ACTUARIAL SOCIETY
OF SOUTH AFRICA

WRITTEN EXAMINATION

Semester 1 2026

MEMORANDUM

Subject A213 — Contingencies

Intermediate Technical

General comments:

Overall performance on the written paper was modest and worse than the CBA, with an average of around 50%-55%, and there was some evidence of time pressure towards the end of the paper. Candidates coped well with the more standard computational and bookwork questions, with Question 2 (the continuously-payable premium and the dependent and independent decrement definitions) and Question 5 (the decreasing-cover design, prospective reserve and suggested policy alterations) the best answered. The weaker performances were concentrated in the more application-heavy questions. Question 3 was poorly handled, with many candidates unable to apply the constant-force assumption at the fractional age on the PMA92C20 basis. Questions 8 and 9 also proved difficult — the last-survivor mortality-profit calculation in Question 8, and in Question 9 the retrospective reserve together with the recursive analysis of the expense and interest surplus.

QUESTION 1

Examiner's comment: *Generally well answered. Most candidates set out the proof correctly, though some lost marks by not justifying the steps or by assuming, rather than using, the independence of the two lives.*

i)

$$\begin{aligned}\mu_{x+t:y+t} &= -\frac{d}{dt}\ln\ (l_{x+t:y+t}) \\ &= -\frac{d}{dt}\ln\ (l_{x+t}l_{y+t}) \\ &= -\frac{d}{dt}\left\{\ln\ (l_{x+t}) + \ln\ (l_{y+t})\right\} \\ &= \mu_{x+t} + \mu_{y+t}\end{aligned}$$

ii)

$$\begin{aligned}\bar{A}_{\overset{1}{x}y} + \bar{A}_{x\overset{1}{y}} &= \int_{t=0}^{\infty} v^t\ {}_tp_{xy}.\mu_{x+t}dt + \int_{t=0}^{\infty} v^t\ {}_tp_{xy}.\mu_{y+t}dt \\ &= \int_{t=0}^{\infty} v^t\ {}_tp_{xy}\left(\mu_{x+t} + \mu_{y+t}\right)dt \\ &= \int_{t=0}^{\infty} v^t\ {}_tp_{xy}.\mu_{x+t:y+t}dt \\ &= \bar{A}_{xy}\end{aligned}$$

QUESTION 2

Examiner's comment: *The best-answered question on the paper. The premium under a constant force of mortality was handled confidently, and the dependent and independent decrement definitions were mostly known, although a minority confused the two.*

i)

$$\mu = 0.02$$

$$\delta = \ln(1.072508) = 0.07$$

EPV of benefits:

$$\bar{A}_{32:\overline{10|}} = \int_0^{10} 0.02e^{-0.09t}\,dt + e^{-(0.09)(10)}$$

$$= 0.02\int_0^{10} e^{-0.09t}\,dt + e^{-0.9}$$

$$= 0.02\left[\frac{1-e^{-0.9}}{0.09}\right] + e^{-0.9}$$

$$= 0.131873 + 0.40657$$

$$= 0.53844$$

$$\text{So EPV of benefits} = (50\ 000)(0.53844)$$

$$= 26\ 922.15$$

$$\text{EPV of premium} = P\bar{a}_{32:\overline{10|}}$$

$$\bar{a}_{32:\overline{10|}} = \int_0^{10} e^{-0.09t}\,dt$$

$$= \frac{1-e^{-0.9}}{0.09}$$

$$= 6.59367$$

$$\text{Now } P\bar{a}_{32:\overline{10|}} = 50\ 000\bar{A}_{32:\overline{10|}}$$

$$\text{Thus } P = 4\ 083.03$$

ii)

The dependent probability $(aq)_x^\alpha$ is the probability that a life aged x in a particular state will be removed from that state within a year by the decrement α, in the presence of all other decrements in the population.

The independent probability q_x^α is the probability that a life aged x in a particular state will be removed from that state within a year by the decrement α, where α is the only decrement acting on the population.

QUESTION 3

Examiner's comment: *Poorly answered. Many candidates could not apply the constant-force assumption over the single year of age and struggled with the fractional-age annuity on the PMA92C20 basis; a common error was to omit the fractional-age adjustment entirely.*

$$a_{71.625}^{(8)} = \frac{1}{8} \cdot {}_{0.125}p_{71.625} . v^{\frac{1}{8}} + \frac{1}{8} \cdot {}_{0.25}p_{71.625} . v^{\frac{2}{8}} + \ddot{a}_{72}^{(8)} \cdot {}_{0.375}p_{71.625} . v^{\frac{3}{8}}$$

$$\begin{aligned} \ddot{a}_{72}^{(8)} &= \ddot{a}_{72} - \frac{7}{16} \\ &= 10.711 - \frac{7}{16} \\ &= 10.2735 \end{aligned}$$

Calculate μ between ages 71 and 72

$$\begin{aligned} p_{71} &= 1 - q_{71} \\ &= 1 - 0.015841 \\ &= 0.98416 \\ &= e^{-\mu} \end{aligned}$$

Hence $\mu = -\ln(0.98416)$

$= 0.01597$

Now ${}_{0.125}p_{71.625} = e^{-0.125(0.01597)} = 0.99801$

${}_{0.25}p_{71.625} = e^{-0.25(0.01597)} = 0.99602$

${}_{0.375}p_{71.625} = e^{-0.375(0.01597)} = 0.99403$

Thus $a_{71.625}^{(8)} = 10.3105$

QUESTION 4

Examiner's comment: *Reasonably answered. The main difficulties were splitting the annuity into its guaranteed and life-contingent portions and applying the monthly (Woolhouse) adjustment; some candidates also mishandled the step-ups in the payment amounts.*

$$\text{EPV} = 10\ 000\ddot{a}_{\overline{5}|}^{(12)} + 15\ 000\ddot{a}_{\overline{5}|}^{(12)}v^5 + 20\ 000\,{}_{10}p_{62}\ddot{a}_{72}^{(12)}v^{10}$$

$$\ddot{a}_{\overline{5}|}^{(12)} = \frac{1-v^5}{d^{(12)}}$$

$$d^{(12)} = 0.039157$$

$$\text{So } \ddot{a}_{\overline{5}|}^{(12)} = \frac{1-(1.04)^{-5}}{0.039157}$$

$$= 4.54766$$

$$\ddot{a}_{72}^{(12)} = \ddot{a}_{72} - \frac{11}{24}$$

$$= 11.67667$$

$$\text{EPV} = 10\ 000(4.54766) + 15\ 000(4.54766)v^5 + 20\ 000\,{}_{10}p_{62}(11.67667)v^{10}$$

$${}_{10}p_{62} = \frac{9\ 193.86}{9\ 804.173}$$

$$= 0.93775$$

$$\text{EPV} = 45\ 476.64 + 56\ 067.73 + 147\ 945.72$$

$$= 249\ 490.09$$

QUESTION 5

Examiner's comment: *Well answered overall. The premium and prospective reserve were generally good; the descriptive parts — the disadvantages of the design and the suggested alterations — were the main differentiators, with weaker candidates offering only vague points.*

i)

$$P\ddot{a}_{55:\overline{10|}} = 100\,000\bar{A}^{\;1}_{55:\overline{5|}} + 20\,000v^5\,{}_5p_{55}\bar{A}^{\;1}_{60:\overline{5|}}$$

$$\text{Now } \ddot{a}_{55:\overline{10|}} = 7.610$$

$$\bar{A}^{\;1}_{55:\overline{5|}} = (1.06)^{\frac{1}{2}}A^{\;1}_{55:\overline{5|}}$$

$$= (1.06)^{\frac{1}{2}}\left(A_{55:\overline{5|}} - {}_5p_{55}v^5\right)$$

$${}_5p_{55} = \frac{l_{60}}{l_{55}}$$

$$= \frac{9\,287.2164}{9\,557.8179}$$

$$= 0.971688$$

$$A_{55:\overline{5|}} = 0.74965$$

$$\text{So } \bar{A}^{\;1}_{55:\overline{5|}} = (1.06)^{\frac{1}{2}}\left(0.74965 - {}_5p_{55}v^5\right)$$

$$= 0.024244$$

$$\bar{A}^{\;1}_{60:\overline{5|}} = (1.06)^{\frac{1}{2}}A^{\;1}_{60:\overline{5|}}$$

$$A^{\;1}_{60:\overline{5|}} = \left(A_{60:\overline{5|}} - {}_5p_{60}v^5\right)$$

$$A_{60:\overline{5|}} = 0.75152$$

$${}_5p_{60} = 0.949828$$

$$\text{So } \bar{A}^{\;1}_{60:\overline{5|}} = 0.04299$$

$$\text{Hence P} = 401.00$$

ii)

$${}_5V^{PRO} = 20\,000\bar{A}^{\;1}_{60:\overline{5|}} - 401\ddot{a}_{60:\overline{5|}}$$

$$\ddot{a}_{60:\overline{5|}} = 4.39$$

$$\bar{A}^{\;1}_{60:\overline{5|}} = 0.04299 \text{ from (i)}$$

$$\text{So } {}_5V^{PRO} = -900.64$$

iii)
Disadvantages:

Policyholder “in debt” at time 5 (with size of debt equal to the negative reserve) as more life cover provided in the first 5 years than is paid for by the level premiums in those years.

If policy is lapsed during first five years (possibly longer) the company will suffer a loss. Not possible to recover this loss from policyholder.

Possible alterations:

Collect the premiums more quickly e.g.

- shorten premium paying term to say five years
- make premiums larger in earlier years, smaller in later years.

Change the pattern of benefits e.g. don’t reduce the sum assured after five years, or don’t reduce it so drastically after five years.

Credit were given for other sensible explanations.

QUESTION 6

Examiner’s comment: *A wide spread of marks. Most candidates set up the equation of value correctly, but errors were common in allowing for the simple, year-end vesting bonus and in incorporating the select mortality and the renewal and claim expenses.*

i)

$$\text{EPV} = 8\,400\ddot{a}_{[35]:\overline{30|}} - 8\,400(0.03)\ddot{a}_{[35]:\overline{30|}} + 8\,400(0.03)$$

$$= 0.97(8\,400)\ddot{a}_{[35]:\overline{30|}} + 8400(0.03)$$

$$\ddot{a}_{[35]:\overline{30|}} = 17.631$$

$$\text{EPV} = 145\,622.6$$

ii)

$$\text{EPV} = 1\,000 + 0.5P + 500A_{[35]:\overline{30|}}$$

$$A_{[35]:\overline{30|}} = 0.32187$$

$$\text{EPV} = 5\,360.94$$

iii)

$$\text{EPV of benefits } = 200\ 000(1-b)A_{[35]:\overline{30}|} + 200\ 000b(IA)_{[35]:\overline{30}|} + 200\ 000bv^{30}\ {}_{30}p_{[35]}$$

$$
\begin{aligned}
{}_{30}p_{[35]} &= \frac{l_{65}}{l_{[35]}} \\
&= \frac{8\ 821.261}{9\ 892.915} \\
&= 0.891675 \\
(IA)_{[35]:\overline{30}|} &= (IA)_{[35]} - v^{30}\ {}_{30}p_{[35]}\left(30A_{65} + (IA)_{65}\right) + 30v^{30}\ {}_{30}p_{[35]} \\
&= 7.47005 - v^{30}\ {}_{30}p_{[35]}(30 \cdot 0.52786 + 7.89442) + 30v^{30}\ {}_{30}p_{[35]} \\
&= 7.47005 - 6.5239 + 8.2476 \\
&= 9.1937 \\
\text{So EPV } &= (1-b)64\ 374 \quad + b \cdot 1\ 838\ 747.49 + b \cdot 54\ 983.99 \\
&= 64\ 374 + 1\ 829\ 357.48b
\end{aligned}
$$

EPV of premium = EPV of benefits + EPV of expenses

$$1\ 829\ 357.48b = 75\ 887.66$$

$$\text{Thus } b = 4.15\%$$

QUESTION 7

Examiner's comment: *Found moderately difficult. The reversionary (widow's) annuity and its monthly payment caused the most trouble, and the premium refund on survival to age 60 was frequently omitted or incorrectly timed.*

i)

$$\text{EPV} = 200\ 000 A^{\ 1}_{50:\overline{10}|}$$

$$A^{\ 1}_{50:\overline{10}|} = A_{50} - v^{10}\ {}_{10}p_{50} A_{60}$$

$${}_{10}p_{50} = \frac{l_{60}}{l_{50}}$$

$$= \frac{9\ 826.131}{9\ 941.923}$$

$$= 0.988353$$

$$A_{50} = 1 - d\ddot{a}_{50}$$

$$= 1 - \frac{0.04}{1.04} \cdot 18.843$$

$$= 0.27527$$

$$A_{60} = 1 - d\ddot{a}_{60}$$

$$= 1 - \frac{0.04}{1.04} \cdot 15.632$$

$$= 0.39877$$

So $A^{\ 1}_{50:\overline{10}|} = 0.00901$

$$\text{EPV} = 1\ 802.52$$

ii)

$$\text{EPV} = 100\ 000 a^{(12)}_{60|50}$$

$$= 100\ 000\left[\left(\ddot{a}_{60} - \frac{11}{24}\right) - \left(\ddot{a}_{60:50} - \frac{11}{24}\right)\right]$$

$$= 100\ 000\left(\ddot{a}_{60} - \ddot{a}_{60:50}\right)$$

$$\ddot{a}_{60} = 16.652$$

$$\ddot{a}_{60:50} = 15.801$$

EPV =85 100

iii)

Let P be the annual premium

$$\text{EPV} = 0.1(5) P v^{10}\ {}_{10}p_{50}$$

$$= 0.33385P$$

iv)

EPV of premiums

$$P\ddot{a}_{50:\overline{5}|} = P\left(\ddot{a}_{50} - v^5\ {}_5p_{50}\ddot{a}_{55}\right)$$

$$\begin{aligned}{}_5p_{50} &= \frac{l_{55}}{l_{50}} \\ &= \frac{9\ 904.805}{9\ 941.923} \\ &= 0.99627\end{aligned}$$

$$\ddot{a}_{55} = 17.364$$

$$\ddot{a}_{50:\overline{5}|} = 4.62$$

Hence $4.62P = 1\ 802.52+85\ 100+0.33385P$

So $P = 20\ 254.67$

Question 8

Examiner's comment: *Among the weaker questions. The prospective reserve was usually attempted, but the mortality-profit calculation across the three scenarios — particularly the last-survivor logic and the associated death strains — defeated many candidates.*

i)

$${}_3V = 500\ 000A_{\overline{58:60}} - 7\ 400\ddot{a}_{\overline{58:60}}$$

$$\begin{aligned}\ddot{a}_{\overline{58:60}} &= \ddot{a}_{58} + \ddot{a}_{60} - \ddot{a}_{58:60} \\ &= 16.356 + 16.652 - 14.549 \\ &= 18.459\end{aligned}$$

$$\begin{aligned}A_{\overline{58:60}} &= 1 - da_{\overline{58:60}} \\ &= 0.29004\end{aligned}$$

$${}_3V = 8\ 422.63$$

ii)

Scenario A

Actual deaths =1

MP=EDS − ADS

DSAR=500 000 − 8 422

=491 578

Now MP $= \left(10\ 000q_{57} \cdot q_{59} - 1\right)\left(491\ 578\right)$

$q_{57} = 0.001558$

$q_{59} = 0.001801$

MP= − 477 784.53

Hence a loss

iii)

Scenario B

Actual deaths = 14. First calculate reserve for female only surviving policies

$$
\begin{aligned}
{}_3V^f &= 500\ 000A_{60} - 7\ 400\ddot{a}_{60} \\
&= 500\ 000\left(1-\frac{0.04}{1.04}\cdot 16.652\right) - 7\ 400\left(16.652\right) \\
&= 56\ 544.43 \\
\text{DSAR} &= 56\ 544.43 - 8\ 422 \\
&= 48\ 122.43 \\
\text{MP} &= \left(10\ 000q_{57}(1-q_{59}) - 14\right)\cdot 48\ 122.43 \\
&= 74\ 683.15
\end{aligned}
$$

iv) Total mortality profit=

Mortality profit from scenario A

\+ Mortality profit from scenario B

\+ Mortality profit from scenario C

Question 9

Examiner's comment: *Poorly answered. The retrospective reserve was often confused with the prospective reserve, and few candidates could set up the recursive relationship to isolate the expense and interest surplus. Method marks were available for a clear, systematic layout.*

i)

$$ {}_5V = \frac{(1.04)^5}{{}_5p_{55}}\left[18\ 520\ddot{a}_{55:\overline{5}|} - 100\ 000A^{\ 1}_{55:\overline{5}|} - 0.5(18\ 520) - 5\ 000 - 1\ 000\left(\ddot{a}_{55:\overline{5}|} - 1\right)\right] $$

$$ {}_5p_{55} = \frac{l_{60}}{l_{55}} $$

$$ = \frac{9\ 287.216}{9\ 557.818} $$

$$ = 0.97169 $$

$$ \ddot{a}_{55:\overline{5}|} = 4.585 $$

$$ A_{55:\overline{5}|} = 0.82365 $$

$$ A^{\ 1}_{55:\overline{5}|} = A_{55:\overline{5}|} - v^5\ {}_5p_{55} $$

$$ = 0.024993 $$

$$ {}_5V = \frac{(1.04)^5}{{}_5p_{55}}(84\ 914 - 2\ 499 - 17\ 845) $$

$$ = 80\ 848.09 $$

ii) 80 848

The gross premium reserve is equal to the retrospective premium reserve since:
1. the retrospective and prospective reserves are calculated on the same basis, and
2. this basis is the same as the basis used to calculate the premiums used in the reserve calculation

iii)

Total profit for 2024

$1\ 000\left({}_4V + P - E\right)(1+i) - S.\text{actual deaths} - {}_5V.\text{number of policies IF}$

$=1\ 000(60350 + 18\ 520 - 1\ 000)(1.038) - 100\ 000(4) - 80\ 848.09(1\ 000\text{-}4)$

$= -95\ 638.19$

Now Expense profit = 0 and

Interest profit = Total profit − Mortality profit − Expense profit

$= -95\ 638 - 60\ 130 - 0$

$= -155\ 768.19$

i.e. loss

END