Actuarial Society of South Africa

Examiners Report

April 2026

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

The examiners mark with the candidates' answers where a question has several parts which follow on each other. It is therefore essential that candidates work consistently with their answers.

Many candidates can also increase their probability of passing this exam by practising good exam technique, which is often missing in its most basic form.

QUESTION 1

i.

a.

$$\left(1+\frac{0.035}{2}\right)^2 = e^{\delta}$$

$$\delta = 2\ln\left(1+\frac{0.035}{2}\right) = 0.034697276 \rightarrow 3.4697\%$$

b.

$$\left(1+\frac{0.035}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$$

$$d^{(4)} = 4\left(1-\left(1+\frac{0.035}{2}\right)^{-\frac{2}{4}}\right) = 0.034547223 \rightarrow 3.4547\%$$

c.

$$\left(1+\frac{0.035}{2}\right)^2 = \left(1+\frac{i^{(12)}}{12}\right)^{12}$$

$$\frac{i^{(12)}}{12} = \left(1+\frac{0.035}{2}\right)^{\frac{2}{12}} - 1 = 0.00289562396 \rightarrow 0.2896\%$$

ii.

These rates are applicable to **the same effective period of six-months**, so for the same present value amount at the start, they should accumulate to the same amount of total interest over the period, i.e. for both rates we must have PV + I = FV

For $\frac{i^{(2)}}{2}$, interest of $I = PV \times \frac{i^{(2)}}{2}$ is added to the principal amount at the end of a six-month period.

For $\frac{d^{(2)}}{2}$, interest of $I = FV \times \frac{d^{(2)}}{2}$ is deducted from the accumulated amount at the start of the period.

Since FV > PV for a positive interest rate, it means that that $\frac{d^{(2)}}{2}$ must be less than $\frac{i^{(2)}}{2}$ for the total interest to be the same.

Alternative

$\frac{d^{(2)}}{2}$ is paid at the start of the half year while $\frac{i^{(2)}}{2}$ is paid at the end of the half-year with time value of money $\frac{d^{(2)}}{2} = \frac{i^{(2)}}{2} \times v^{0.5}$. Therefore $\frac{d^{(2)}}{2} < \frac{i^{(2)}}{2}$.

Part (i) was answered well.
In part (ii) candidates struggled to explain the time value of money.

QUESTION 2

i.

$$FV = \int_0^4 (140 + 4t) \exp\left(\int_t^8 (0.07 + 0.002s)ds\right) dt$$

where

$$\begin{aligned}
&\exp\left(\int_t^8 (0.07 + 0.002s)ds\right) \\
&= \exp(0.07s + 0.001s^2)\,|_t^8 \\
&= \exp(0.07(8 - t) + 0.001(8^2 - t^2)) \\
&= \exp(-0.07t - 0.001t^2 + 0.624)
\end{aligned}$$

$$\begin{aligned}
FV &= \int_0^4 (140 + 4t) \exp(-0.07t - 0.001t^2 + 0.624)\, dt \\
&= -2000 \times \exp\,(0.624) \int_0^4 (-0.07 - 0.002t) \exp(-0.07t - 0.001t^2)\, dt
\end{aligned}$$

$Let\ u = f(t) = -0.07t - 0.001t^2$

$\therefore du = (-0.07 - 0.002t)dt$

$0 \le t \le 4 \rightarrow 0 \le u \le -0.296$

$$\begin{aligned}
FV &= -2000 \times \exp(0.624) \int_0^{-0.296} \exp(u)\, du \\
&= -2000 \times \exp(0.624) \times [\exp\,(u)]_0^{-0.296} \\
&= -2000 \times \exp(0.624) \times (\exp(-0.296) - 1) \\
&= 956.379346 \rightarrow \text{R}956.40
\end{aligned}$$

Alternative approach for 2nd integral:

$f(t) = -0.07t - 0.001t^2$
$f'(t) = -0.07 - 0.002t$

$$\begin{aligned}
FV &= -2000 \times \exp(0.624) \int_0^4 (-0.07 - 0.002t) \exp(-0.07t - 0.001t^2)\, dt \\
&= -2000 \times \exp(0.624) \times [\exp(-0.07t - 0.001t^2)]_0^4 \\
&= -2000 \times \exp(0.0624) \times (\exp(-0.07 \times 4 - 0.001 \times 4^2) - 1) \\
&= -2000 \exp(0.624)\, (\exp(-0.296) - 1) \\
&= 956.379346
\end{aligned}$$

ii.

Total Interest equals Accumulated Value minus Total Paid

$$Total\ Interest\ = 956.379346 - \int_0^4 (140 + 4t)dt$$

$$Total\ Interest = 956.379346 - (140t + 2t^2)|_0^4 = 956.379346 - 592$$
$$= \text{R}364.40$$

The integral in part (i) has been covered extensively in the notes. However, candidates still struggled to deal with this kind of integral.

Common mistakes were:

- *Boundaries in the 2nd (inside) integral.*
- *Confusion between the two alternative methods of substitution for the integral. Candidates must choose a method and practise that method often.*

Part (ii) was often not attempted and when attempted a common mistake was to integrate the delta instead of the annual rate of the continuous cashflow. The annual rate of the continuous cashflow should be integrated to calculate the total amount of money received.

QUESTION 3

$$(I\bar{a})_{\overline{n|}} = \int_0^1 1v^t dt + \int_1^2 2v^t dt + \int_2^3 3v^t dt + \cdots + \int_{n-1}^n nv^t dt$$
$$= \int_0^1 1v^t dt + 2v\int_0^1 1v^t dt + 3v^2\int_0^1 1v^t dt + \cdots + nv^{n-1}\int_0^1 1v^t dt$$
$$= 1\bar{a}_{\overline{1|}} + 2v\bar{a}_{\overline{1|}} + 3v^2\bar{a}_{\overline{1|}} + \cdots + nv^{n-1}\bar{a}_{\overline{1|}}$$
$$= \bar{a}_{\overline{1|}}(1 + 2v + 3v^2 + \cdots + nv^{n-1})$$
$$= \bar{a}_{\overline{1|}}(I\ddot{a})_{\overline{n|}}$$
$$= \frac{1-v}{\delta}\left(\frac{\ddot{a}_{\overline{n|}} - nv^n}{d}\right)$$
$$= \frac{\ddot{a}_{\overline{n|}} - nv^n}{\delta}$$

where $1 - v = d$

Question 3 was the worst answered question in the paper with many candidates not even attempting the proof. This is a standard proof from the notes.

When candidates did attempt the question, many did the incorrect proof that also allowed for continuous increases.

Some candidates failed because they could not reproduce this theory from the notes.

QUESTION 4

i.

$(1+y_2)^2 \exp\left(3 \times F_{2,3}\right) = (1+y_5)^5$

$(1+0.055)^2\, exp\left(3 \times F_{2,3}\right) = (1+0.0625)^5$

$F_{2,3} = 0.065347191 \rightarrow 6.5347\%$

ii.

$1 = yc_n(1.05^{-1} + 1.055^{-2} + 1.0575^{-3} + 1.06^{-4} + 1.0625^{-5}) + 1(1.0625)^{-5}$

$yc_n = \dfrac{1 - 1.0625^{-5}}{4.227023323} = 0.061861931 \rightarrow 6.1862\%$

iii.

The gross redemption yield will be higher than the par yield.

The par yield gives the coupon rate at which the price of the bond equals the par value.

If a bond trades at a price lower than par and with coupons higher than the par yield, it follows that the gross redemption yield must be greater than the par yield.

Parts (i) and (ii) were answered well.

Part (iii) was not answered well, which is often the case with higher order questions. The relationship between the par yield and gross redemption yield is not well understood by candidates.

QUESTION 5

$$PV_L = 40{,}000a_{\overline{5|}}$$
$$= 40{,}000(4.067471638)$$
$$= 162{,}698.8655$$

$$PV_A = Xv^{\frac{8}{12}} + Yv^9$$
$$= 162{,}698.8655\ (PV_L = PV_A)(Eq.1)$$

$$v_L(numerator) = 40{,}000v(Ia)_{\overline{5|}}$$
$$= 40{,}000v(11.63046871)$$
$$= 433{,}568.2651$$

$$v_A(numerator) = X\left(\frac{8}{12}\right)v^{\frac{20}{12}} + Y(9)v^{10}$$
$$= 433{,}568.2651\ (v_L = v_A)(Eq.2)$$

Multiply Eq. 1 by 9v

$$9v(162{,}698.8655) = 9Xv^{\frac{20}{12}} + 9Yv^{10}$$

Subtract Eq. 2 from Eq. 1

$$9v(162{,}698.8655) - 433{,}568.2651 = \left(8\frac{4}{12}\right)Xv^{\frac{20}{12}}$$

$$X = 125{,}654.3221$$

Substitute into Eq. 1

$$(125{,}654.3221)v^{\frac{8}{12}} + Yv^9 = 162{,}698.8655$$

$$Y = 80{,}713.50007$$

Question 5 was one of the best answered questions in the paper.

As an alternative to volatility, the discounted mean term (DMT) could have been used.

Candidates must take care with notation. If only the numerator of the volatility or DMT is used, it must be labelled as such.

QUESTION 6

i.

Working in millions:

$$3\left(1+\frac{0.04}{4}\right)^{4\times 3} = 4Xa^{(4)}_{\overline{5}|@i^{(4)}=0.08} + v^{5}_{@i^{(4)}=0.08}\left(4X\, a^{(4)}_{\overline{7}|@i^{(4)}=0.07} + 0.005(Ia)_{\overline{7\times 4}|@\frac{i^{(4)}}{4}=\frac{0.07}{4}}\right)$$

where

$$a^{(4)}_{\overline{5}|@i^{(4)}=0.08} = 4.087858336$$
$$a^{(4)}_{\overline{7}|@i^{(4)}=0.07} = 5.49673684$$
$$(Ia)_{\overline{7\times 4}|@\frac{i^{(4)}}{4}=\frac{0.07}{4}} = 294.0191009$$
$$\ddot{a}_{\overline{7\times 4}|@\frac{i^{(4)}}{4}=\frac{0.07}{4}} = 22.37172644$$

so that

$$3.38047509 = 4X(4.087858336) + (0.672971333)\big(4X(5.49673684) + 0.005(294.0191009)\big)$$

$$\rightarrow X = \frac{2.391142959}{31.14801862} = 0.07676708m = R76{,}767.08$$

ii.

Working in millions:

$$L_{after\ 14th\ pmt} = 3.38047509\left(1+\frac{0.08}{4}\right)^{14} - 4(0.07676708)s^{(4)}_{\overline{3.5}|@i^{(4)}=0.08}$$

where

$$s^{(4)}_{\overline{3.5}|@i^{(4)}=0.08} = 3.993484538$$

$$\begin{aligned} L_{after\ 14th\ pmt} &= 4.46046509 - 1.2262726602 \\ &= R3.2341924307m \end{aligned}$$

$$\begin{aligned} Interest\ portion\ of\ 15th\ pmt &= R3.2341924307m \times \frac{0.08}{4} \\ &= 0.0646838486m \\ &= R64{,}683.85 \end{aligned}$$

$$\begin{aligned} Capital\ portion\ of\ 15th\ pmt &= 0.076767085m - 0.0646838486m \\ &= 0.01208324m \\ &= \text{R}12{,}083.24 \end{aligned}$$

The question was done reasonably well as the underlying theory is well understood.

However, this question contained many details to keep track of, this confused some candidates.

Even if the initial equation of value was incorrect, candidates could gain many marks by calculating their annuities and final answer correctly with the incorrect equation of value.

Common errors in part (i) were:

- *Using incorrect terms for the annuities.*
- *Not realising that the increases happened at each payment (thus quarterly) which simplified the question.*

QUESTION 7

i.

$$
\begin{aligned}
P &= 8 \times \frac{113}{115} \times \frac{109}{109(1.02)} \times (1.05)^{-1} \\
&\quad + 8 \times \frac{111}{115} \times \frac{109}{109(1.02^2)} \times (1.05)^{-2} \\
&\quad + 8 \times \frac{109}{115} \times \frac{109}{109(1.02^3)} \times (1.05)^{-3} \\
&\quad + 8 \times \frac{109(1.02)}{115} \times \frac{109}{109 \times 1.02^4} \times (1.05)^{-4} \\
&\quad + 8 \times \frac{109}{115(1.02)^3} \times (1.05)^{-5} \\
&\quad + \ \ldots \\
&\quad + 8 \times \frac{109}{115(1.02)^3} \times (1.05)^{-10} \\
&\quad +103 \times \frac{109}{115(1.02)^3} \times (1.05)^{-10}
\end{aligned}
$$

Annuity approach:

$P = 7.339747493 \ + 6.731877178 + \ 8 \times \frac{109}{115(1.02)^3} \times (1.05)^{-3} \times \ddot{a}_{\overline{8|}@d=0.047619047}$

$+ \ 56.47709843$

where $\ddot{a}_{\overline{8|}@d=0.047619047} = 6.786373397$

$P = 112.436548$

Alternative geometric approach:

$$P = 7.339747493 + 6.731877178 + 8 \times \frac{109}{115(1.02)^3} \times (1.05)^{-3} \times \left(\frac{(1-1.05^{-8})}{1-1.05^{-1}}\right) + 56.47709843$$

$$\left(\frac{(1-1.05^{-8})}{1-1.05^{-1}}\right) = 6.786373397$$

$$P = 112.436548$$

ii.

An index-linked bond with no lag will keep the real value of coupon payments at 8%.

but because of the decreasing inflation index (negative inflation rate) over 2022 to 2025 and stable positive inflation over the rest of the term the real coupons will be below 8%.

and hence the present value of the real coupons of 8% discounted at a 5% real rate will be lower than the present value of money coupons of 8% discounted at a 5% money rate.

Hence the price of the fixed interest bond would be higher.

This was one of the worst answered questions in the paper. This is a section of the work that candidates continue to struggle with.

Common errors were:

- *Assuming that the effect of the lag in the index will cancel out fully due to the later constant rate of increase in the inflation index and hence treating the problem like a fixed interest bond.*
- *Adding inflation to the coupons and redemption amounts incorrectly i.e. using incorrect index values and inverting the index values. This demonstrates a lack of understanding of money versus real cashflows and their connection to money and real rates in an equation of value.*

QUESTION 8

i.

Initial outflow of purchase price. Listed price observable at time of purchase. Hence timing and amount certain.

Regular dividend income. Only paid if declared, so timing uncertain. Amount dependent on future unknown profits, so amount uncertain.

Final inflow of sale price. Sale price only known at date of sale, and timing dependent on market movements. Amount and timing uncertain.

ii.

$$P = 2.4(0.6)\{(1.025 \times v^{0.75} + 1.025^{1.5} \times v^{1.25} + \cdots + 1.025^{4} \times v^{3.75}) \\ + 1.025^{4} \times v^{4}(1.05^{0.5} \times v^{0.25} + 1.05^{1} \times v^{0.75} + \cdots)\}$$

$$P = 2.4(0.6)\left\{1.025 \times v^{0.75} \times \frac{\left(1-\left(\frac{1.025}{1.11}\right)^{0.5\times 7}\right)}{1-\left(\frac{1.025}{1.11}\right)^{0.5}} \right.$$

$$\left. + 1.025^{4} \times v^{4.25} \times 1.05^{0.5} \times \frac{1}{1-\left(\frac{1.05}{1.11}\right)^{0.5}}\right\}(geom)$$

$$P = 2.4(0.6)\{1.025 \times v^{0.75} \times (6.23127145) + 1.025^{4} \times v \times^{4.25} \times 1.05^{0.5} \times (36.493205422)\}$$

$$P = 46.65031182 = R46.65$$

Alternative annuity approach

$$P = 2.4(0.6)\left\{1.025 \times v^{0.75} \times \left(2\ddot{a}^{(2)}_{\overline{3.5|}@j_1}\right) + 1.025^{4} \times v^{4.25} \times 1.05^{0.5} \times \left(2\ddot{a}^{(2)}_{\overline{\infty|}@j_2}\right)\right\}(annuity)$$

where

$$\begin{aligned} j_1 &= \frac{1.11}{1.025} - 1 \\ &= 0.082926829 \\ \ddot{a}^{(2)}_{\overline{3.5|}@j_1} &= 3.115635725 \\ j_2 &= \frac{1.11}{1.05} - 1 \\ &= 0.057142857 \\ 2\ddot{a}^{(2)}_{\overline{\infty|}@j_2} &= 36.49320542 \end{aligned}$$

iii.

The time period spans two years and ten months. In this time there are five dividends starting on 1 Jan 2023 and ending on 1 Jan 2025.

$$46.6503 = 2.4(0.6)\left\{1.025 \times v^{0.75} \times 2\ddot{a}^{(2)}_{\overline{2.5|}@j_1}\right\} + 45v^{2\frac{10}{12}}$$

<u>NO CGT</u>

Let $i = 0.04$

$$2\ddot{a}^{(2)}_{\overline{2.5|}@j_1} = \frac{\left(1-\left(\frac{1.025}{1.04}\right)^{0.5\times5}\right)}{1-\left(\frac{1.025}{1.04}\right)^{0.5}} = 4.928145$$

$$RHS = 7.063090962 + 40.2672 = 47.3302867$$

Let $i = 0.05$

$$2\ddot{a}^{(2)}_{\overline{2.5|}@j_1} = \frac{\left(1-\left(\frac{1.025}{1.05}\right)^{0.5\times5}\right)}{1-\left(\frac{1.025}{1.05}\right)^{0.5}} = 4.881661$$

$$RHS = 6.946435375 + 39.19008 = 46.13651685$$

$$\frac{i-0.04}{0.05-0.04} = \frac{46.6503-47.3303}{46.1365-47.3303}$$

$$IRR = 0.045696$$

iv.

Tax on dividend income will now be deferred (i.e. paid later), so the tax outflows will be discounted over longer time periods and hence carry a smaller weight in the present value expression.

If the discount rate remains unchanged the net present value of cashflows will increase.

Hence to make the net present value equal the purchase price, the discount rate must increase, and hence the IRR will be higher than if taxes are paid immediately with every cash inflow.

Part (i) was done reasonably well.

In part (ii) the underlying theory was well understood.

However, this question contains many details to keep track of and requires careful dealing with timing. Students struggled to construct the first principles equation for the cashflows of a share purchased at a non-dividend date.

Again, if the initial equation of value was incorrect candidates could gain many marks by calculating their annuities and final answer correctly and consistently with the incorrect equation of value.

A common error in part (ii) was to apply the growth to the dividends yearly instead of half-yearly.

In part (iv), many candidates struggled to explain that deferring a cash outflow reduces its present value in magnitude and therefore has a positive effect on the net present value (NPV), all else being equal. They also struggled to relate this to the internal rate of return (IRR), where such a rate exists, by not connecting this to the fact that the IRR is the discount rate at which the NPV is zero.