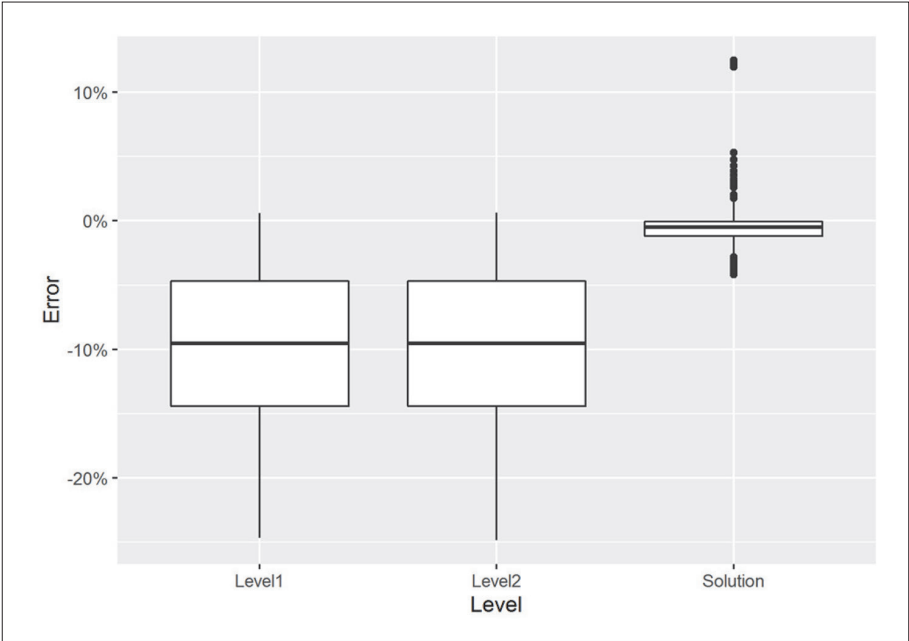


FIGURE 11 Box plots for risk margin errors using the optimal choices in Level 1 and 2 compared to the proposed solution

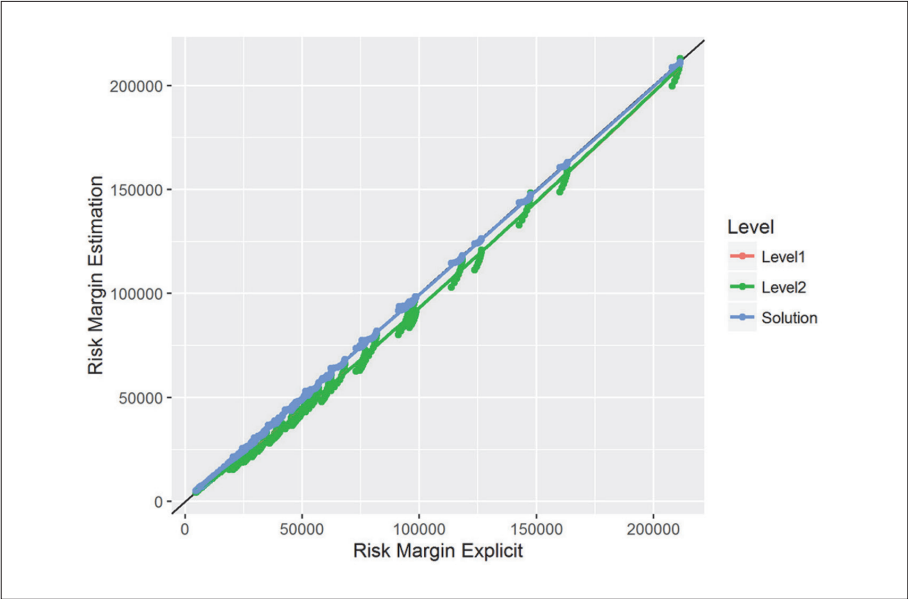


FIGURE 12 Scatter plot (trended) of the risk margin, under Level 1 and 2’s optimal choices compared to the proposed solution

4.5.3 Figures 13, 14 and 15 show the risk margin error present in Level 1, 2 and the proposed solution under the various sensitivity tests given in Table 3.

TABLE 3 Key for the sensitivities carried out and the specific scenarios considered under each assumption change

Key	Meaning
Base	Original calculation of the risk margin, under the specific level or proposed solution
IntDown	Interest rates down 200 basis points
IntUp	Interest rates up 200 basis points
LongImpUp	Longevity improves by 100%
MortDown	Mortality deteriorates by 10%
MortUp	Mortality increases by 10%
RenExpDown	Renewal decreases increase by 80%
RenExpUp	Renewal expenses increase by 200%

4.5.4 It can be seen from Figure 13 that when the Level 1 calculations of the risk margin are made on different assumptions, the estimation error for the risk margin overall actually improves. However, the variance seem to increase slightly when

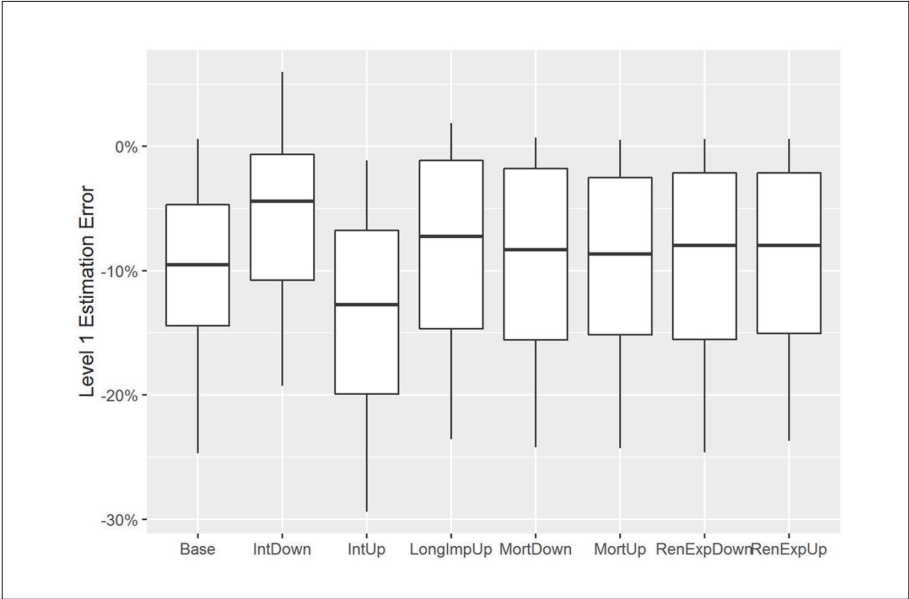


FIGURE 13 Box plots for risk margin errors under Level 1 for various sensitivity tests

exposed to different sensitivity tests. The exception to this pattern is when interest rate is flexed and this leads to a large increase in the estimation error. The outliers that are seen in Figure 15, are caused by a cohort of individuals aged 90 and older 15 years after the balance sheet date. However, the risk margins for ages 90 and older are insignificant, so these points being outliers are not a major consideration and therefore are not be focused on.

4.5.5 It can be seen from Figure 14 that Level 2’s sensitivity tests follow in much the same fashion as Level 1’s sensitivity tests.

4.5.6 It can be seen in Figure 15 that overall the sensitivity tests increase the estimation error for the proposed solution. However, the increase in the estimation error seems to be insignificant. In addition, the variances of the calculations seem to increase generally but insignificantly.

4.5.7 It can be seen from the Figures 13 to 15 that the estimation error increases under Level 1 and 2 of the simplifications whereas under the proposed solution, although the estimation errors also increase in the sensitivities, the resulting errors remain insignificant. Therefore, the proposed solution retains its accuracy in the estimation of the explicit calculation of the risk margin.

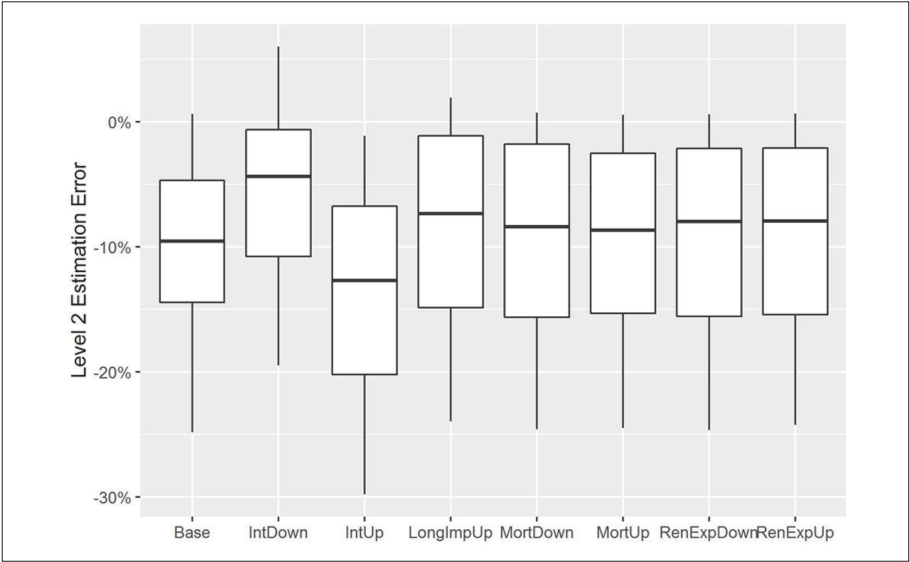


FIGURE 14 Box plots for risk margin errors under Level 2 for various sensitivity tests

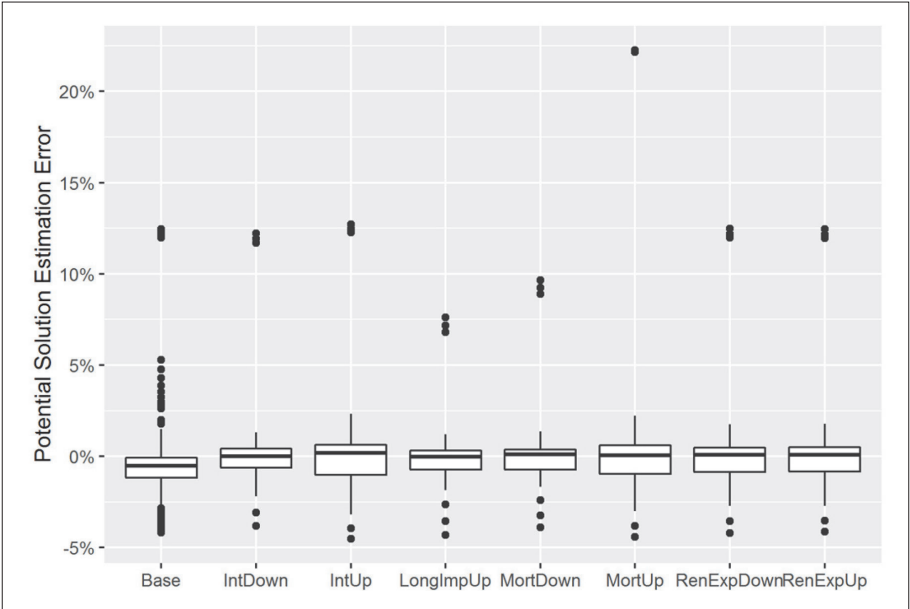


FIGURE 15 Box plots for risk margin errors under the proposed solution for various sensitivity tests

4.5.8 Figure 16 shows how Level 1 and 2 simplifications and the proposed solution behave under a variety of alternate assumption sets. The Level 1 and Level 2 errors are mostly identical and so Level 1 errors aren't visible on the plot. Overall, it can be seen that Level 1 and 2 are still systematically underestimating the risk margin (the sole exception to this is under the interest rate downward movement), whereas the proposed solution maintains its accuracy and remains tightly tracked to the explicit calculation under all alternate assumptions tested. This shows that the proposed solution is robust and produces high quality estimates under in most cases.

5. RESULTS AND DISCUSSION: CREDIT LIFE

The results of the modelling for credit life products are discussed in this section. The results are dealt with for credit life Level 1 in section 5.1 and Level 2 in section 5.2. In section 5.3 the reason for the BEL being a poor driver for predicting the run-off of the SCR is discussed. The Level 1 and Level 2 simplifications are compared in section 5.4. Section 5 concludes with sensitivity testing of the results derived in sections 5.1

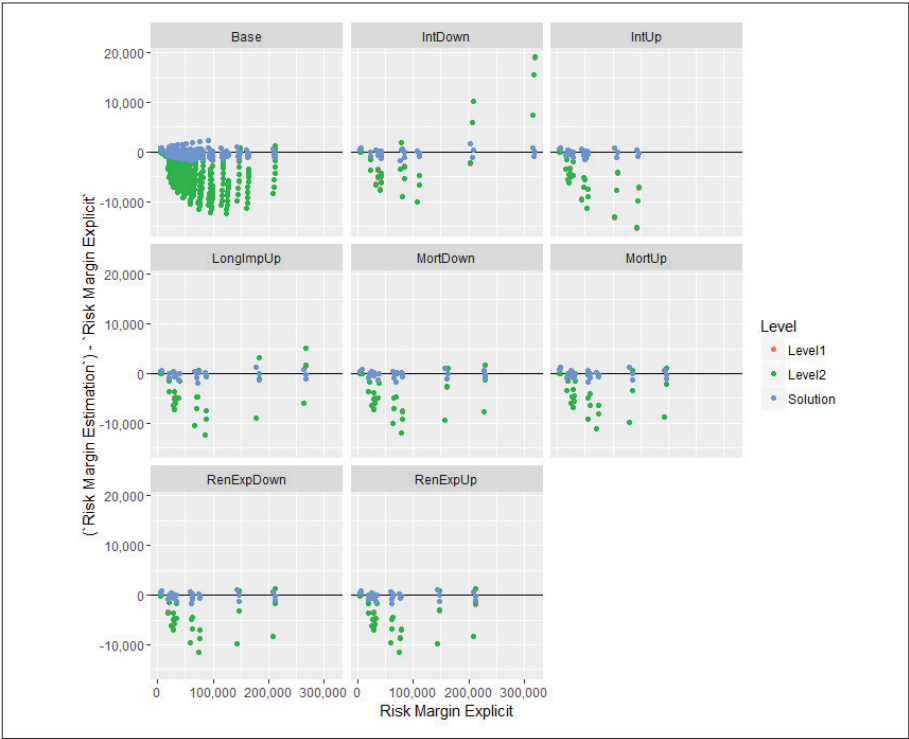


FIGURE 16 Scatter plot of the risk margin estimation error under Level 1 and 2's optimal choices and the proposed solution, when exposed to various sensitivity tests. Level 1 points are hidden by Level 2 due to their similarity.

and 5.2. The standard simplifications produce reasonable estimates and so we do not propose an alternative approach.

5.1 Credit Life Level 1 Simplifications

5.1.1 The optimal Level 1 simplification requires the determination of optimal capital drivers for each of the relevant capital requirements: mortality risk, expense risk and lapse risk. The drivers considered are given in Table 4. Figures 17 to 21 display the box plots of the errors between the comprehensive calculation and the simplification for each risk capital component. As discussed in §3.9.1, a driver that has a tighter variance, that is closer to 0% error and that acts more or less consistently as the size of the risk margin increases is considered superior.

5.1.2 Figures 17 and 18 show the same information, except that the BEL driver has been excluded from Figure 18. The BEL driver has material errors and distorts the scale of the graph. For this reason we can exclude it and focus only on the errors that are better fits. For this reason Figures 18 to 21 do not plot the estimation errors for the BEL driver.

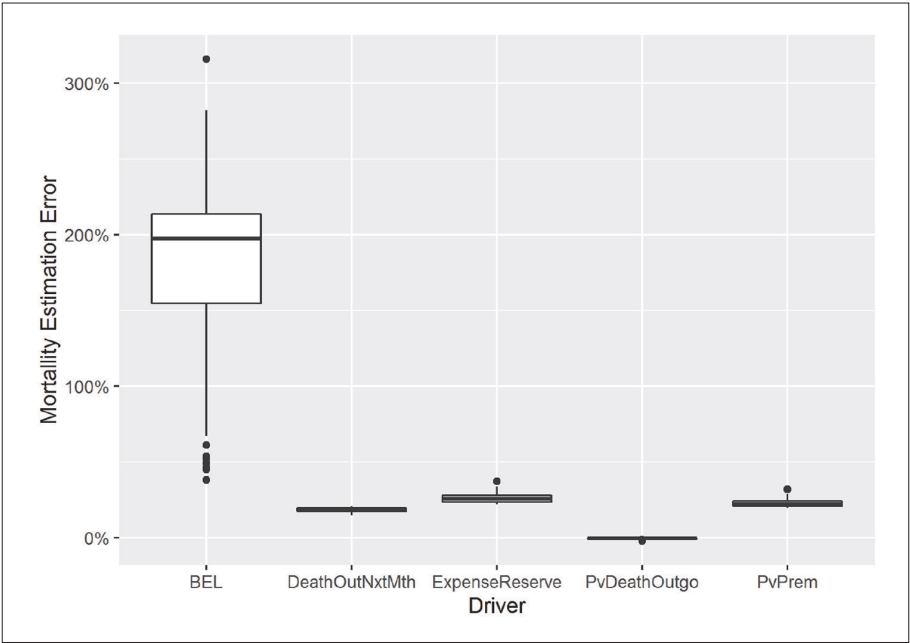


FIGURE 17 A box plot with the estimation error of the mortality risk capital component for each driver (x-axis) when comparing the simplified approach against the explicit calculation

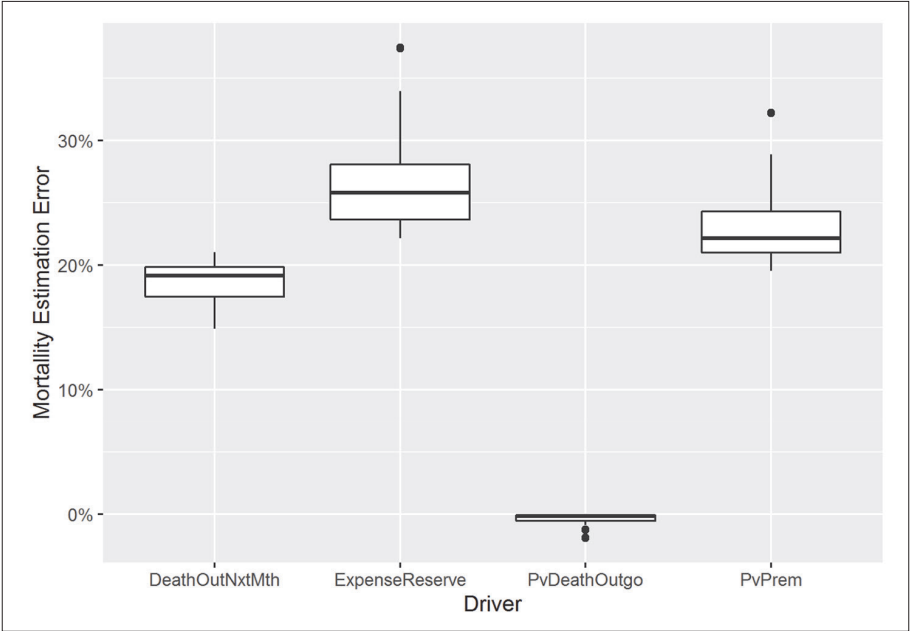


FIGURE 18 A box plot of the estimation errors for the mortality risk capital component for four drivers

TABLE 4 A key for drivers used

Driver name	Driver description
BEL	BEL including the following cashflows: future expected death benefits, maintenance expenses and premiums.
DeathOutNxtMonth	Death benefit payable in the following month
Expense Reserve	Expense reserve
PV Death Outgo	Expected present value of the future death benefits
PV Prem	Expected present value of future premiums

5.1.3 For the mortality risk capital component, PV Death Outgo is, by visual inspection, the preferred driver. For the expense risk capital component, PV Prem is the preferred driver, whereas for mortality catastrophe risk DeathOutNxMth is the preferred driver. The preferred driver for Lapses was PV Prem. While it does not appear to be the best driver from the box-plot in Figure 21, the scatter plot in Figure 22 shows that it is the superior driver as the risk margin starts to increase in size.

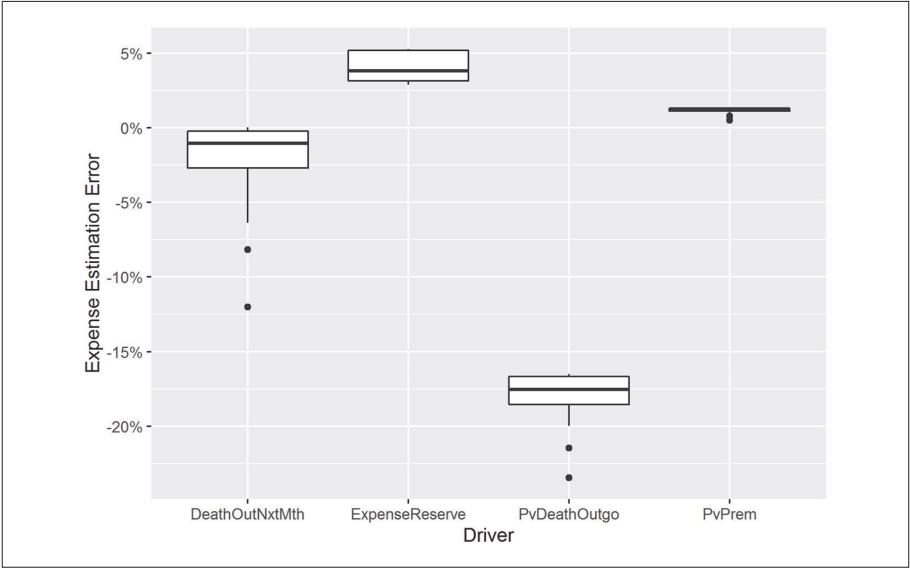


FIGURE 19 A box plot of the estimation errors for the expense risk capital component for four drivers

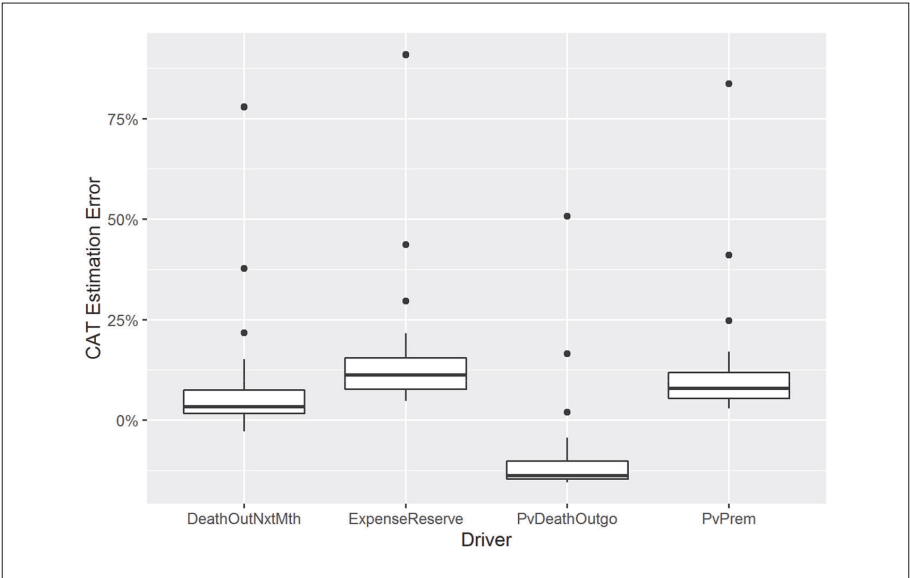


FIGURE 20 A box plot of the estimation errors for the mortality catastrophe risk capital component

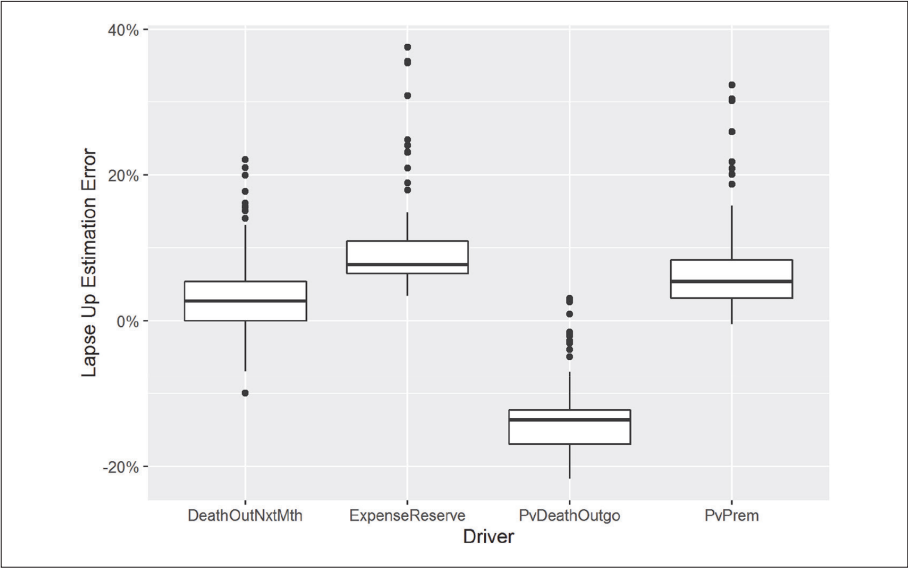


FIGURE 21 A box plot of the estimation errors for the Lapse Up risk capital component

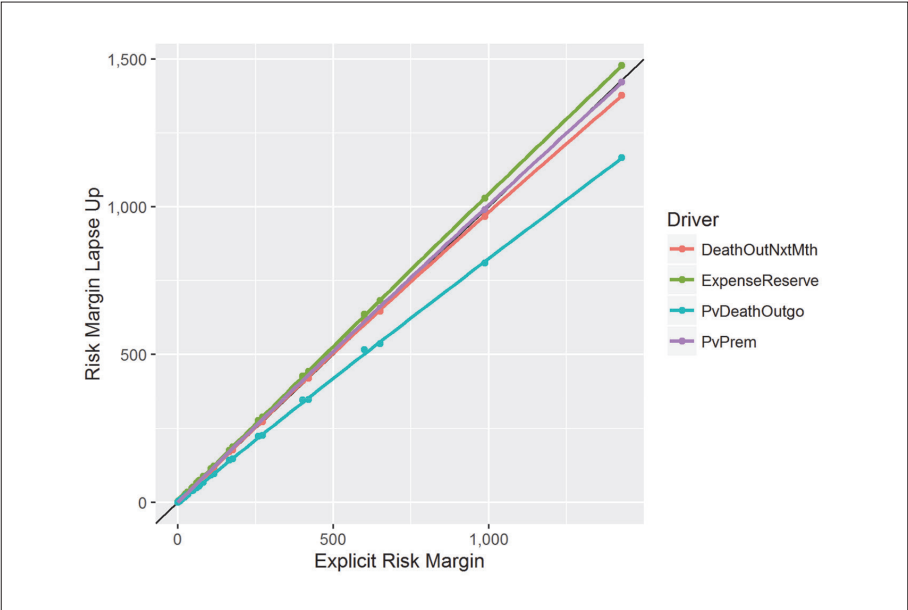


FIGURE 22 A scatter plot of the Lapse Up simplification using the four major drivers (y-axis) against the explicit calculation (x-axis)

5.1.4 The drivers chosen for each risk capital component are summarised in Table 5.

TABLE 5 Credit life drivers

Risk capital component	Driver name
Mortality	PV Death Outgo
Expense	PV Prem
Mortality catastrophe	DthOutNxtMth
Lapse	PV Prem

5.1.5 Figure 23 compares the estimates from these drivers against the explicit risk margin calculation. From this we can see that the Level 1 simplification is accurate most of the time with a few cases of underestimation.

5.1.6 Figure 24 shows the sensitivity of the Level 1 risk margin estimation error to the age of the policyholder on the balance sheet date. There is low variation in the errors across ages except for policies that are nearest to completion.

5.1.7 Figure 25 shows the sensitivity of the Level 1 risk margin simplification to the duration in force. A credit life policy with a loan term of five years and loan interest

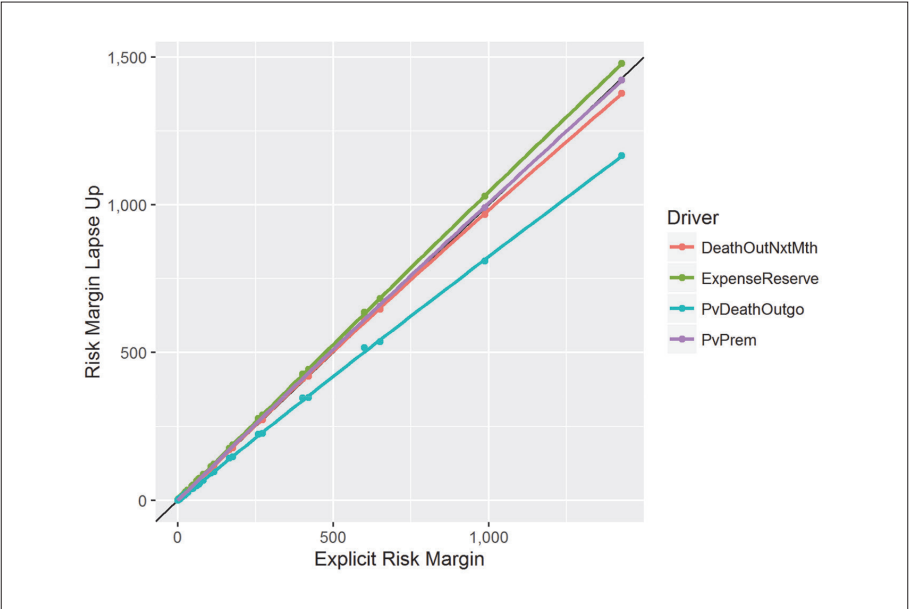


FIGURE 23 A scatter plot of the Level 1 risk margin estimate (y-axis) and the explicit risk margin calculation (x-axis)

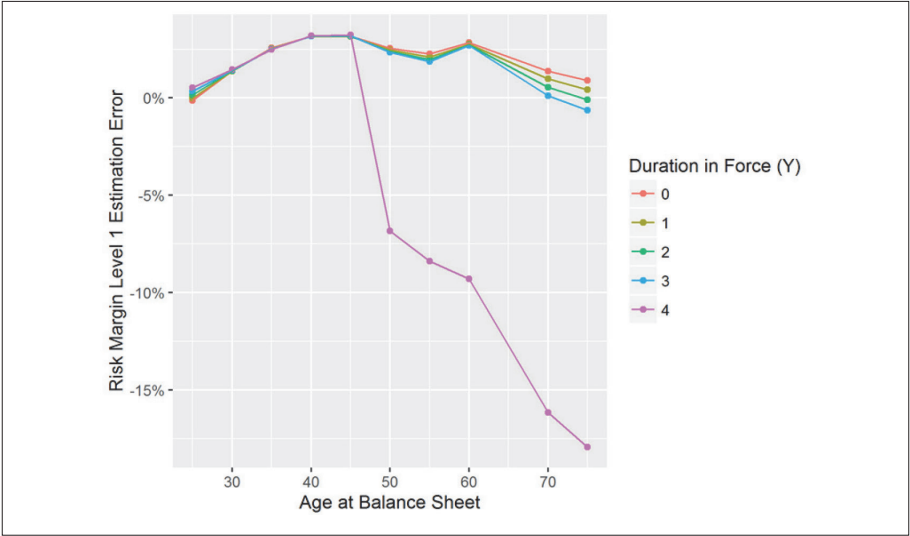


FIGURE 24 The relative error of the Level 1 simplification (*y*-axis) plotted against the age of the policyholder on the balance sheet date (*x*-axis) at sample duration in force

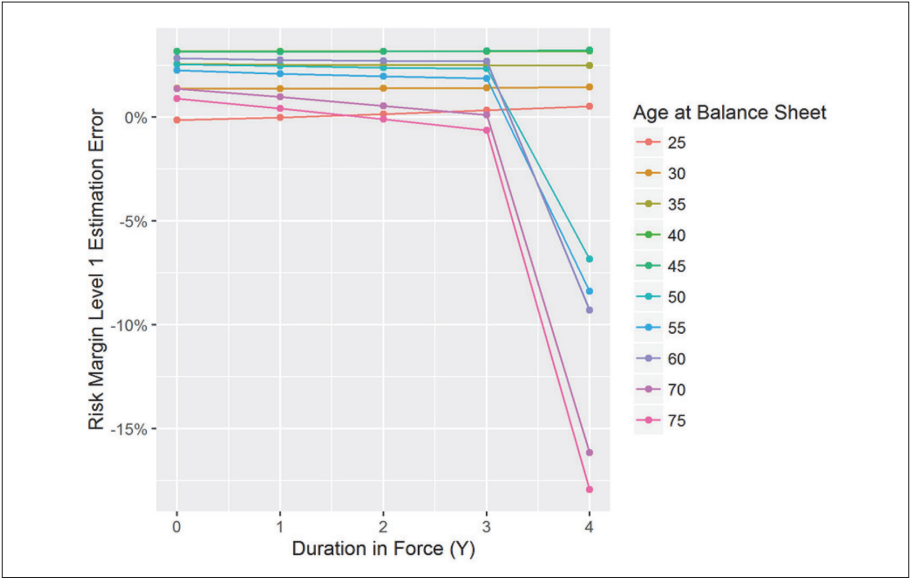


FIGURE 25 The relative error of the Level 1 simplification (*y*-axis) plotted against the duration in force of the policyholder (*x*-axis) for a variety of ages on the balance sheet date

rate of 10% was used. The estimation error remains reasonably small except for policies with a short outstanding term and where the policyholder is older.

5.1.8 Figure 26 shows the sensitivity of the Level 1 risk margin simplification to the term of the loan which shows that the overestimation tends to increase as the loan term increases except for very young policyholders; however the variation is quite small. A credit life policy with a duration in force of zero and interest rate of 10% was use.

5.1.9 The graphs indicates that there is a reasonable level of accuracy for the Level 1 estimator and there is little to suggest how the bias can be predicted.

5.2 Credit Life Level 2 Simplification

5.2.1 The optimal Level 2 simplification requires the determination of an optimal driver for projecting the overall capital requirements. The drivers considered are given in Table 5. Figure 27 displays a box plot of these errors between the comprehensive calculation of the risk margin and the Level 2 simplification. The same data is plotted in a scatter plot in Figure 28. As discussed in ¶3.9.1, a driver that has a tighter variance, that is closer to 0% error and that acts more or less consistently as the size of the risk margin increases is considered superior.

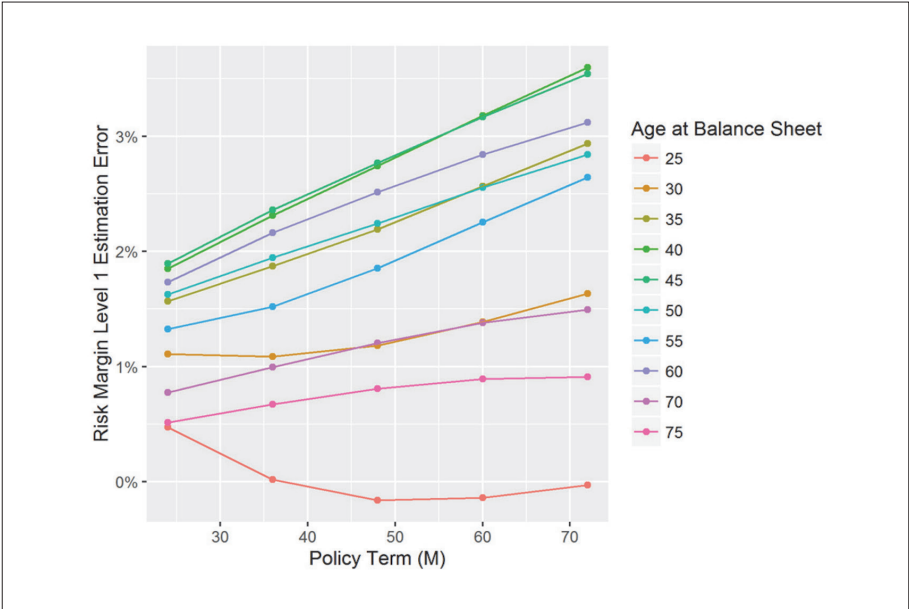


FIGURE 26 The relative error of the Level 1 simplification (y-axis) plotted against the policy term (in months) for a variety of ages

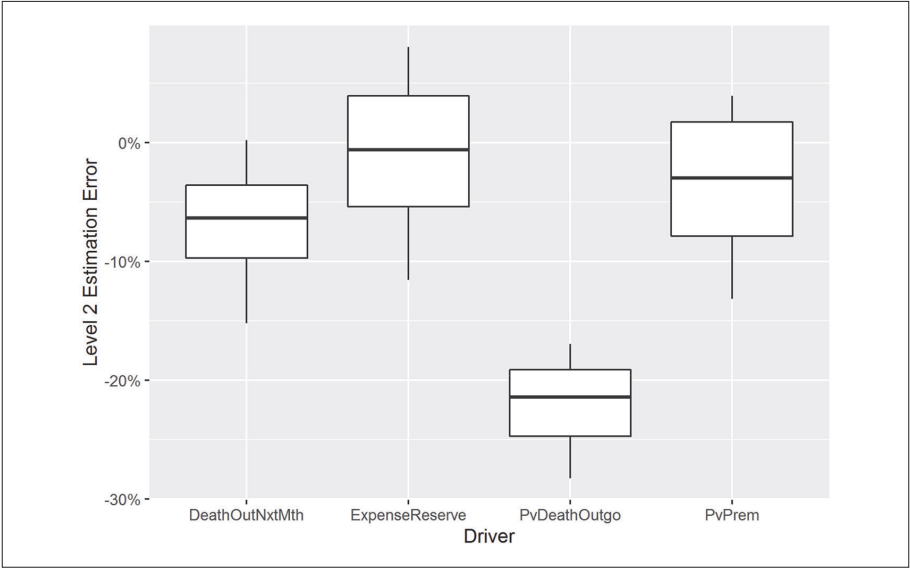


FIGURE 27 A box plot of the estimation error of the Level 2 risk margin simplification for four different drivers

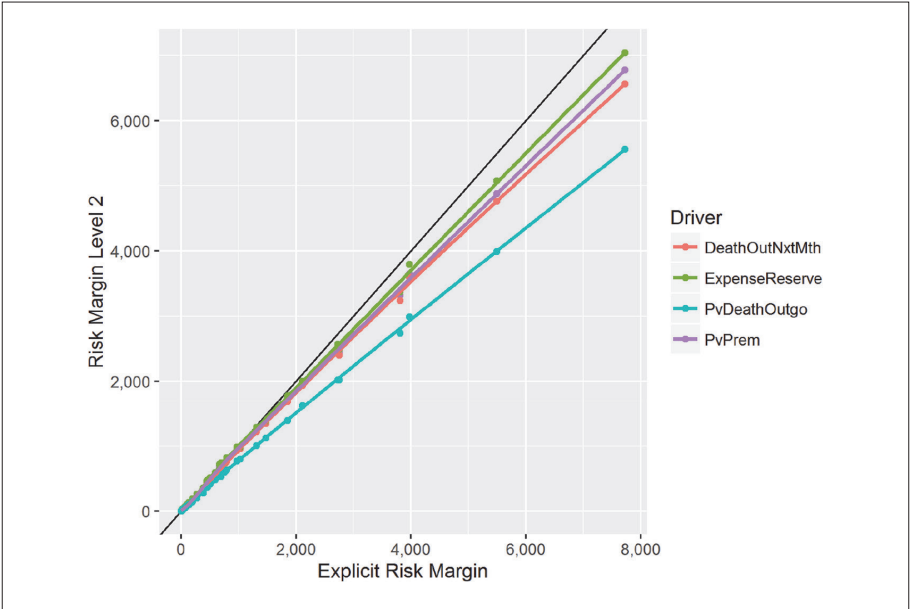


FIGURE 28 A scatter plot of the Level 2 risk margin simplification (y-axis) against the explicit calculation (x-axis) for four different possible drivers

5.2.2 The expense reserve is shown to be the preferred driver under the Level 2 simplification. This is a curious result as it was not the preferred driver for any of the individual risk capital components in the Level 1 calculation. However it can be seen that all drivers underestimate the risk margin with expense reserve underestimating the least, especially at larger risk margins. Figure 29 shows a scatter plot of the simplification against the explicit risk margin calculation. While there are cases where the simplification overestimates, the majority of cases have an underestimation.

5.3 BEL as a Driver for Credit Life

5.3.1 The BEL is considered a poor driver for estimating the risk margin for credit life products.

5.3.2 Figure 30 shows the projected BEL of a sample credit life product and the run-off of the SCR which shows that the BEL runs off far slower than the SCR.

5.4 Comparison of Credit Life Level 1 and Level 2 Simplifications

5.4.1 Figures 31 and 32 show two plots of the relative errors of the Level 1 and Level 2 risk margin simplifications. The Level 1 risk margin produces a better quality estimate for the risk margin.

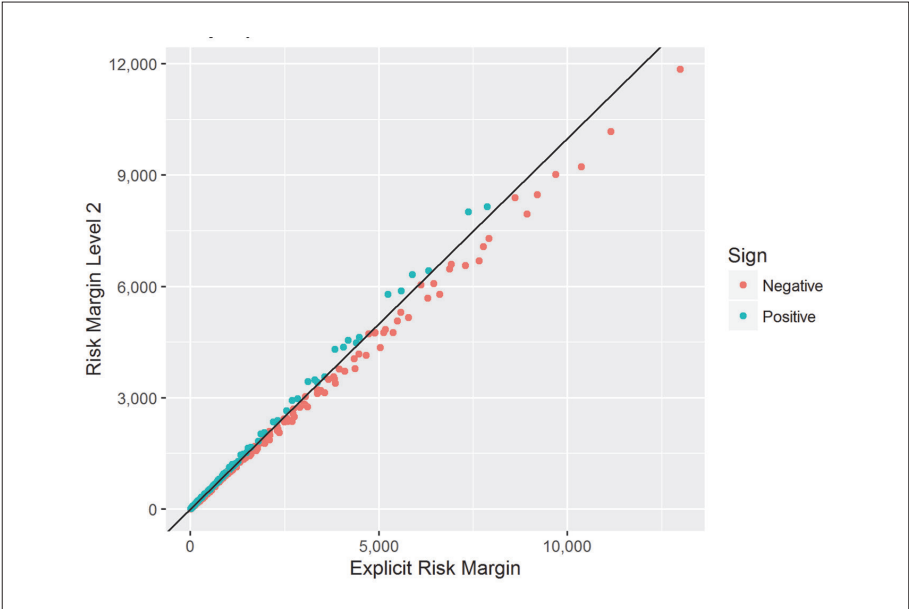


FIGURE 29 A scatter plot of the Level 2 simplification (y-axis) against the explicit risk margin calculation (x-axis) for a set of policies

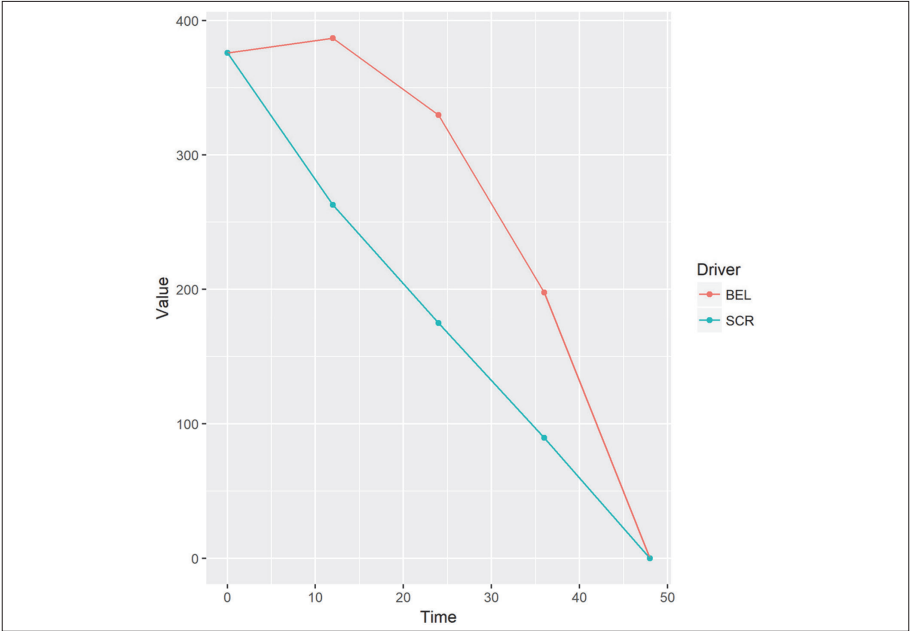


FIGURE 30 The run-off of the SCR and BEL

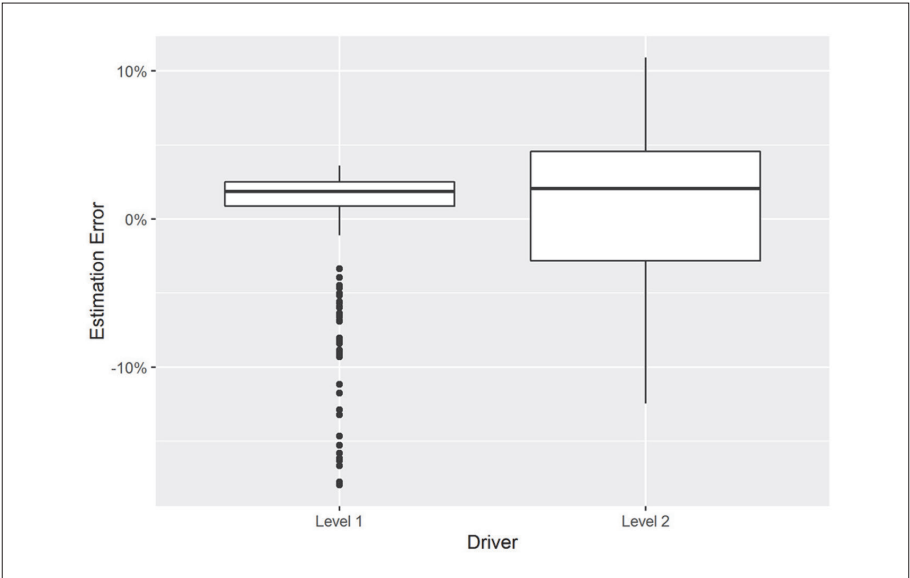


FIGURE 31 A box plot of the relative estimation errors using the two risk margin simplifications

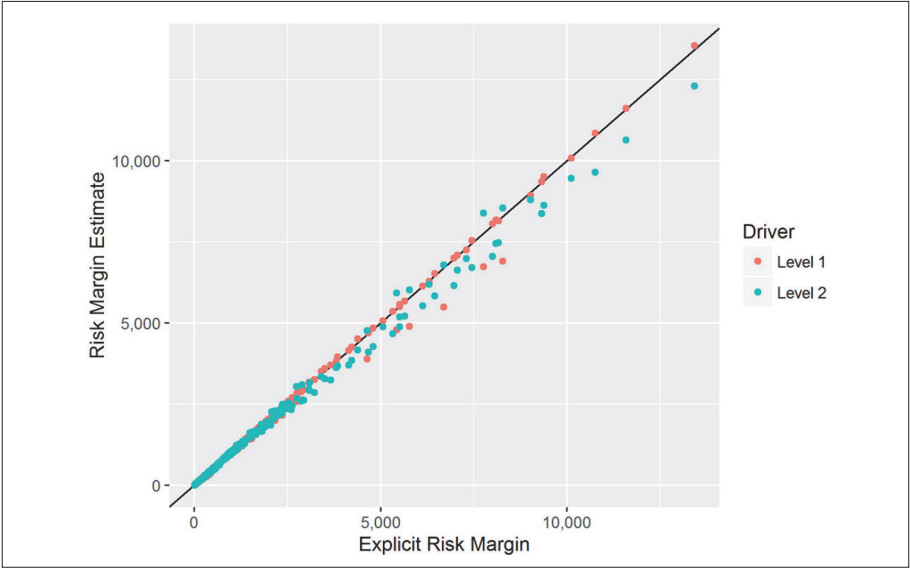


FIGURE 32 A scatter plot of the Level 1 and Level 2 simplifications (y -axis) against the explicit risk margin calculation (x -axis)

5.5 Sensitivity Testing

Figure 33 shows a comparison of the level of error of the Level 1 and Level 2 risk margin simplifications in absolute rand terms against the explicit risk margin calculation for a subset of policies on a variety of different assumptions shown in Table 6. The initial finding of section 5 is that the Level 1 simplification is better than the Level 2 simplification; and that Level 1 simplifications produce reasonable results – and this can be shown to hold across all alternative input assumptions.

6. RESULTS AND DISCUSSION: WHOLE LIFE ASSURANCE

The results of the modelling for a whole-of-life assurance will be discussed in this section. The results are dealt with for Level 1 in section 6.1 and Level 2 in section 6.2. In section 6.3 we look at the reason that BEL is a poor driver for predicting the run-off of the SCR. A potential solution is proposed in section 6.4 and compared against the Level 1 and Level 2 simplifications. Section 6.5 concludes with sensitivity testing of the results derived in sections 6.1, 6.2 and 6.4.

6.1 Whole-of-Life Assurance Level 1 Simplifications

6.1.1 The optimal Level 1 simplification requires the determination of optimal capital drivers for each of the relevant capital requirements: mortality risk, expense risk and lapse risk. The drivers considered are given in Table 7. Figures 34 to 36 display the box plots of the errors between the comprehensive calculation and the simplification

TABLE 6 Key for the sensitivities carried out and the specific scenarios considered under each assumption change

Key	Meaning
Base	Original calculation of the risk margin, under the specific level or proposed solution
IntDown	Interest rates down 200 basis points
IntUp	Interest rates up 200 basis points
MortDown	Mortality deteriorates by 10%
MortUp	Mortality increases by 10%
ExpUp	Renewal expenses increase by 1000%
Inflation Up	Expense inflation increases by 200 basis points
Inflation Down	Expense inflation decreases by 200 basis points
LapseDown	Lapse rates decrease by 50%
LapseUp	Lapse rates increase by 50%
Loan Rate Up	Interest rate on loan increases by 500 basis points
Loan Rate Down	Interest rate on loan decreases by 500 basis points

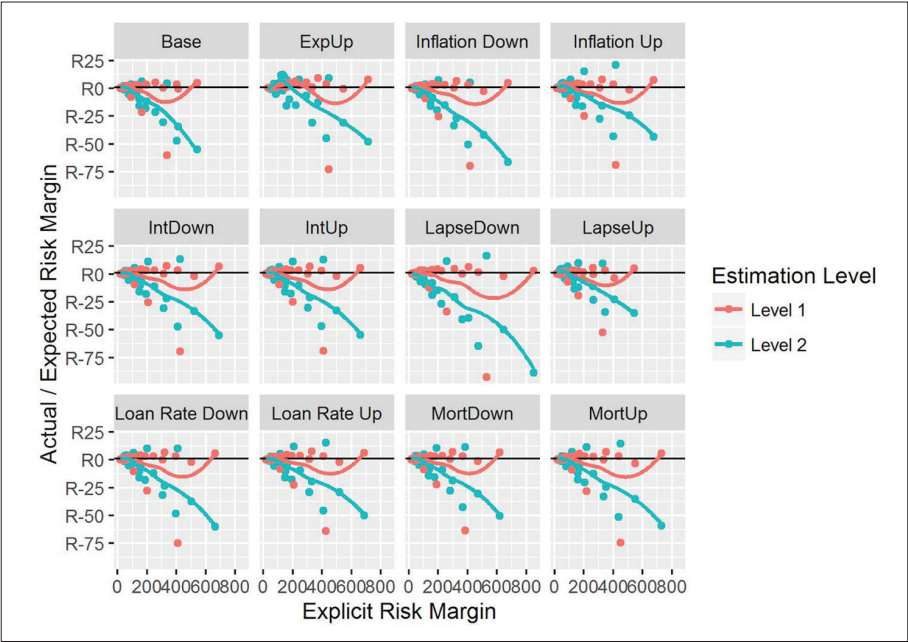


FIGURE 33 Comparisons of the level of error of the Level 1 and Level 2 risk margin simplifications in absolute rand terms against the explicit risk margin calculation for a subset of policies on a variety of different assumptions

for each risk capital component. As discussed in ¶3.9.1, a driver that has a tighter variance, that is closer to 0% error and that acts more or less consistently as the size of the risk margin increases is considered superior.

6.1.2 Figures 34 and 35 show the same information, except that the BEL driver has been excluded from Figure 35. The BEL driver has material errors and distorts the scale of the graph. For this reason we can exclude it and focus only on the errors that are better fits. For this reason Figures 35 to 39 do not plot the estimation errors for the BEL driver.

TABLE 7 A key for drivers used

Driver name	Driver description
BEL	BEL including the following cashflows: future expected death benefits, maintenance expenses and premiums.
DeathOutNxtMonth	Death benefit payable in the following month
Expense reserve	Expense reserve
minBel0	Minimum of the BEL and zero
PV Death Outgo	Expected present value of the future death benefits
PV Prem	Expected present value of future premiums

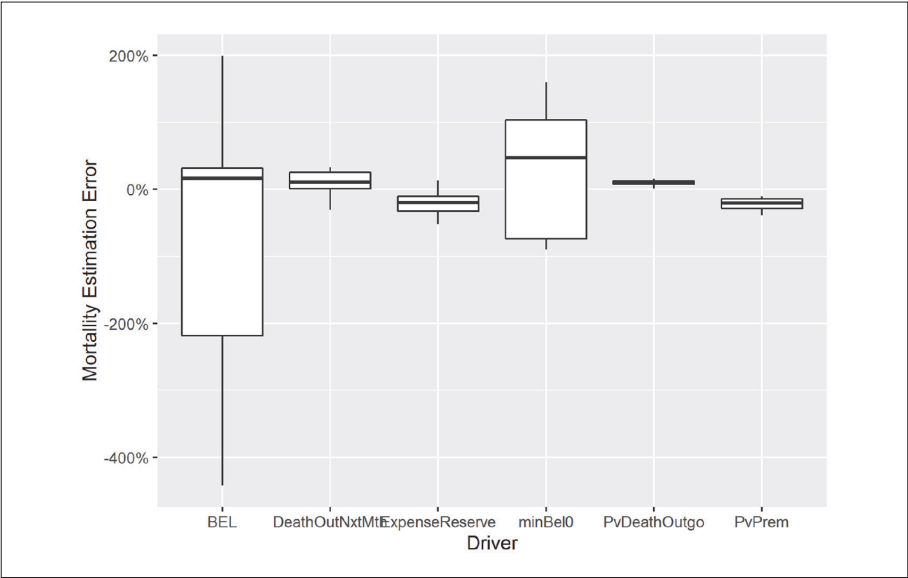


FIGURE 34 A box plot with the estimation error of the mortality risk capital component for a whole-of-life assurance for each driver (x-axis) when comparing the simplified approach against the explicit calculation

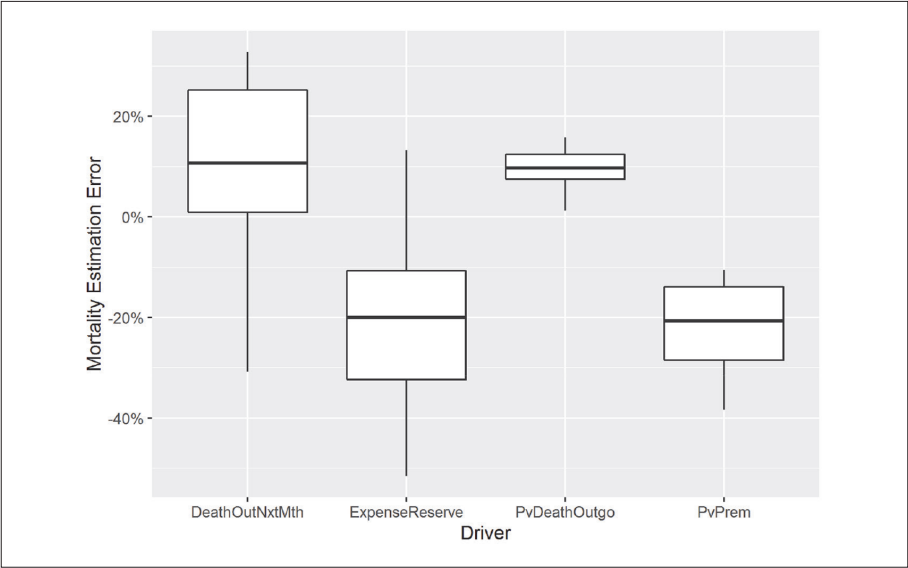


Figure 35 The box plot of the estimation errors for the mortality risk capital component of a whole-of-life assurance for four drivers

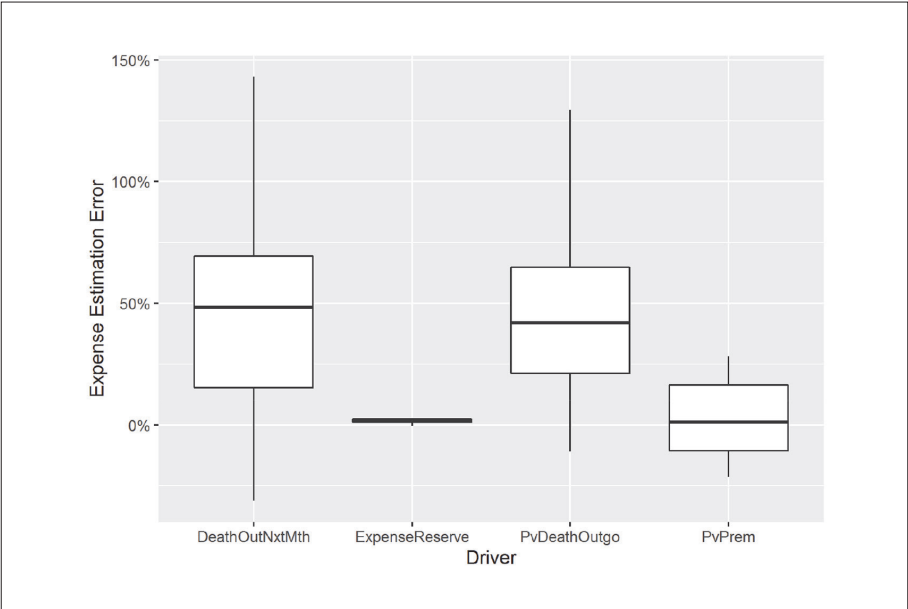


FIGURE 36 The box plot of the estimation errors for the expense risk capital component of a whole-of-life assurance for four drivers

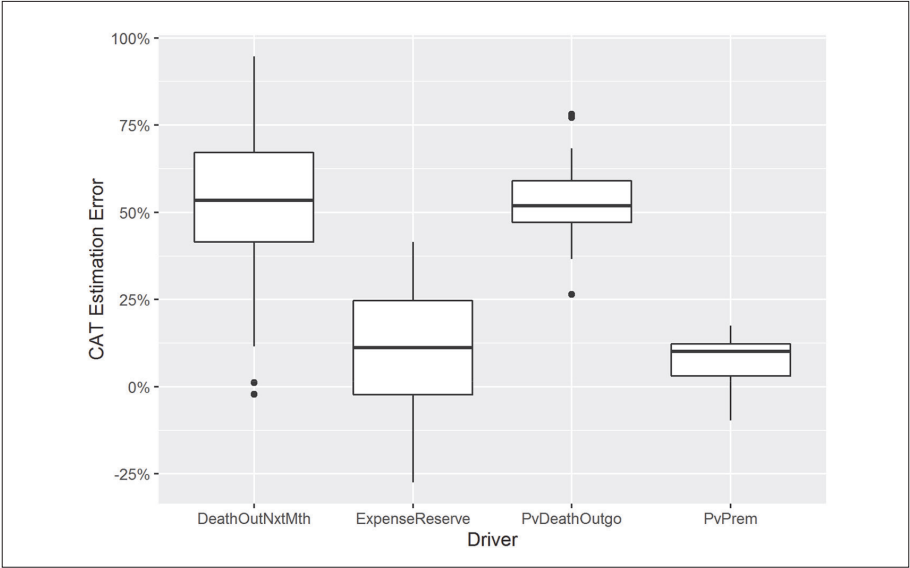


FIGURE 37 The box plot of the estimation errors for the mortality catastrophe risk capital component of a whole-of-life assurance

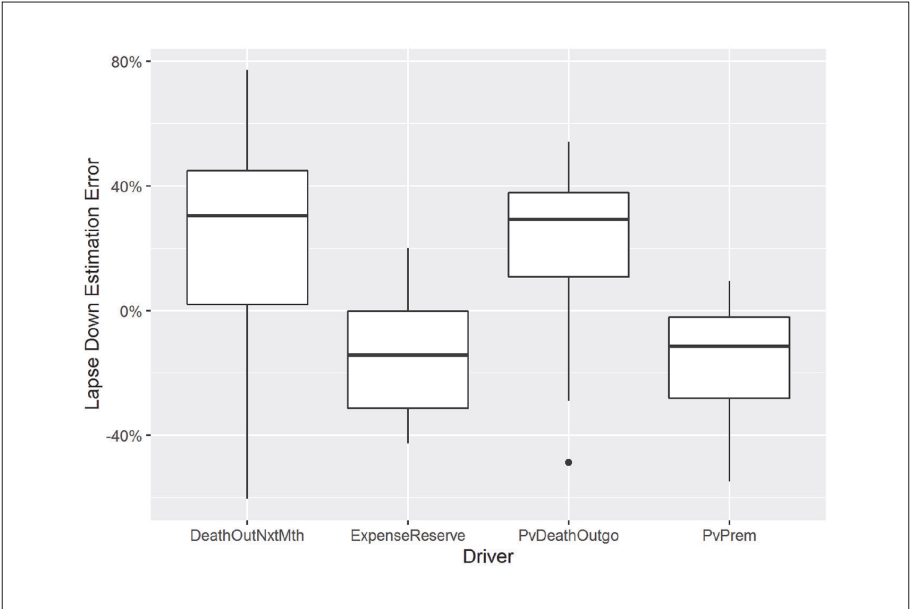


FIGURE 38 The box plot of the estimation errors for the Lapse Down risk capital component of a whole-of-life assurance

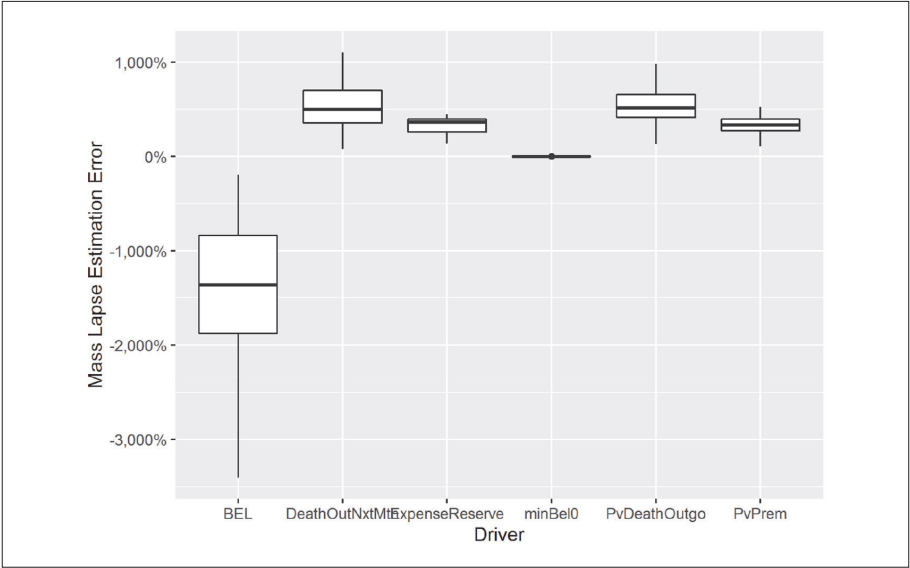


FIGURE 39 The box plot of the estimation errors for the Mass Lapse risk capital component of a whole-of-life assurance

6.1.3 The drivers chosen for each risk capital component are summarised in Table 8.

TABLE 8 Whole-of-Life Assurance Drivers

Risk capital component	Driver name
Mortality	PV Death Outgo
Expense	Expense Reserve
Mortality catastrophe	PV Prem
Lapse Down	PV Prem
Mass Lapse	minBel0

6.1.4 Figure 40 compares the estimates from the Level 1 simplification against the explicit risk margin calculation. From this we can see that the Level 1 simplification has a tendency to underestimate the risk margin. The underestimation increases as the value of the risk margin increases.

6.1.5 Figure 41 shows the sensitivity of the Level 1 risk margin estimation error to the age of the policyholder on the balance sheet date. It shows that there is an underestimation of the risk margin at low ages and an overestimation at higher ages.

A whole-of-life policy with a sum assured of R100 000 and premium and sum assured escalation rate of 0% was used.

6.1.6 Figure 42 shows the sensitivity of the Level 1 risk margin simplification to duration in force. It shows that there tends to be an underestimation at early durations and overestimation at later durations. A whole-of-life policy with a sum assured of R100 000 and premium and sum assured escalation rate of 0% was used.

6.1.7 Figure 43 shows the sensitivity of the Level 1 risk margin simplification to the escalation rate of the premium and sum assured. As the escalation rate increases so the relative underestimation increases. A whole-of-life policy with a sum assured of R100 000 at duration in force of 0 years was used.

6.1.8 The sensitivity graphs shown show that there are some clear drivers of bias. A whole-of-life assurance book with younger ages, no escalations and an early duration in force will tend to underestimate the risk margin. This would be typical of a funeral cover book of business. The underestimation will be worse if there are escalations on the policies as are commonly sold in the South African market.

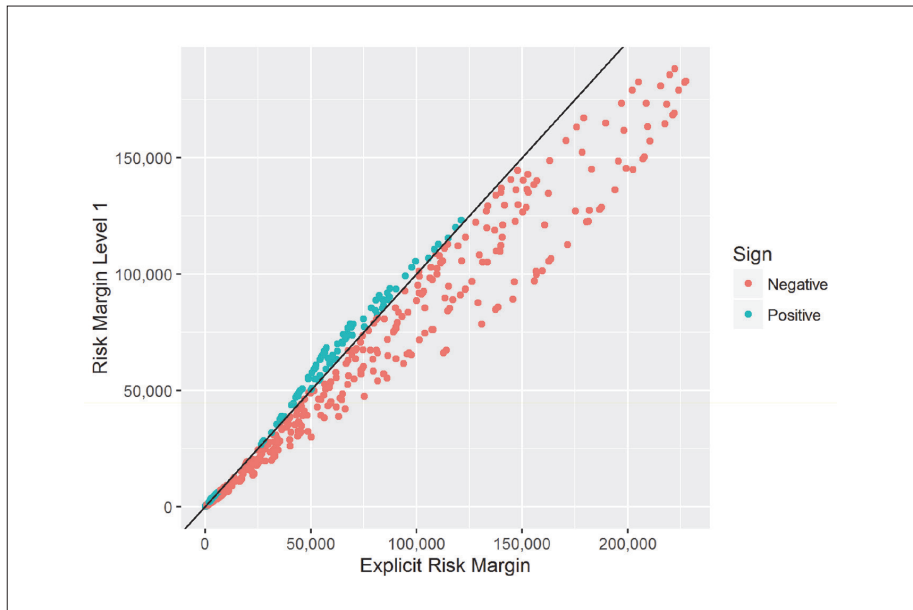


FIGURE 40 A scatter plot of the Level 1 risk margin estimate (y-axis) and the explicit risk margin calculation (x-axis)

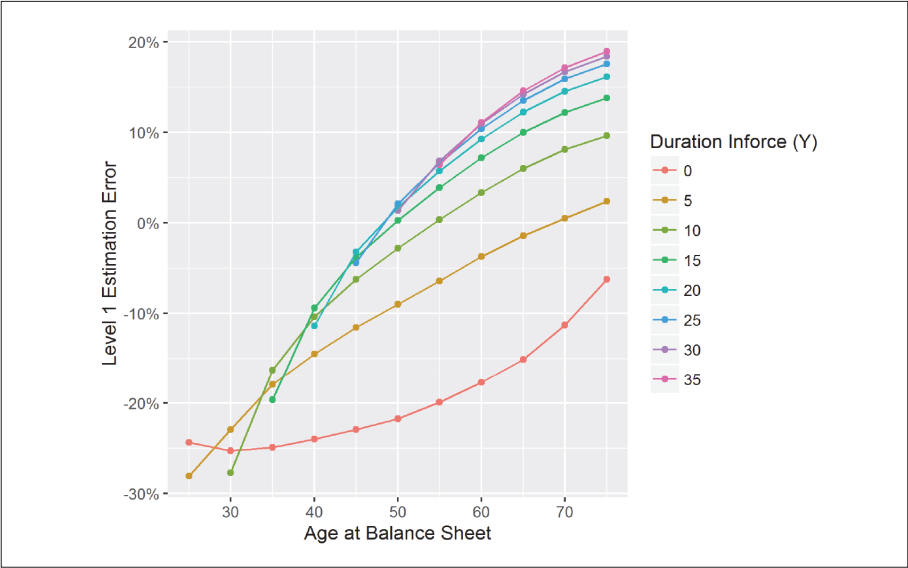


FIGURE 41 The relative error of the Level 1 simplification (y -axis) against the age of the policyholder on the balance sheet date (x -axis) for a sample of durations in force

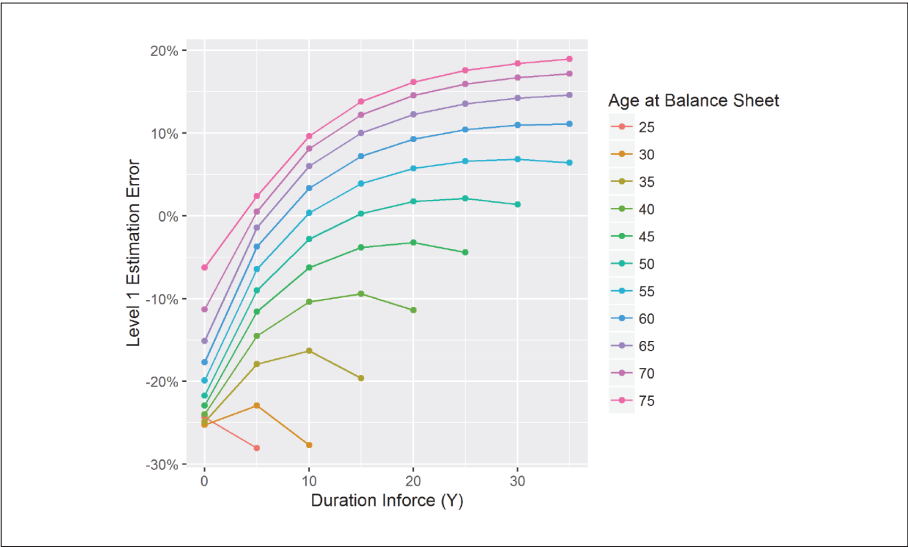


FIGURE 42 The relative error of the Level 1 simplification (y -axis) plotted against the duration in force of the policyholder (x -axis) for a variety of ages on the balance sheet date

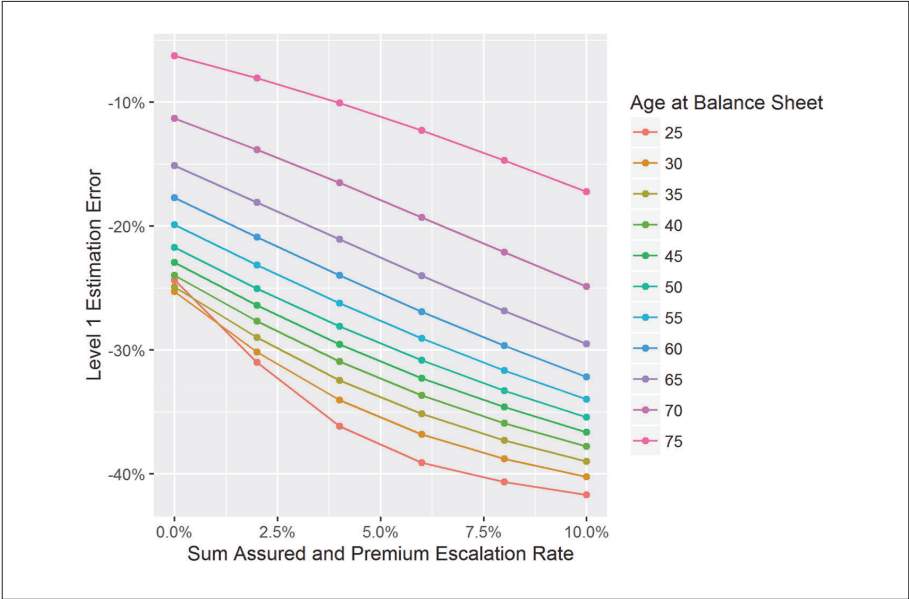


FIGURE 43 The relative error of the Level 1 simplification (y -axis) plotted against the annual escalation rate for the premium and sum assured

6.1.9 The early duration underestimation bias may influence the allocation of capital during a new business planning exercise or at the time of pricing. A book with older ages, and later durations in force will tend to overestimate the risk margin. This would be typical of a mature whole-of-life assurance book.

6.2 Whole-of-Life Assurance Level 2 Simplification

6.2.1 The optimal Level 2 simplification requires the determination of optimal driver for projecting the overall capital requirements. The drivers considered are given in Table 7. Figure 44 displays a box plot of these errors between the comprehensive calculation of the risk margin and the Level 2 simplification. The same data is plotted in a scatter plot in Figure 45. A driver that has a tighter variance, that is closer to 0% error and that acts more or less consistently as the size of the risk margin increases, was the driver that was chosen, as set out in §3.9.1.

6.2.2 The present value of premiums driver is the preferred driver under the Level 2 simplification. This driver was also the preferred driver for some of the individual risk types. Figure 46 shows a scatter plot of the simplification against the explicit risk margin. The plot shows that the majority of cases result in an underestimation that is often quite severe with a smaller proportion of cases producing an overestimation.

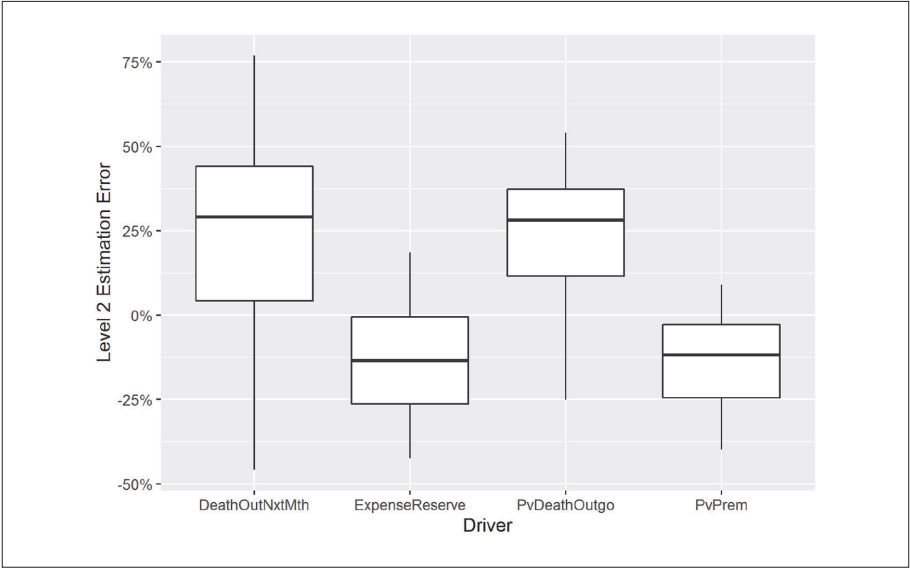


FIGURE 44 A box plot of the estimation error of the Level 2 risk margin simplification for four different drivers

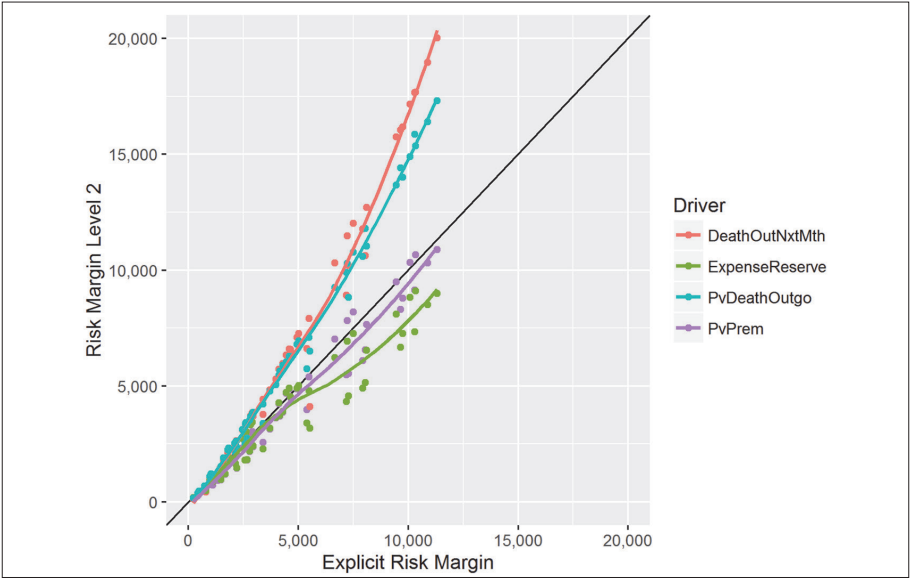


FIGURE 45 A scatter plot of the Level 2 risk margin simplification (y -axis) against the explicit calculation (x -axis) for four different possible drivers for a sample set of policies

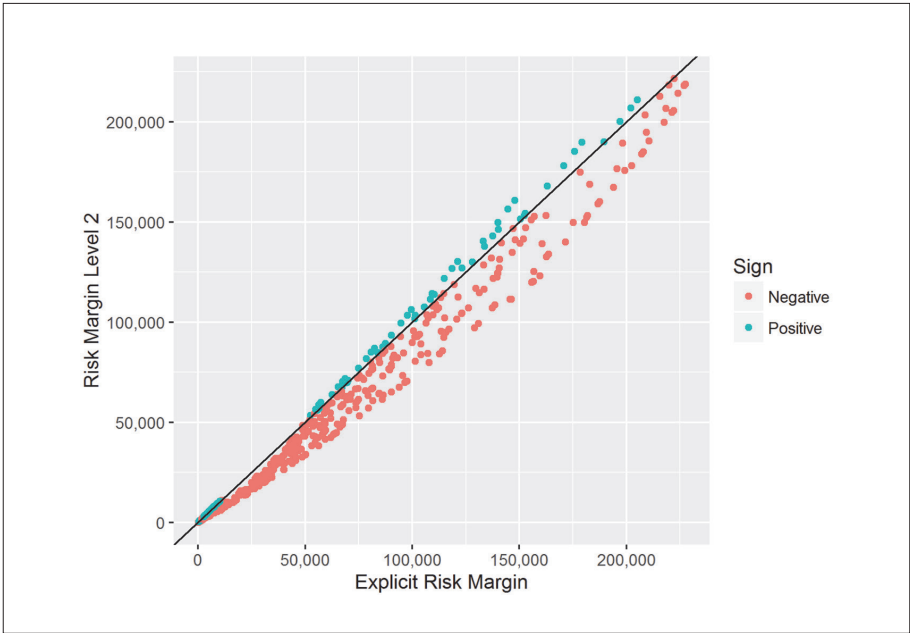


FIGURE 46 A scatter plot of the Level 2 simplification (y -axis) against the explicit risk margin calculation (x -axis) for a comprehensive set of policies

6.3 BEL as a Driver for Whole-of-Life Assurance Products

6.3.1 The BEL is considered a poor driver for estimating the risk margin on whole-of-life assurance products.

6.3.2 Figure 47 shows the projected BEL run-off profile of a sample whole-of-life assurance product and the run-off of the SCR from the same product. The BEL run-off profile appears odd at first, but makes sense when one considers that – in absolute terms – the BEL begins as a negative value and then crosses to zero and becomes positive. This means that the run-off profile (where the BEL is expressed as a percentage of the starting BEL) becomes negative and results in a very different shape to the SCR which runs off smoothly over time.

6.4 A potential solution

6.4.1 The ratio of SCR and present value of future premiums is plotted in Figure 48 and shows that there is an increasing approximately linear relationship between SCR/PVPrem over time. The Level 2 simplification assumes that the ratio of SCR/PVPrem will remain constant over time. From Figure 48 we can see that this assumption does not hold true.

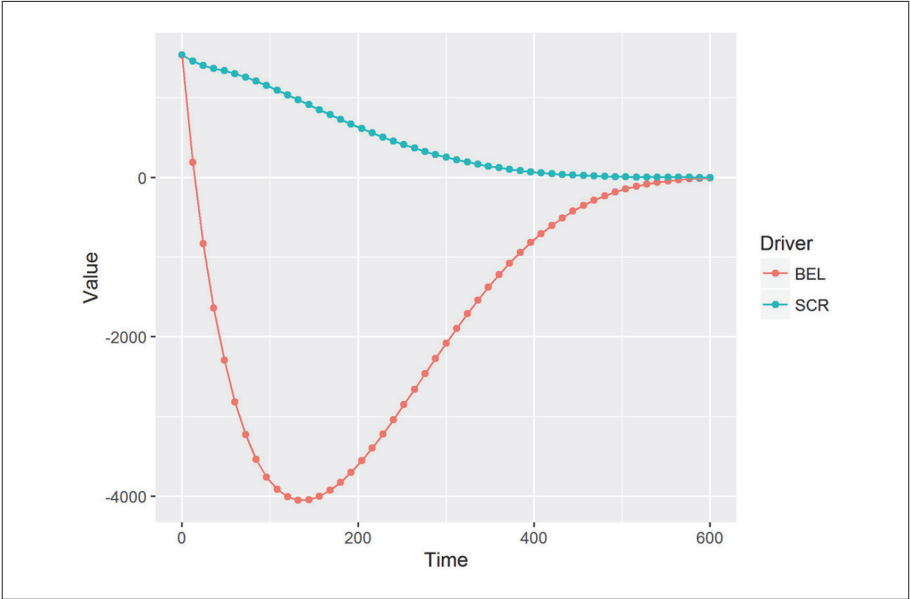


FIGURE 47 A scatter plot of the run-off profile of the SCR and the BEL

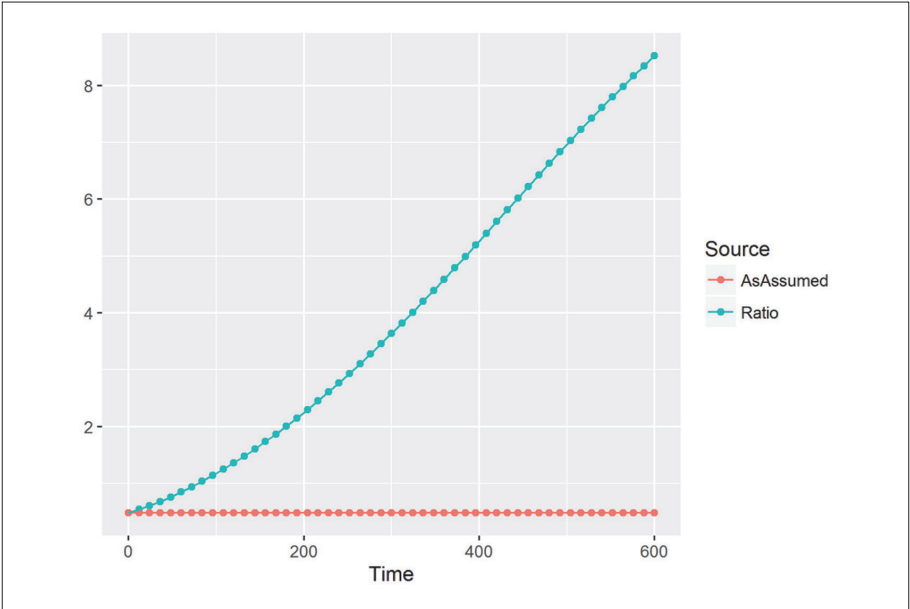


FIGURE 48 The ratio of SCR/PVPrem (y-axis) over time in months (x-axis) for a whole-of-life assurance

6.4.2 The solution to reduce the estimation error proposed for annuity products can also be applied to whole-of-life assurance products. This approach relies on the calculation at some point in the future, f , of one additional SCR, $SCR(f)$. $SCR(f)/PVPrem(f)$ and $SCR(0)/PVPrem(0)$ are used to fit a straight line and then predict future $SCR(t)/PVPrem(t)$ values using this fitted line. The value of $PVPrem(t)$ can easily be projected by most cashflow models. Knowing the value of $PVPrem(t)$ allows the value of $SCR(t)$ to be calculated.

6.4.3 For whole-of-life assurance products a value of ten years was used for f to predict the future SCR and estimate the risk margin. Figure 49 shows the quality of the estimation and Figure 50 allows for a comparison of the errors for proposed solution as well as the Level 1 and Level 2 simplifications.

6.4.4 The proposed solution shows a small overestimation bias in the majority of cases. The size of the error and the smaller variability of the error suggest that the proposed solution is a superior to the Level 1 and Level 2 simplification. The Level 1 simplification is superior to the Level 2 simplification.

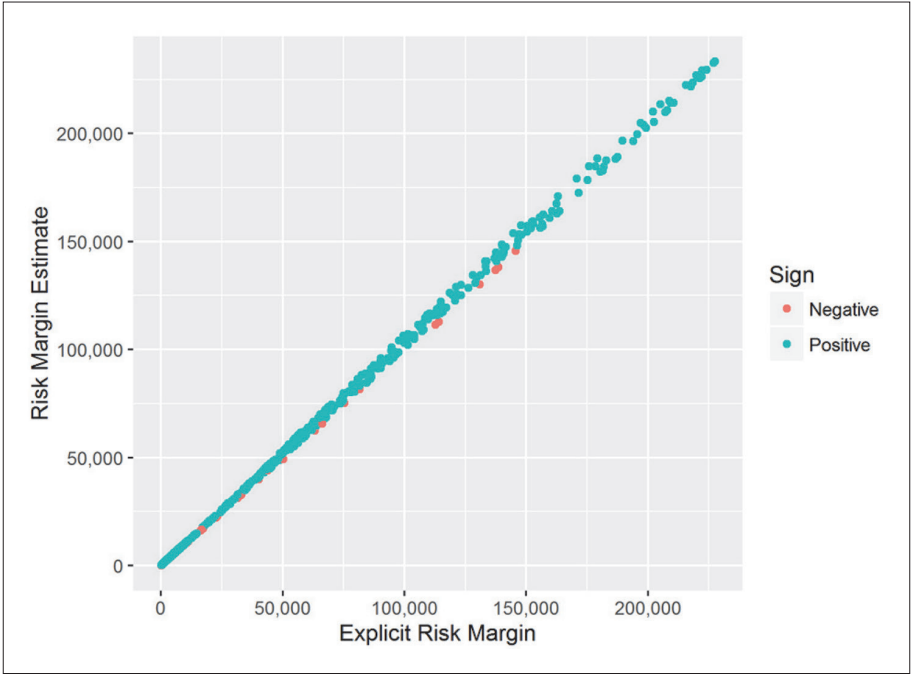


FIGURE 49 A scatter plot of the risk margin estimated by the proposed solution (y -axis) against the explicit risk margin calculation (x -axis)

6.5 Sensitivity Testing

6.5.1 Figure 51 shows a comparison of the absolute errors of the proposed solution, Level 1 and Level 2 risk margin simplifications against the explicit risk margin calculation for a subset of policies on a variety of different assumptions. The initial finding of section 6 is that the Level 1 simplification is better than the Level 2 simplification and the proposed solution is better than both. This can be shown to hold across all alternative input assumptions. Table 6 from the Credit Life section lists the sensitivities tested. It should be noted how the Level 1 and Level 2 estimate become increasingly poor as lapse rates decrease.

6.5.2 Figures 52 to 54 show the same information as Figure 51, as box plots allowing for a better comparison of the estimate errors under different assumptions. The range of Level 1 and Level 2 relative errors are reasonably consistent for all sensitivities. The proposed solution is sensitive to the lapse assumptions but overall remains less sensitive than both the Level 1 and Level 2 simplifications. The initial finding of section 6 is that the Level 1 simplification is better than the Level 2 simplification and the proposed solution is better than both. This can be shown to be hold across all alternative input assumptions.

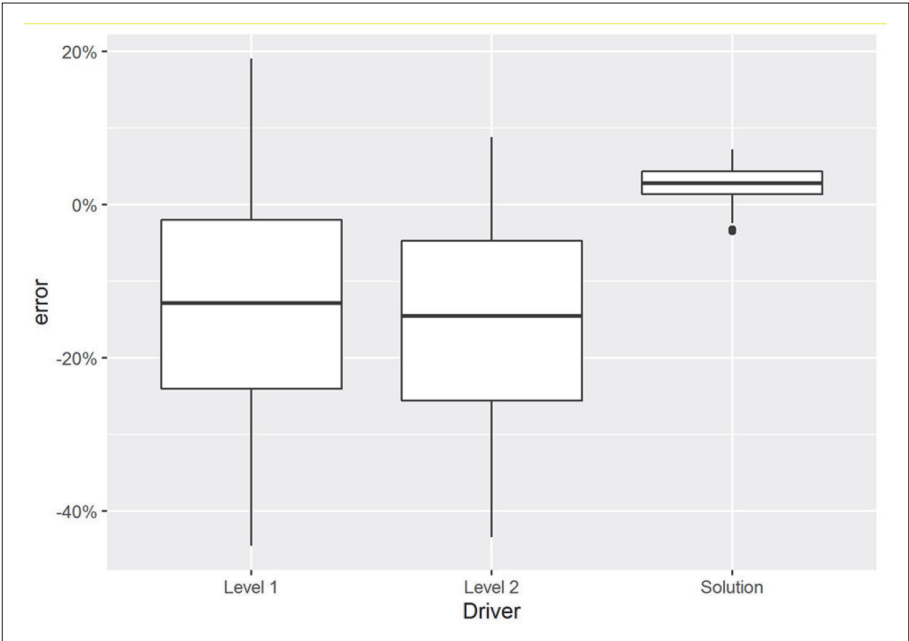


FIGURE 50 A box plot of the relative errors of the Level 1 and Level 2 simplification as well as of the proposed solution

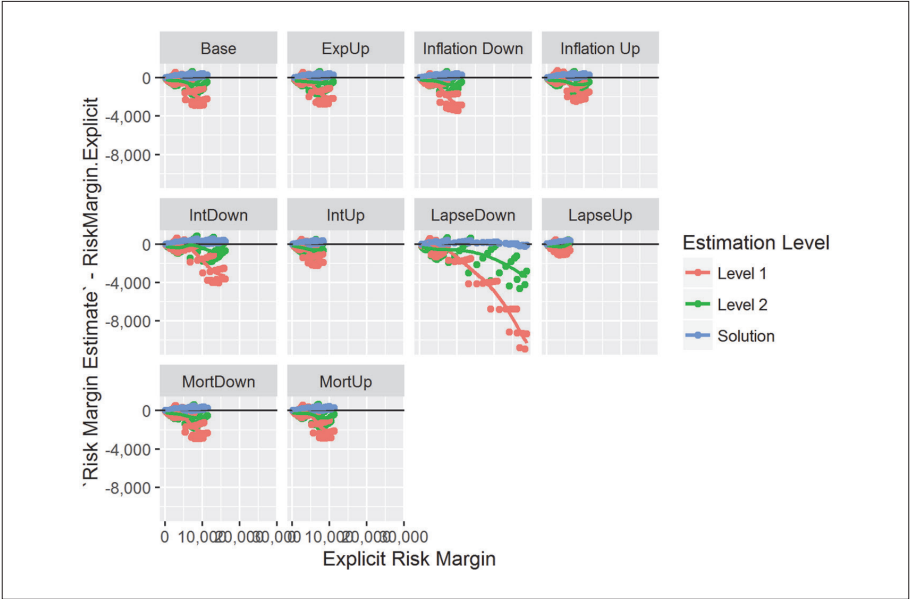


FIGURE 51 The Level 1 and Level 2 simplifications plotted against the actual risk margin calculation, across a number of sensitivities

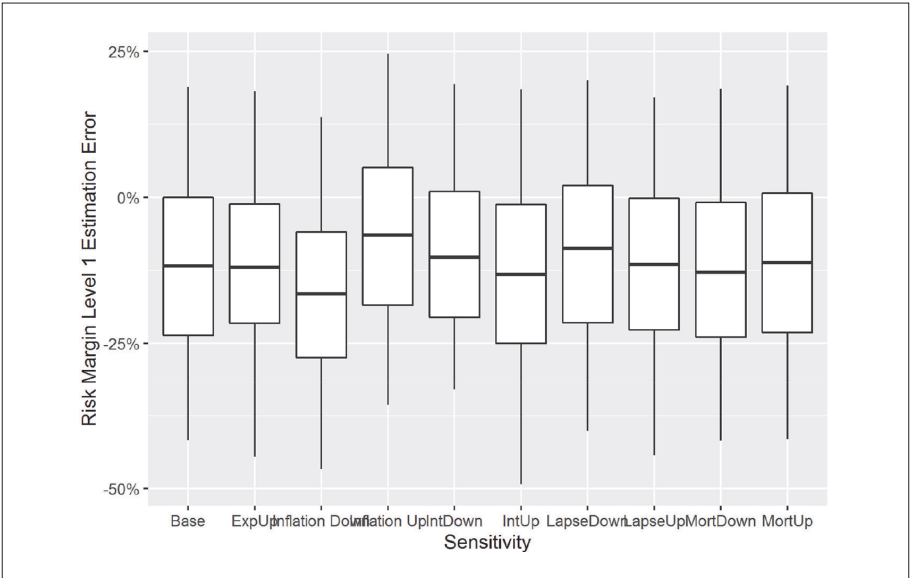


FIGURE 52 A box plot of the estimation error of the Level 1 simplification on different input assumptions

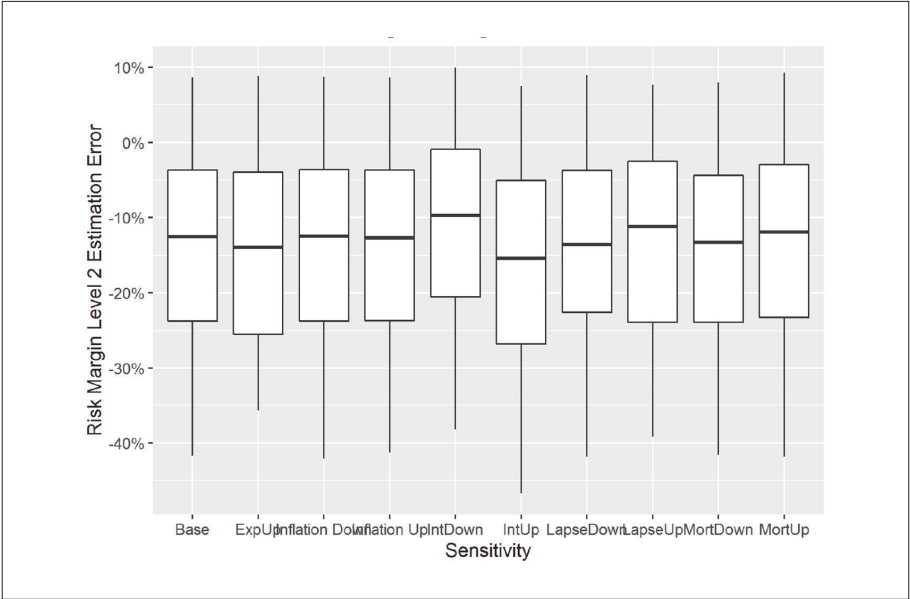


FIGURE 53 A box plot of the estimation error of the Level 2 simplification on different input assumptions

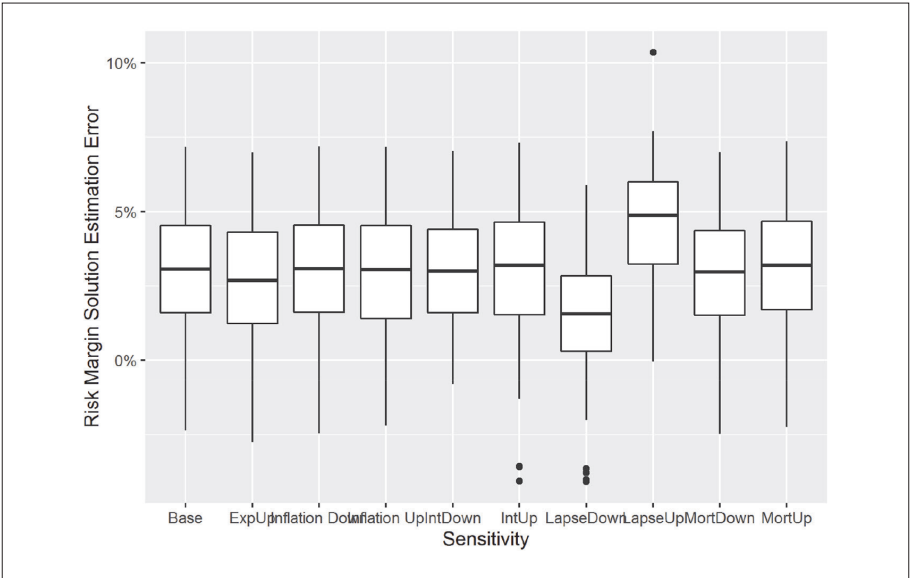


FIGURE 54 A box plot of the estimation error of the proposed solution on different input assumptions

7. SUMMARY

7.1 The FSI GN 2.2, introduces a hierarchy of simplifications to calculate the SCR values. These simplifications introduce biases. It was found that these biases are present due to the drivers not accurately representing the run-off patterns of the SCR.

7.2 The simplifications used to estimate the risk margins for annuity products show a systemic bias towards underestimation of the risk margin. The BEL and expense reserve are the preferred drivers for a Level 1 driver approach and BEL for a Level 2 driver approach. We propose a solution based on the relationship between the SCR and the BEL which produces a more accurate estimate of the risk margin and show this solution to be robust under a number of different assumptions.

7.3 For credit life products, the simplifications making use of the BEL as a driver to estimate the risk margin do not work well. Both Level 1 as well as Level 2 simplifications produce reasonable results, although the Level 1 simplification is better in most scenarios.

7.4 For whole-of-life assurance products the simplifications used to estimate the risk margin do not work well when the BEL is chosen as the driver. Using the Level 2 simplification (with the present value of future premiums as a driver) provides a biased underestimation. A more accurate estimate can be obtained through the more complex calculation of the Level 1 simplification. A solution is proposed based on the relationship between the SCR and the present value of future premiums, as it produces a more accurate estimate of the risk margin. It is then showed that this solution is robust under a number of assumptions.

8. RECOMMENDATIONS FOR FURTHER RESEARCH

8.1 It was seen that there is a lack of literature around the hierarchy of simplifications in general and the assessment of its suitability in particular. The purpose of this research was to address this gap; however, there are numerous areas that requires further thought.

8.2 The first main area of additional research that can be carried out is to analyse the risks that were not included in this research. Mainly this would entail analysing the suitability of the hierarchy of simplifications for the full set of life insurance products. This includes: health and care, unit-linked and index-linked products. The study of joint life products (for example, joint life annuities), should also be carried out, in order to assess the suitability of the hierarchy of simplifications.

8.3 In addition, the full set of market risks can be included in the SCR calculations which will be relevant for embedded value calculations.

8.4 Furthermore, the question of the suitability of the hierarchy of simplifications could be assessed with respect to short-term products as well.

8.5 The research and modelling that were carried out assumed an absence of all forms of reinsurance and alternative risk transfer measures. The addition of these factors may affect the calculations using the hierarchy of simplifications.

8.6 The consequences of how drivers do not accurately match the run-off of risk capital has potential implications for the unit of measure under IFRS17 and the calculation of an embedded value of an insurer. Further research should be done in order to examine the impact under these two different paradigms.

8.7 Lastly an attempt to find more effective drivers that provide a more accurate fit of the risk margin could be of significant use to the industry. Thus, a full assessment of multiple drivers and their respective fits could be worthwhile for future use.

ACKNOWLEDGEMENTS

The authors would like to thank Andrew Rayner for his guidance on this paper, and Megan Carswell for her input and support during the research and production of this paper.

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APPENDIX A

Simplifications Insurers are using in the South African Insurance Market

The table below summarises the simplifications used by insurers during the QIS3 exercise. At the time of QIS3, a full calculation of the risk margin was labelled as a 'Level 1 simplification' and a 'Level 2 simplification' was what is now (under the FSI) considered a Level 1 simplification.

TABLE A.1 A full distribution of methods used by South African insurers to calculate their reserves as of 2015²⁷

Methodology	Number
Full calculation	10
Risk approximation (Level 2)	50
SCR approximation (Level 3)	118
Duration approach (Level 4)	17
% Best estimate	2
Other	14

27 FSB 17th Annual report of the Registrar of Long-term Insurance, *supra*

APPENDIX B
Summary of model points used

The tables below summarise the model points used in the modelling of the annuity, credit life and whole-of-life assurance products. In creating model points, a selection of policy factors was chosen, and then each combination of these factors was modelled, subject to certain constraints.

TABLE B.1 Model points used in modelling annuity policies. The annuity products were subject to the constraint of no guarantee period after 80 years old.

Factor	Values used
Policyholder age at balance sheet	60 to 95 in five-year intervals
Annuity escalation rate (annual)	0% to 8% in 1% intervals
Guarantee period remaining (years)	0 to 10 years in one-year intervals

TABLE B.2 Model points used in modelling credit life policies, subject to the constraint of no policies with a duration inforce greater than the original term

Factor	Values used
Policyholder age at balance sheet	25 to 75 in five- intervals
Sum assured at start of policy	R10 000, R100 000 and R2 000 000
Original term (years)	2 to 7 years in one-year intervals
Duration inforce (years)	0 to 5 years in one-year intervals

TABLE B.3 Model points used in modelling whole-of-life assurance policies

Factor	Values used
Policyholder age at balance sheet	25 to 75 in five-year intervals
Sum assured at start of policy	R10 000, R100 000 and R2 000 000
Premium and sum assured escalation rate (annual)	0% to 10% in 2% intervals
Duration inforce (years)	0 to 35 years in five-year intervals

APPENDIX C
Lapse rates used in modelling credit life as well as whole-of-life assurances

TABLE C.1 Annual lapse rate used in modelling whole-of-life assurance and credit life policies

Months in force	Annual lapse rate
0–12	21.5%
13–24	17.8%
25–36	14.0%
37+	12.0%