

Living too long for your living annuity: Exploring dynamic drawdown strategies

By Kyle Hulett, FFA, FASSA, CFA and Anton Swanepoel, CFA

Presented at the Actuarial Society of South Africa's 2024 Convention
Cape Town International Convention Centre, 20–21 November 2024

ABSTRACT

Living annuities are a well-supported product offering in South Africa, with the Association for Savings and Investment South Africa (ASISA) estimating assets under management of R626bn at the end of 2023 (ASISA, 2023). Living annuities are versatile but are riskier than other guaranteed annuity options, because the annuitant takes on the investment and longevity risk. Six of seven reviewed papers concluded that living annuities are inferior to traditional annuities. The studies may not be appropriate, however, as they used living annuities with static drawdowns, where the drawdown path is set at retirement; using dynamic drawdowns allows the path to be changed during retirement in response to market conditions. This paper explores how to improve the retirement outcomes of living annuity users, investigating in a systematic manner the overall outcome of a static drawdown relative to dynamically varying annuity drawdown rates. It compares different dynamic drawdown methods, considering a variety of drawdown rates, and different dynamic methods, triggers and thresholds. Lastly, it identifies optimal asset allocations for South African investors in two forward-looking economic scenarios for different levels of drawdowns.

KEYWORDS

Dynamic drawdown strategies; living annuities; maximising periodic payouts; post-retirement spending needs

CONTACT DETAILS

Mr Kyle Hulett, Sygnia Asset Management, Cape Town; Email: khulett@sygnia.co.za
Mr Anton Swanepoel, Sygnia Asset Management, Cape Town; Email: aswanepoel@sygnia.co.za

1. INTRODUCTION

1.1 The South African annuity market

1.1.1 South Africans typically choose between a guaranteed (non-profit) life annuity, a (guaranteed) with-profit life annuity, a living annuity or a hybrid of these when they retire.

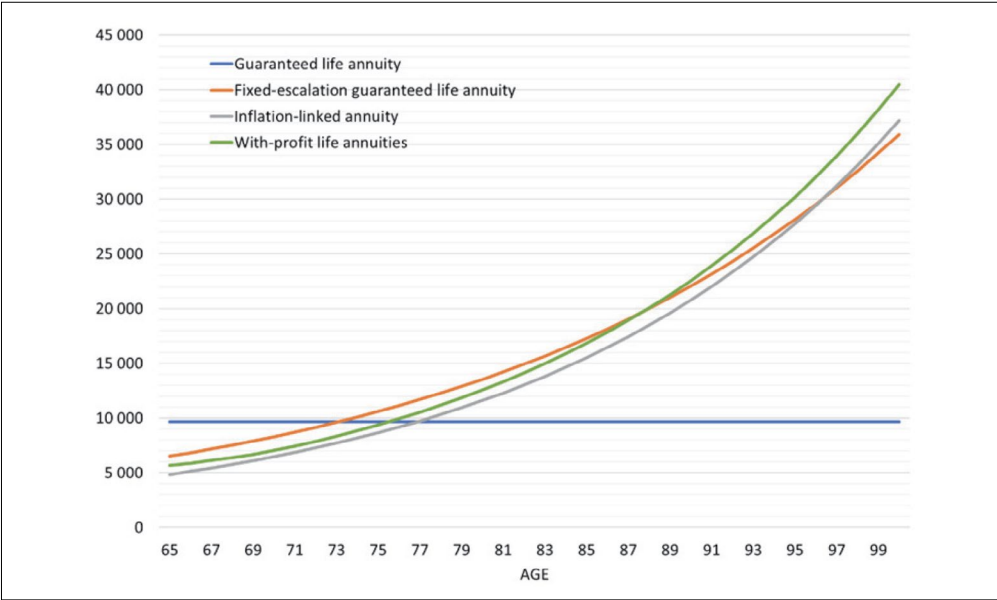


FIGURE 1. Payout profile over time of different South African annuity products
(Source: Sygnia/Just SA, February 2024)

1.1.2 Guaranteed annuities usually provide retirees with a level payout for the duration of their lives, and the insurer bears the longevity and investment risk. The downside for the retiree is that the annuity does not increase with inflation and no money can be left to heirs. Fixed escalation and inflation-linked options are available but have much lower starting points than the level-guaranteed annuities. For example, a 5% fixed escalation annuity typically has a starting value 30% lower than a level guaranteed annuity, while an inflation-linked starting point is about 50% lower. Fixed escalating annuities have recently been increasing in popularity, while inflation-linked annuities remain hardly used. In 2019, 78% of guaranteed annuities were level, but this has dropped to 52% in 2024, with escalating annuities now making up 46%. Inflation-linked guaranteed annuities are still

too expensive and have very little market share at both time points (Chartered Financial Analyst Society of South Africa (CFASSA), 2024).

1.1.3 With-profit annuities invest in a riskier asset mix to provide a stable income and a bonus in good years, so the annual payout increases to a new minimum to alleviate some inflationary pressures. The insurer takes on some of the investment risk and charges the client a capital guarantee charge, typically around 1% per annum. The starting point in our pricing analysis was 45% lower than a level annuity (pricing is from Just SA on their StableGro in February 2024, as per Figure 1). With the current high interest rates and steep yield curve in South Africa, guaranteed non-profit life annuities look attractive relative to with-profit annuities. However, with-profit annuities can be invested in equities and global stocks as a hedge against a weak rand. The rand may weaken faster than the inflation differentials or purchasing power parity (PPP) might suggest for long periods of time, meaning that the fixed escalation and inflation-linked solutions are unable to keep up with dollar returns. This is because sentiment, growth and productivity influence currencies as well as inflation differentials. Over the last 20 years, the Organisation for Economic Co-operation and Development's PPP for South Africa has weakened by 109%, while the rand has weakened by 220%. It is noted that the rand could enter a period of relative strength if commodities rally and reforms are enacted.

1.1.4 Living annuities give retirees more flexibility and control over their retirement investment and income—they can vary their drawdown between 2.5% and 17.5% of their capital to meet changing needs (alterable once a year)—and any excess savings can be left to their heirs. Furthermore, their asset allocation is not limited and can be fully offshore or in equities. Given South Africa's high-inflation environment and structurally weaker economy relative to developed markets, the freedom of asset allocation lends itself to living annuities. Income for life is not guaranteed in these funds, so there is a risk that annuitants may outlive their savings. Perhaps as a result of higher bond yields or demand for higher drawdown rates, or possibly an awareness of ongoing advice fees in living annuities, life annuities have been gaining market share from living annuities. In 2015, 90% of retirement savings were going into living annuities, but by 2023 living annuities had fallen to 56% of new business (CFASSA, 2024).

1.1.5 Investors can switch from a living annuity into a life annuity but not the other way around. A hybrid annuity is a life annuity held inside a living annuity, providing the combined benefits of a guaranteed income for life (longevity protection), the flexibility of a living annuity and the ability to bequeath any remaining capital at death.

1.2 Living annuity drawdown selection

1.2.1 Retirees with living annuities often consult financial advisors to determine their optimal level of drawdown (income) and asset allocation based on their budget and capital. Retirees generally have the following goals for their retirement investment (Guyton & Klinger, 2006):

- maximise income withdrawals,
- protect purchasing power from inflation, and
- avoid undesired fluctuations in that income.

1.2.2 Meeting these goals simultaneously is difficult, but the liability side of the equation can help the overall solution. Saving more or retiring later—and consequently building up a larger capital position at retirement—can help to meet these retirement needs.

1.2.3 Looking purely at investment risk, maintaining purchasing power generally requires exposure to real assets such as equities, which have high levels of volatility. To achieve lower fluctuations in income, one must either invest in less-risky assets and give up potential returns or invest in risky assets and pay a life insurer or bank for protection (thereby also giving up some returns). Looking at the underlying investments of annuities, investing in less-risky assets is similar to the guaranteed non-profit annuity options, while investing in risky assets with a protection payment would be akin to the guaranteed with-profit annuity. Living annuities are often seen as too risky, with too high a risk of income running out, but it may be possible to improve living annuity outcomes if the client is prepared to adapt their behaviour in response to market movements and accept fluctuations in income.

1.3 Sustainable static drawdown rates

1.3.1 ASISA 2023 data indicate that the average drawdown rate in the South African retirement industry is 6.7%.

1.3.2 Maré (2016) has shown that in South Africa, static drawdown rates above 6% result in less than 60% success rates for moderate equity holdings and suggests that even a 5% withdrawal rate is only sustainable over 15 years or less. Similarly, Van Appel et al. (2021) found that static withdrawal rates in excess of 5% are not sustainable over a 30-year period, regardless of the asset allocation and, similarly, that asset allocation has a negligible influence on the success of the portfolio for lower withdrawal rates of 3% and 4%.

1.3.3 The forced selling of capital in fallen markets can permanently erode the spending power of retirement assets. For example, if equities fall for the first five years, the capital base will be depleted by drawdowns and will struggle to recover with the market, making it likely that an annuitant will quickly outlive their savings. This is known as sequencing risk.

1.3.4 Higher static drawdown percentages encounter a matching problem. For example, a low drawdown rate of 3% can be matched by the yield on growth assets such as equities. Assuming dividends are not cut, the asset class thus provides the annual cash flow required and grows with inflation despite market volatility. For higher yield requirements such as 7%, however, the annual drawdown can be met with the addition of nominal assets that provide higher yields, such as cash and bonds—but these do not grow with inflation.

Higher returns are needed to sustain higher withdrawal rates, requiring a higher percentage of low-yielding risky growth assets. However, these assets provide insufficient yield, so capital must be sold more regularly to cover drawdown needs, putting more capital at the mercy of market fluctuations.

1.4 Living annuities vs life annuities

1.4.1 Given the living annuities' asset liability mismatch for higher drawdown rates, it is no surprise that they are not the preferred annuity option. Of the seven papers we reviewed, only De Villiers-Strijdom (2013) concluded that living annuities are superior to composite annuities (guaranteed and living annuities), which in turn outperform guaranteed annuities. Four papers concluded that guaranteed and with-profit annuities are better than living annuities:

- Goemans & Ncube (2008) found that with-profit and escalating annuities outperform living annuity strategies.
- Lodhia & Swanepoel (2012) found that for members of at least average health, guaranteed annuities are better than living annuities. Living annuitants could only match the income provided by an inflation-linked annuity until the age of 75, at which point the drawdown cap is reached and income starts to decrease.
- Theron (2013) concluded that life annuities outperform living annuities in all cases.
- Bester (2016) concluded that living annuities should be used only as an additional source of income on top of a guaranteed annuity or other form of income.

1.4.2 Two papers recognised that the asset liability mismatch impacts living annuities at higher drawdown rates but does not impact them at low drawdown rates, i.e. living annuities are better suited to individuals with a buffer between desired (comfort) income and necessity income.

- Levitan et al. (2009) found that a living annuity was the superior option when tested against a *comfort* level of income, whereas the inflation-linked annuity performed best when measured against a *necessities* level of income.
- Butler et al. (2013) found that living annuities were only preferred using ruin theory at the comfort level of income and one of the discounted utility measures at necessity level. Level and escalating guaranteed annuities were preferred using ruin theory at necessity level and two of the three discounted utility functions at both the comfort and necessity level.

1.4.3 Living annuities with static percentage drawdowns (increased with inflation every year regardless of market conditions) certainly have their limitations. Scott et al. (2009) are critical of static drawdowns from an inherently volatile portfolio. Similarly, comparing living annuities that have static drawdowns to with-profit annuities or combination products that have a guaranteed annuity and with-profit annuity is not

an effective comparison. With-profit annuities are really a form of dynamic drawdown: the declared bonus is highly variable and may at times be zero—which in real terms is a negative move.

1.4.4 While it would be worth running a comparison of life annuities against living annuities with dynamic drawdowns, the present analysis is limited to whether dynamic drawdowns can improve the outcome of *living annuities*.

2. BUILDING THE ASSET MODEL

2.1 Asset classes

2.1.1 We forecast data to:

- investigate the impact of economic environment and asset allocation on the expected and variation of time to ruin for various drawdown levels, and
- identify ways to actively manage the drawdown rate to enable a higher allocation to higher-risk asset classes (equities), thereby generating higher returns while reducing the impact of sequencing risk.

2.1.2 The choice between buying a guaranteed annuity or living annuity depends greatly on the expected excess return of equities over bonds (Loeys & Wise, 2024). As such, and for simplicity, three South African asset classes were used to build the portfolios:

- risk-free asset class (cash),
- income asset class (bonds), and
- growth asset class (equities).

2.1.3 Because stability is important and bonds and cash are used primarily as an income source, we focused only on South African (SA) bonds and SA cash. Equities are naturally volatile, so adding foreign exchange volatility is tolerable and possibly even welcome to diversify growth risks. We thus also look at global equity performance when setting our long-term return forecasts, as, while we model SA equities, SA and global equities could be grouped.

2.1.4 In reality, the living annuity investment could be far more diversified through many more growth asset classes, including real estate and private equity. The asset classes could also be dynamically managed or have hedge overlays, which would improve diversification and reduce the portfolio's tail risk—and thus to some extent counter the limitation of the assumed normal distribution in our simulation model. However, a more advanced investment portfolio is beyond the scope of this paper.

2.1.5 The starting point for our 30-year forward-looking asset model is the historical performance of the last 30 years. Using current market valuations, Butler et al. (2015) showed that the Shiller cyclically adjusted price-to-earnings ratio (which takes the 10-year earnings average) assists forward return assumptions in the United States of America (US). That said, 30 years is approximately three or four economic cycles, and

equity valuations do naturally fluctuate widely within a cycle. So while we started with the historical 30-year returns and covariance matrix (30 June 1994 to 30 June 2024), we took the following into account when adjusting for our forward-looking numbers:

- Inflation targeting has had a major impact on real bond and cash returns since it began in the early 1990s. It was adopted first in New Zealand, Canada and the United Kingdom, with the US unofficially adopting it in the late 1990s. South Africa only started to use it in 2000. Thus, a time period shorter than 30 years may be appropriate.
- 2020 marked the end of a 40-year interest rate decline. The US 10-year interest rate peaked in 1981 at 16% and fell to 0.5% in 2020. The US dollar is the reserve currency of the world, with 58.4% of central banks' reserves in US dollars in 2023. Dollar interest rates thus impact all asset prices directly or indirectly and, as interest rates are used to discount present value cash flows, they impact the valuation of all assets. Interest rates are bound by zero and are more likely to be higher than lower over the next 30 years, for similar reasons that growth will be lower—namely debt, demographics, deglobalisation and decarbonisation.

2.1.6 To expand, the headwinds to growth and bond yields are:

- **Debt** Falling interest rates over the last 40 years encouraged consumers and governments to take on debt and spend their future earnings, thereby boosting growth. Debt-to-GDP levels have grown to historic highs, however, lowering the amount of growth going forward as the interest burden reduces spending power. Government debt is expected to reach a record \$56 trillion in 2024 (Bloomberg, 2024). The Congressional Budget Office (CBO) estimates that, relative to a stable debt scenario, current rising US government debt trajectories will slow income growth by 12% over the next three decades and could slow income growth by 33% if a rapidly rising debt scenario occurs (CBO, 2024). Rising fiscal debt and deficits are likely to put upward pressure on real bond yields due to additional debt issuance and/or inflationary concerns caused by the higher fiscal spend (Wise, 2023). Fiscal reform is a risk to this expectation.
- **Demographics** A shrinking labour force and increasing dependency ratio as a result of ageing populations reduces potential GDP growth and profit margins (Berezin, 2019). The impact on interest rates is mixed. Longer life expectancies mean that people work longer and accumulate more savings during their lifetimes, placing downward pressure on real interest rates. The slowing working population growth rate requires less investment in infrastructure, similarly pushing rates lower. However, the rising dependency ratio means that an increasing proportion of the population is not of working age, reducing their savings and applying upward pressure to real interest rates (Peta, 2024).
- **Deglobalisation** Supply chains focusing on national security and/or national growth over efficiency will raise inflation and reduce earnings. Shifting patterns

- of trade could put upward pressure on real yields. De-risking supply chains, on-shoring and friend-shoring all come at an additional cost and require additional investments. Geopolitical tension driven by power competition is likely to persist or even intensify. More trade wars with escalating tariffs would further reduce the efficiency of supply chains, pushing up costs and lowering earnings.
- **Decarbonisation/climate change** Correctly pricing environmental costs into services/products will similarly raise costs and reduce earnings. Mitigating climate change will require substantial investment and will be compounded by rebuild costs after an increase in destructive climate events. This additional demand for investment will push up interest rates (Wise, 2023).
 - These negative impacts may to some extent be balanced by an improvement in productivity from artificial intelligence. In addition, the declining price of capital goods and the increased spend by consumers on services relative to goods will see a reduction in investment, which would support lower bond yields (Peta, 2024).

These issues are not specifically included in our adjustments, but they do support the lower return and at least the stable bond yield environment going forward—a more stagflationary environment.

2.1.7 We thus focus on when the US 10-year interest rate was last at the 30 June 2024 level (4.3%), which was in mid-2007—a primary lookback period of 17 years. This covers a period of stable interest rates (point to point) rather than falling interest rates over the last 30 years. Over the last 17 years, US economic growth, profit growth and equity market returns have been exceptional and have outperformed the rest of the world, driven by the global (and unique) success of US technology companies. As such, while we review the MSCI World Index returns, we also look at MSCI World ex US index returns.

TABLE 1. Historic linear returns, volatility and covariance matrix as at June 2024 in ZAR terms, over the previous 17-year and 30 year periods (Source: Refinitiv)

17 years							30 years					
	SWIX	ALBI	STeFI	MSCI ex US	MSCI World	SA CPI	SWIX	ALBI	STeFI	MSCI ex US	MSCI World	SA CPI
12-m return	10.2%	8.7%	6.9%	9.9%	13.9%	5.6%	15.0%	11.7%	9.2%	12.5%	15.3%	5.7%
Volatility	16.1%	8.5%	0.6%	15.9%	16.8%	1.5%	20.4%	9.5%	1.1%	17.9%	18.6%	1.7%
Covariance table	SWIX	ALBI	STeFI				SWIX	ALBI	STeFI			
SWIX	3.9%	0.6%	0.0%				2.8%	0.6%	0.0%			
ALBI	0.6%	0.9%	0.0%				0.6%	0.8%	0.0%			
STeFI	0.0%	0.0%	0.0%				0.0%	0.0%	0.0%			

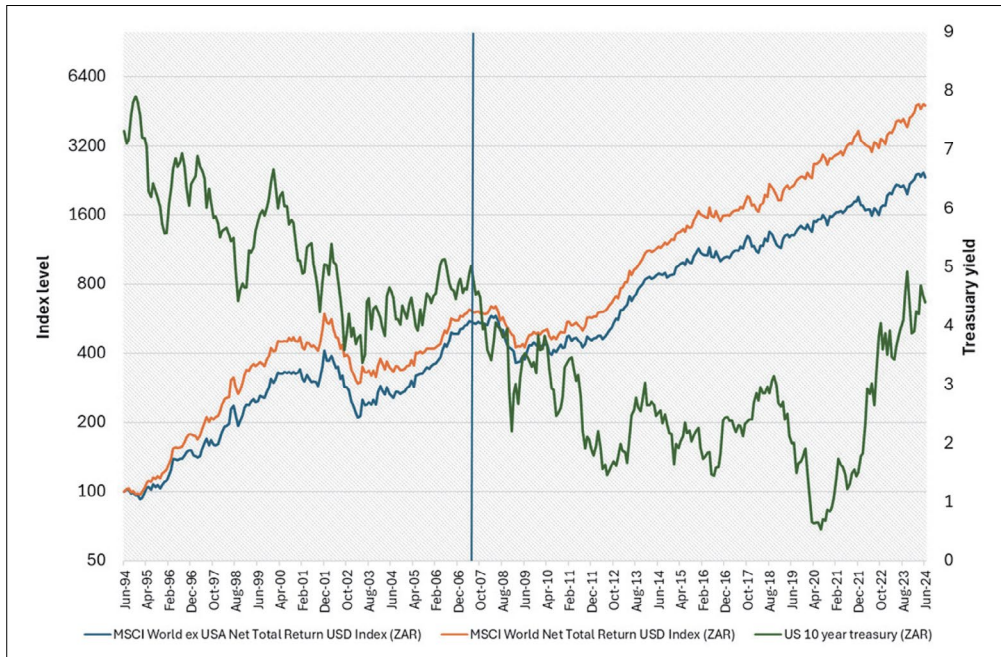


FIGURE 2. US yields and world equity returns over the last 30 years, as at July 2024
(Source: Bloomberg)

Where:

- SWIX – FTSE/JSE Shareholder Weighted Total Return Index (ZAR)
- ALBI – FTSE/ JSE All Bond Composite Total Return Index (ZAR)
- STeFI – SAFE South Africa Short-Term Fixed Interest Rate – Composite Index (ZAR)

2.1.8 We note:

- 17-year average returns and volatility are lower than for the 30-year period.
- Over the last 17 years, SA equity performance (represented by SWIX) and MSCI World ex US performance are very similar.
- MSCI World performance was 40% higher than MSCI World ex US over the last 17 years.
- Inflation in SA is roughly the same over both periods.

2.1.9 Two economic scenarios were modelled to measure the robustness of our dynamic drawdown models in a high and low return environment. Our base case low return environment is modelled on the past 17 years, while our secondary higher return scenario uses average returns that are approximately equal to one-third of the 30-year period and two-thirds of the 17-year period (using MSCI World ex US as a proxy for

equity returns). Historical volatilities from the two time periods for SA assets were used. The scaling down of the return on the high return scenario reduces the Sharpe ratio for that scenario.

2.1.10 Low return scenario (inflation rate 5.5%)

- Cash: 7% (1.5% real)
- Bonds: 8.5% (3% real)
- Equities: 10% (4.5% real)

2.1.11 High return scenario (inflation rate 5.5%)

- Cash: 7.5% (2% real)
- Bonds: 9.5% (4% real)
- Equities: 11.5% (6% real)

Any reasonable set of return assumptions could be used; for this analysis, the important factor is that the assumptions are consistent with the dynamic drawdown process.

2.2 Generating optimum portfolios

2.2.1 We estimated two sets of forward returns, one for each of the two scenarios—low return and high return—and for each of the asset classes—SA equities, SA cash and SA bonds.

2.2.2 To determine the optimal portfolios for the analysis, we started with the historical monthly total returns for the three asset classes in each time frame and converted them to monthly logarithmic returns. Using these monthly log returns, we constructed two separate covariance matrixes for both scenarios' chosen sample timelines. These were multiplied by 12 to scale to the correct time frame. The scaled 12-month log estimates were then converted into linear estimates by adjusting the log returns by a factor of their volatility and taking the exponent of the sum. Consider an asset whose price we denote P_t . Logarithmic returns $r_{t,\tau}$ at time t with horizon τ are defined in such a way that

$$P_{t+\tau} = e^{r_{t,\tau}} P_t$$

$$r_{t,\tau} = \ln \left(\frac{P_{t+\tau}}{P_t} \right)$$

2.2.3 The Markowitz efficient frontier is formulated in linear returns and not log returns. Using logarithm returns leads to suboptimal portfolios, particularly over long time horizons (Meucci, 2001). In this review, the portfolios were annually rebalanced to the target weight, so the effective optimisation time horizon is 12 months despite the 30-year term. The 12-month strategy was selected as it better reflects reality (where the annuitant reviews and rebalances their portfolio annually).

2.2.4 With the forward-looking linear 12-month return and covariance calculated, we used an optimisation to find the efficient frontier of the optimal portfolios. We calculated the portfolio return and volatility for every possible combination of the three asset classes, with asset allocation increments set at 2.5%. Twenty optimal portfolios were selected for each of the scenarios, as that number scaled well with the increment size while giving sufficient granularity in results. For ease of reference, we kept the number of portfolios consistent across the low return and high return scenarios. The optimal portfolios were determined by selecting the portfolio with the lowest volatility in each bucket. These represented every portfolio on the efficient frontier line.

TABLE 2. Mean-variance optimal portfolios for the low return market scenario

	Return	Volatility	SWIX	ALBI	STeFI
1	7.0%	0.5%	0.0%	0.0%	100.0%
2	7.2%	0.8%	2.5%	5.0%	92.5%
3	7.3%	1.3%	5.0%	10.0%	85.0%
4	7.5%	1.9%	7.5%	15.0%	77.5%
5	7.6%	2.5%	12.5%	15.0%	72.5%
6	7.8%	3.1%	15.0%	20.0%	65.0%
7	7.9%	3.7%	17.5%	25.0%	57.5%
8	8.1%	4.6%	20.0%	35.0%	45.0%
9	8.3%	5.2%	22.5%	40.0%	37.5%
10	8.4%	5.9%	27.5%	40.0%	32.5%
11	8.6%	6.5%	30.0%	45.0%	25.0%
12	8.7%	7.1%	32.5%	50.0%	17.5%
13	8.9%	7.7%	35.0%	55.0%	10.0%
14	9.1%	8.5%	37.5%	62.5%	0.0%
15	9.2%	9.2%	47.5%	52.5%	0.0%
16	9.4%	10.0%	57.5%	42.5%	0.0%
17	9.5%	11.0%	67.5%	32.5%	0.0%
18	9.7%	12.0%	77.5%	22.5%	0.0%
19	9.8%	13.1%	87.5%	12.5%	0.0%
20	10.0%	14.5%	100.0%	0.0%	0.0%

Source: Sygnia

TABLE 3. Mean-variance optimal portfolios for the high return market scenario

	Return	Volatility	SWIX	ALBI	STeFI
1	7.5%	1.0%	0.0%	0.0%	100.0%
2	7.7%	1.2%	2.5%	5.0%	92.5%
3	7.9%	1.7%	5.0%	10.0%	85.0%
4	8.1%	2.3%	7.5%	15.0%	77.5%
5	8.3%	3.0%	10.0%	20.0%	70.0%
6	8.6%	3.9%	12.5%	27.5%	60.0%
7	8.8%	4.6%	15.0%	32.5%	52.5%
8	9.0%	5.3%	17.5%	37.5%	45.0%
9	9.2%	6.0%	20.0%	42.5%	37.5%
10	9.4%	6.7%	22.5%	47.5%	30.0%
11	9.6%	7.6%	25.0%	55.0%	20.0%
12	9.8%	8.3%	27.5%	60.0%	12.5%
13	10.0%	9.0%	30.0%	65.0%	5.0%
14	10.2%	9.8%	35.0%	65.0%	0.0%
15	10.4%	10.7%	45.0%	55.0%	0.0%
16	10.7%	12.0%	57.5%	42.5%	0.0%
17	10.9%	13.2%	67.5%	32.5%	0.0%
18	11.1%	14.5%	77.5%	22.5%	0.0%
19	11.3%	15.9%	87.5%	12.5%	0.0%
20	11.5%	17.6%	100.0%	0.0%	0.0%

Source: Sygnia

2.3 Forecasting asset returns

2.3.1 Once the efficient frontier had been created and we had determined the optimal asset allocations, we generated 1 000 sets of 30-year return time series for the three

asset classes, which were then combined with the set of optimal portfolio weights to create equivalent portfolio return time series.

2.3.2 We used multivariate geometric Brownian motion (GBM) and Euler’s method (as per Jones, 2019) to model and forecast asset prices. GBM assumes that the logarithm of the asset price follows geometric Brownian motion, so the increments in the log prices are independent and identically normally distributed. As increments in the log price are equal to the log returns, GBM effectively assumes the log returns are identically normally distributed.

2.3.3 To include the correlation, the CoRand function was used in Excel from the SIMSTOOLS add-in published by Roger Myerson at the University of Chicago. This links the random probabilities for each equity and bond price change. This generated correlated normal variables to represent the monthly log returns for bonds and equities; correlation was not taken into account for cash estimates. The process generated 360 monthly returns and was repeated 1 000 times, with the simulation results in line with expectations. Table 4 shows the results for the base line scenario. Figure 3 shows 255 of the 1 000 sample paths generated by GBM for the equity asset class.

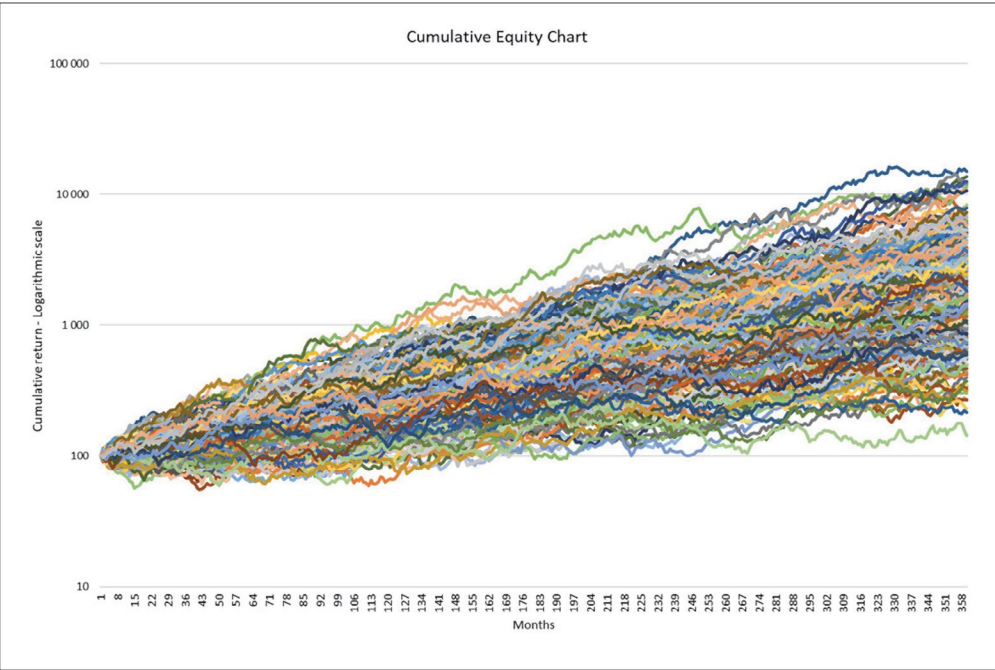


FIGURE 3. Simulated equity return paths generated using inputs from Table 1
(Source: Sygnia)

TABLE 4. Input parameters and simulation results for three asset classes, using low returns scenario

Correlation			
Target	0.35		
Average simulation	0.35		
	Cash	Bonds	Equity
Target return	7.00%	8.50%	10.00%
Average CAGR	6.98%	8.54%	10.28%
Median CAGR	6.99%	8.48%	10.21%
Minimum CAGR	6.63%	2.82%	−0.11%
Maximum CAGR	7.40%	14.02%	18.85%
	Cash	Bonds	Equity
Target std dev	0.57%	8.46%	16.08%
Average annualised std dev	0.57%	8.44%	16.09%
Median annualised std dev	0.57%	8.44%	16.08%
Minimum annualised std dev	0.50%	7.46%	13.95%
Maximum annualised std dev	0.64%	9.39%	18.08%
Number of return sets	1 000	1 000	1 000

Source: Sygnia

Where CAGR = Compound annual growth rate

3. BUILDING THE LIABILITY MODEL

3.1 Assumptions

3.1.1 On the liability side, the following assumptions were used to represent the annuitant:

- Annual rebalance of portfolio to target asset allocation
- Payout to annuitant reviewed annually and increased with inflation (where applicable for the dynamic drawdown methods)
- Monthly payment in arrears
- No tax on capital growth or interest income (withdrawn income is taxed in the hands of the individual and is beyond the scope of this analysis)
- No fees (asset returns are net of fees)
- No longevity risk (each liability term is 30 years)—South African conditional median life expectancy, given age 60, is 76 years for both sexes (Maré, 2016). While the average is 16 years, we have modelled 30 years to provide a wider analysis
- Models were built for 3%, 5%, 7% and 9% drawdowns.

3.1.2 This represents a typical client process, in which the financial advisor meets with the client annually and reviews the drawdown and asset allocation. There is

often sufficient cash in the cash account to meet the next year's drawdowns, depending on the withdrawal rate, as cash is generated during the year via interest, bond coupons and equity dividends.

3.2 Combining the asset and liability models to examine sequencing risk/time to ruin

3.2.1 The return series from the 20 optimal asset allocation portfolios per scenario was applied to the drawdown requirements (3% to 9%) to investigate the time to ruin.

3.2.2 The first analytical tool used to analyse the success of an asset allocation for a particular drawdown rate is the average months to ruin (AMTR). The model has a maximum time frame of 30 years or 360 months, so any of the 1 000 scenarios in which the capital does not run out will reflect 360 months. AMTR gives the average of the months to ruin for each of the 1 000 scenarios.

3.2.3 While this establishes how well the asset allocation meets the drawdown on average, the key concern for many individuals is the worst-case scenario. If an investor's default is a guaranteed annuity, where their pension never runs out, understanding the worst possible scenario (within the limits of the assumed normal distribution) is key. As such, we focus on minimum months to ruin (MMTR) at the 1% threshold. This takes the worst 1% case—that is, of the 1 000 scenarios, the ten shortest times the capital lasted. The 1% can be varied according to the client's risk threshold: some clients may be comfortable with a 5% threshold.

3.2.4 The final analysis tool is net present value. This is useful because some dynamic drawdown methods are designed to ensure that the retirement capital never runs out, which would provide a good MMTR—but it is not a good solution if the drawdown is two cents on the rand in 30 years' time. Net present value is the sum of all drawdowns across each of the scenarios, including any terminal value at 360 months, discounted back at the inflation rate of 5.5%, and is used relative to the static drawdown method as a comparison.

4. STATIC DRAWDOWN ANALYSIS

4.1 Results

4.1.1 We start the results with the analysis of the time to ruin for the static drawdown across the 3%, 5%, 7% and 9% drawdowns, the high and low return scenarios and the various asset allocation portfolios.

- The left block of Table 5 shows the AMTR for each level of asset allocation and drawdown. Here we identify the asset allocation portfolio with the best (highest) AMTR.
- The AMTR table shows that the optimal required blend of risky assets increases as the drawdown rate increases. As the required return increases with higher

- drawdown rates, so the optimal solution becomes a portfolio with a higher expected return (hence the downward-sloping green-shaded area in the left-hand table).
- The block on the right shows the MMTR, which is the worst-case outcome at the 1% level. The variation in time to ruin widens as the allocation to equities and bonds is increased. As a result, the MMTR is worse for the higher-equity solutions and worse for higher drawdowns.
 - The high return scenario shows a similar pattern but with slightly higher AMTR and slightly worse MMTR, which is expected due to the lower Sharpe ratio of the higher return scenario.

TABLE 5. Average months to ruin and minimum months to ruin at 1% level
for each simulated drawdown rate

Low returns		Average months to ruin			
Equity weight		3%	5%	7%	9%
0.0%	360	301	206	157	
2.5%	360	310	210	159	
5.0%	360	320	214	162	
7.5%	360	329	219	165	
10.0%	360	335	225	168	
12.5%	360	338	231	171	
15.0%	360	340	237	174	
17.5%	360	342	246	180	
20.0%	360	342	252	183	
22.5%	360	342	259	188	
25.0%	360	342	263	192	
27.5%	360	342	268	197	
30.0%	360	341	271	201	
35.0%	360	341	274	204	
45.0%	360	340	277	210	
57.5%	359	339	279	214	
67.5%	359	337	279	218	
77.5%	358	335	279	221	
87.5%	357	332	278	222	
100.0%	356	328	275	223	

Low returns		Minimum months to ruin at 1%			
Equity weight		3%	5%	7%	9%
0.0%	360	288	199	153	
2.5%	360	291	199	152	
5.0%	360	285	196	149	
7.5%	360	278	191	146	
10.0%	360	274	188	144	
12.5%	360	267	183	141	
15.0%	360	260	179	139	
17.5%	360	249	172	133	
20.0%	360	243	168	130	
22.5%	360	235	165	128	
25.0%	360	229	161	125	
27.5%	360	223	157	122	
30.0%	360	217	154	119	
35.0%	360	212	151	116	
45.0%	360	204	147	114	
57.5%	346	196	139	111	
67.5%	328	187	132	106	
77.5%	299	172	127	100	
87.5%	277	164	121	96	
100.0%	251	153	113	91	

Source: Sygnia

4.1.2 Higher drawdowns thus require higher returns and lower volatility to avoid running out of capital. On average, one can receive higher returns by investing in riskier assets, but the variation in returns also increases. This is by design in our asset model. When returns are below expectations, higher drawdowns result in capital depletion at the wrong time, permanently impacting a portfolio’s spending power and resulting in a worse outcome than not having risky assets.

4.1.3 A retirement industry risk—and possible reason for a high 6.7% living annuity drawdown level—is that investors who need the highest returns are likely to opt for living annuities with a risky asset mix, as they cannot afford the lower starting income

of a with-profit annuity or inflation-linked life annuity. With a static drawdown, however, this is likely to have poor outcomes.

4.1.4 One way to combat the worst-case scenario (MMTR) in the living annuity is for the annuitant to dynamically manage their drawdown, lowering their spending when markets are down to create a type of self-insured with-profit annuity.

5. DYNAMIC ADJUSTED DRAWDOWN RATES

5.1 Dynamic drawdowns

5.1.1 Scott et al. (2009) are critical of static drawdowns from an inherently volatile portfolio. We investigated methods in which clients take on higher-growth assets, which are more volatile, and dynamically manage their drawdown rates to manage the volatility. The clients take on more market risk, giving them a higher probability of achieving their results in the long term on average, with the caveat that if the market underperforms in the short term, the client will have to accept a lower income to avoid depleting capital.

5.1.2 This is effectively a type of self-insured with-profit annuity. The client saves the capital charge (cost of the guarantee, which is approximately 1% a year) and instead takes on the cost of ongoing financial advice and some flexibility in their annual income with the overall outcome better by the cost differentials, and a potentially higher return from being able to take on a more aggressive asset allocation.

5.1.3 We kept the methods simple, with only annual changes to drawdown to reflect when a person might see an advisor or realistically be able to update their drawdowns. A more frequent review, for example twice a year, would likely lead to improved results, but we leave this for future research. We reviewed two broad model methods, both of which require forecasted market returns.

5.1.4 Developing assumed returns is vital to the process. Firstly, it forces retirees to engage with a realistic drawdown rate relative to the set of market return assumptions. Secondly, it allows the client to systematically change their behaviour based on market cycles, enabling them to take on a higher proportion of risk assets and receive a better pension. The anticipated capital base projection is determined by factoring in assumed returns, drawdown rate, inflation rate and asset allocation for each scenario. As no volatility is factored into these calculations, it is feasible to precisely forecast how many months' planned drawdowns are sustainable until the portfolio is exhausted. All scenarios commence with a capital balance of R1 million. While assumed returns remain constant throughout a simulation, the drawdown amount incrementally increases with inflation adjustment. Consequently, the estimated projected capital base for most scenarios experiences a gradual rise, peaking when the monthly drawdown amount relative to the portfolio size matches the deannualised weighted average return of the portfolio. After this peak, the estimated capital base undergoes a sharp decline until it reaches zero. As drawdown amounts are only adjusted for inflation annually, and only the drawdown *amounts* (not the drawdown *rate*) are subject to inflation adjustment, a portfolio could

hypothetically never exhaust its funds following this principle if the drawdown amount grows at a slower pace than the portfolio.

5.2 Dynamic drawdown method 1: Annual annuity adjustment

5.2.1 The first dynamic drawdown method relies on the work of Waring & Siegal (2015: 91), who suggest that if investors recalculate their drawdown rate each year based on market conditions, they “will experience consumption that fluctuates with asset values, but they can never run out of money”.

5.2.2 The annual annuity adjustment method works as follows: The market value of available capital is reviewed using the assumed market return assumptions for each asset class, and the optimal amount that could be paid annually from then until the end of the analysis is calculated (maximum 30-year period or end of estimated period if drawdown is too high). When the annual drawdown (from the annual annuity method) is above the estimated amount at the annual review, the annual drawdown is increased with inflation. However, when low capital values cause the annual annuity adjustment method to calculate a drawdown amount lower than the estimated value, the annual annuity adjustment value is used. Simply put, when a payout is less than expected, the client works backwards from their end date, dividing their capital equally across the remaining years to ensure it will last.

$$\text{Adjusted drawdown} = \frac{\text{PV} \left((1+R)^{\frac{1}{12}} - 1 \right)}{1 - \left((1+R)^{\frac{1}{12}} - 1 \right)^{-n}}$$

PV = Current market value of capital

R = Assumed annual growth rate (weighted return based on optimised asset allocation split and assumed asset class returns)

n = Months until ruin, based on initial total expected months, adjusted for months transpired

5.2.3 In essence, this is like the investor buying a guaranteed level annuity every year, but without the guarantee, and the income is only valid for a year before it is recalculated. The upside to this method is that the annuitant will never run out of money; the downside is that the income they receive could become effectively meaningless, as the payment could drop as low as, for example, two cents on the rand. The other downside is that the drawdown can be very volatile as the capital fluctuates.

5.2.4 Figure 4 shows the ideal planned monthly income drawdown profile in orange; the static drawdown in blue is a scenario in which the money does not last until the expected date. The green line depicts the annual annuity adjustment drawdown, which lasts to the expected date but is volatile, with very low relative payouts at times.

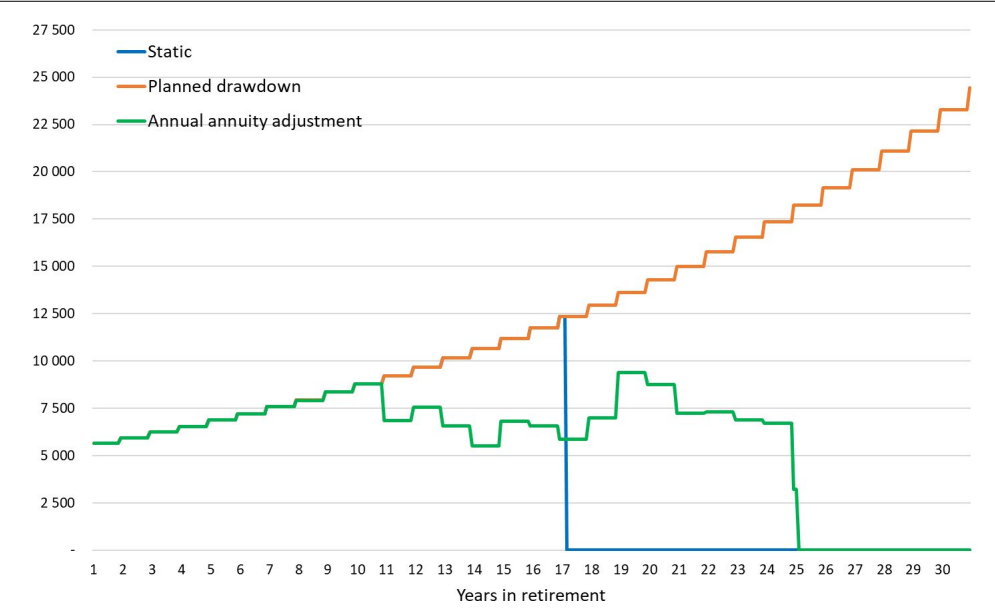


FIGURE 4. Monthly income profile over time of planned drawdowns versus static and annual annuity adjustment drawdowns (Source: Sygnia)

5.2.5 The AMTR and MMTR are meaningless in this method, as this capital will always last until the expected term. We thus focus on the net present value (NPV) of total drawdowns and terminal value at 360 months (where applicable), discounted at the inflation rate. (See Table 6.)

5.2.6 The median NPV shows that the annually corrected method increases the NPV relative to a static drawdown, but only at higher equity allocations and drawdowns. At the highest drawdown and equity allocation, the NPV increased by 2.05%, while the average increase was 0.06%, 0.61% and 0.84% for the 5%, 7% and 9% drawdowns respectively. Thus, while the average and minimum number of months improves to the target, the client is not significantly better off using this method. While the NPV improves, so does the complexity and volatility of the drawdowns, and the annual payout may become meaningless.

5.2.7 Similar results were experienced with the high return scenario, with an average increase of 2.0% to NPV median using the annually corrected method.

5.3 Dynamic drawdown method 2: Payout based on capital value relative to estimate

5.3.1 The second dynamic drawdown method creates a dynamic drawdown trigger based on the market value of the annuitant’s capital relative to the estimated projected capital base.

TABLE 6. Net present value median comparison of annually corrected method versus static drawdown across all simulated portfolios and drawdowns

Low returns	Net present value median							
	Annually corrected		Static drawdown		Annually corrected		Static drawdown	
Equity weight	3% drawdown		5% drawdown		7% drawdown		9% drawdown	
0.0%	R1 312 341	R1 312 341	R1 185 871	R1 186 210	R1 128 177	R1 132 431	R1 088 983	R1 101 788
2.5%	R1 370 965	R1 370 965	R1 225 790	R1 225 790	R1 154 499	R1 154 499	R1 115 980	R1 115 980
5.0%	R1 430 805	R1 430 805	R1 261 387	R1 261 387	R1 176 178	R1 176 178	R1 137 031	R1 137 031
7.5%	R1 495 256	R1 495 256	R1 300 317	R1 300 793	R1 201 388	R1 203 610	R1 150 909	R1 150 909
12.5%	R1 565 727	R1 565 727	R1 344 783	R1 348 080	R1 225 580	R1 225 580	R1 171 496	R1 171 496
15.0%	R1 631 462	R1 631 462	R1 390 188	R1 391 399	R1 252 497	R1 252 497	R1 188 366	R1 185 440
17.5%	R1 699 370	R1 699 370	R1 431 659	R1 431 220	R1 279 529	R1 274 764	R1 216 729	R1 206 680
20.0%	R1 812 956	R1 812 956	R1 499 581	R1 499 581	R1 312 786	R1 312 786	R1 234 561	R1 234 561
22.5%	R1 889 898	R1 889 898	R1 553 208	R1 553 208	R1 343 425	R1 340 388	R1 255 147	R1 255 147
27.5%	R1 964 408	R1 964 408	R1 602 637	R1 602 637	R1 379 513	R1 367 668	R1 277 342	R1 276 203
30.0%	R2 045 737	R2 045 737	R1 657 604	R1 657 604	R1 395 697	R1 397 706	R1 306 322	R1 297 348
32.5%	R2 132 816	R2 132 816	R1 708 910	R1 708 000	R1 433 135	R1 427 885	R1 336 379	R1 311 289
35.0%	R2 218 003	R2 218 003	R1 764 185	R1 764 185	R1 470 110	R1 449 371	R1 363 706	R1 328 536
37.5%	R2 311 479	R2 311 479	R1 810 101	R1 808 157	R1 476 857	R1 471 638	R1 341 480	R1 338 799
47.5%	R2 411 319	R2 411 319	R1 861 377	R1 861 377	R1 505 189	R1 496 214	R1 347 813	R1 345 600
57.5%	R2 507 217	R2 507 217	R1 934 124	R1 927 427	R1 535 671	R1 517 798	R1 381 591	R1 363 395
67.5%	R2 573 963	R2 573 963	R1 970 867	R1 970 867	R1 560 401	R1 537 262	R1 396 472	R1 366 935
77.5%	R2 636 798	R2 636 798	R2 001 497	R2 001 497	R1 575 080	R1 548 247	R1 415 433	R1 370 459
87.5%	R2 663 993	R2 663 993	R2 026 979	R2 020 585	R1 599 319	R1 564 530	R1 394 940	R1 373 983
100.0%	R2 725 034	R2 725 034	R2 036 519	R2 022 046	R1 601 429	R1 569 925	R1 394 940	R1 366 935

% difference between annually corrected and static					
Equity weight	3% drawdown	5% drawdown	7% drawdown	9% drawdown	
0.0%	0.00%	-0.03%	-0.38%	-0.25%	
2.5%	0.00%	0.00%	0.00%	0.00%	
5.0%	0.00%	0.00%	0.00%	0.00%	
7.5%	0.00%	-0.04%	-0.18%	0.00%	
12.5%	0.00%	-0.24%	0.00%	0.00%	
15.0%	0.00%	-0.09%	0.00%	0.25%	
17.5%	0.00%	0.03%	0.37%	0.83%	
20.0%	0.00%	0.00%	0.00%	0.00%	
22.5%	0.00%	0.00%	0.23%	0.00%	
27.5%	0.00%	0.00%	0.87%	0.09%	
30.0%	0.00%	0.00%	-0.14%	0.69%	
32.5%	0.00%	0.05%	0.37%	1.91%	
35.0%	0.00%	0.00%	1.43%	2.65%	
37.5%	0.00%	0.11%	0.35%	0.20%	
47.5%	0.00%	0.00%	0.60%	0.16%	
57.5%	0.00%	0.35%	1.18%	1.33%	
67.5%	0.00%	0.00%	1.51%	2.16%	
77.5%	0.00%	0.00%	1.73%	3.28%	
87.5%	0.00%	0.32%	2.22%	1.53%	
100.0%	0.00%	0.72%	2.01%	2.05%	
Average	0.00%	0.06%	0.61%	0.84%	

Source: Sygnia

5.3.2 Markets, like the economies they are based on, are cyclical, and it is this cyclicity that the dynamic capital method attempts to harness. When markets turn down and capital value decreases, prudent spending will help the capital recover faster until the economy recovers, without permanent damage to the capital values and spending ability of the client. Linking this understanding directly to return estimates can offer the investor clarity into what is required for them to retire, and with what timing.

5.3.3 If the market value of the annuitant’s portfolio is above that based on the return assumptions, then the annual income is increased with inflation at the annual review as per the static drawdown method. However, if the capital has not grown to meet the assumed returns, a variety of options can be applied to dynamically reduce the drawdown.

5.3.4 In calculating the dynamic drawdown method, we need two additional sets of variables:

- **Trigger:** When to reduce the drawdown, and
- **Haircut:** How much to reduce the drawdown by.

5.3.5 For the trigger, we initially considered using standard deviations of returns relative to estimate as a threshold (e.g. one standard deviation). This would effectively delay the process for longer for more volatile portfolios before cutting the payout, so the client would accept more volatility. But there is good reason to say that the opposite is true—that the more volatile the investment, the sooner we should protect capital and cut the payout. We instead decided that static thresholds of 0% and 10% and 20% below the estimate value were better.

- The 0% threshold is the first trigger. As capital falls below that required, the first threshold is triggered.
- The 10% threshold is halfway between the first and third thresholds. The S&P 500 has historically fallen more than 10% in one out of every two years over the last 20 years (Schwab, 2022), with an average drawdown of 15%.
- The 20% threshold is linked to the market fall that defines a recession.

5.3.6 There are three haircut options, each matched to a trigger. This allows clients to choose how much pain they want to take:

- The first haircut is no inflationary increase (linked to the 0% threshold)—it is not a reduction in nominal income but a loss in real income. The client will not receive an inflationary increase on their annual drawdown if the market value is less than the estimate. Inflationary increases resume when the market value catches up to the estimated value, but the payments do not catch up retroactively. This is the easiest haircut for the client to accept but is expected to preserve capital the least. This is similar to a zero bonus in a with-profit annuity.
- The second haircut is a 5% cut to annual income and occurs when the second threshold is met, when the market value is less than 90% of the estimate. Inflationary increases kick in again when the market value catches up to the estimated value.
- The third haircut is an additional 5% cut to annual income (taking it to a cumulative 10% cut). This occurs if the market value is less than 80% of the estimate. Inflationary increases start again when the market value catches up to the estimated value. Anything more than a 10% haircut is felt to be unsustainable by the client, and the options start to resemble the annual annuity adjustment method.

Figure 5 shows the actual value of the client's portfolio relative to the estimated value under each threshold.

5.3.7 Figure 6 shows the annual payout for each method. When returns are better than the expected average, dynamic drawdowns are not needed. Dynamic drawdowns are only needed when returns are worse than expected. Figure 6 thus illustrates when returns are less than expected and the static drawdown runs out before the expected date. The payout increases with inflation until the capital falls below the estimate and the payouts

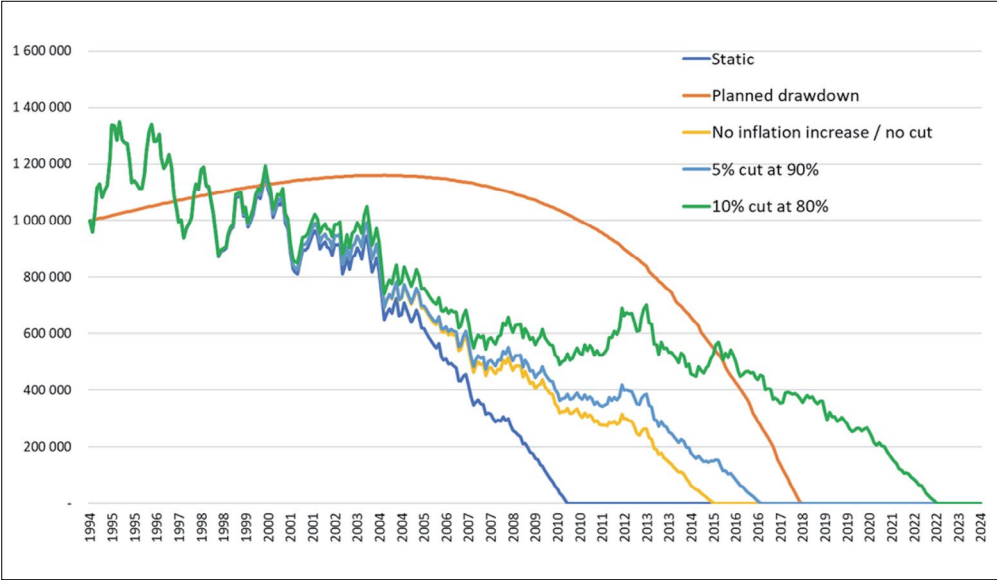


FIGURE 5. Living annuity asset value over 30 years using the dynamic drawdown method (1994–2024) (Source: Sygnia)

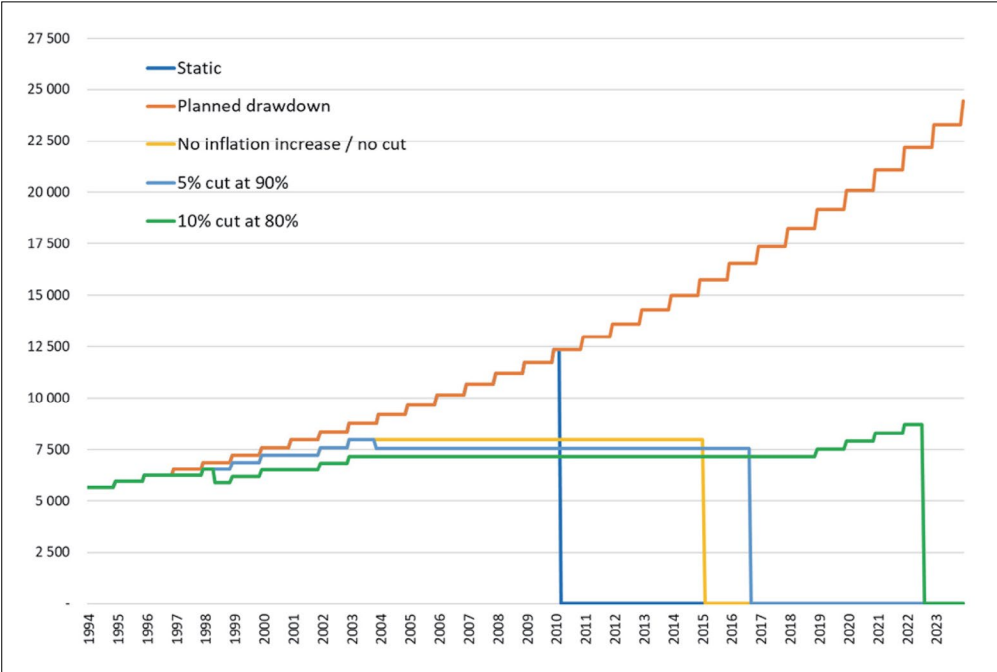


FIGURE 6. Monthly income profile for 5% drawdown using dynamic drawdowns (1994–2024) (Source: Sygnia)

become level or cut. The static drawdowns continue with inflationary increases until the capital runs out (blue line). Then the first dynamic drawdown option of no inflationary increases runs out, followed by the 5% haircut and finally the 10% haircut.

5.3.8 The yellow, light blue and green haircut lines remain flat from 2004/5 onwards, rather than continuing to be cut in consecutive years. This is because the maximum cut is 10%, and the payout stays flat unless the portfolio value catches up to the estimated value, which happens in Figure 6 in 2019.

5.3.9 Figure 6 is just one example, using one optimal portfolio for one drawdown rate.

5.4 Dynamic drawdown capital method: Detailed analysis

5.4.1 A detailed results analysis of the 1 000 simulations was performed for each of the four drawdown levels in each of the two scenarios. This allowed us to identify the asset allocation and dynamic threshold that allow the optimal outcome. “Optimal” is here defined as:

- having a higher median NPV and AMTR than the static method;
- while not allowing the worst case 1% MMTR to be worse than the static method.

When results are “close”, the investor will prefer smaller/no haircuts to larger haircuts. This is subjective but speaks directly to points 1 and 3 of the retiree goals defined in Section 1.2.

5.5 Low return analysis: 3% drawdown

5.5.1 The dynamic drawdown method at the 3% drawdown improves the median NPV by between 5.02% and 5.45% on average relative to the static method.

5.5.2 The AMTR also improves, but only marginally, as the 3% drawdown lasts the full 30 years on most asset allocations with the static drawdown.

5.5.3 The dynamic drawdown method allows a more aggressive asset allocation without sacrificing MMTR. Only the first trigger is required (no inflation increases—reflected as “no cut” on the figures), which is the same as a zero bonus in a with-profit annuity. This sacrifice in income stability allows a higher equity allocation of 100% (relative to 37.5% in the static drawdown), allowing a higher NPV while still meeting the MMTR 30-year payout period. While the 10% cut does allow a marginal increase in NPV over no cut, it does not allow a higher risk asset allocation, and the marginal increase in NPV is not worth the additional 10% income variability.

5.5.4 Figure 7 shows that with dynamic drawdowns, one could achieve a 27.7% higher NPV by investing in 100% equities (instead of 37.5% equities) while maintaining the 360-month MMTR by using the “no inflation increase” haircut.

5.5.5 For the high return scenario, the median NPV can be increased by 31% while maintaining the 360-month MMTR by using the 100% equity asset allocation.

TABLE 7. Median NPVs of the dynamic drawdown methods relative to static method for 3% drawdown (Source: Sygnia)

Low returns								
3% drawdown		% difference from static						
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 312 341	R1 330 143	R1 329 575	R1 329 575	1.36%	1.31%	1.31%	
2.5%	R1 370 965	R1 392 810	R1 392 884	R1 392 884	1.59%	1.60%	1.60%	
5.0%	R1 430 805	R1 462 814	R1 461 152	R1 461 152	2.24%	2.12%	2.12%	
7.5%	R1 495 256	R1 533 368	R1 532 425	R1 532 425	2.55%	2.49%	2.49%	
12.5%	R1 565 727	R1 616 506	R1 616 820	R1 616 820	3.24%	3.26%	3.26%	
15.0%	R1 631 462	R1 692 108	R1 694 522	R1 693 588	3.72%	3.87%	3.81%	
17.5%	R1 699 370	R1 771 728	R1 773 246	R1 771 293	4.26%	4.35%	4.23%	
20.0%	R1 812 956	R1 890 796	R1 899 036	R1 898 124	4.29%	4.75%	4.70%	
22.5%	R1 889 898	R1 977 700	R1 987 483	R1 987 275	4.85%	5.16%	5.15%	
27.5%	R1 964 408	R2 068 612	R2 077 513	R2 072 841	5.30%	5.76%	5.52%	
30.0%	R2 045 737	R2 155 596	R2 170 518	R2 166 775	5.37%	6.10%	5.92%	
32.5%	R2 132 816	R2 249 006	R2 259 362	R2 258 111	5.45%	5.93%	5.87%	
35.0%	R2 218 003	R2 335 741	R2 356 288	R2 351 009	5.31%	6.23%	6.00%	
37.5%	R2 311 479	R2 444 526	R2 459 487	R2 452 108	5.76%	6.40%	6.08%	
47.5%	R2 411 319	R2 563 054	R2 576 270	R2 571 758	6.29%	6.84%	6.65%	
57.5%	R2 507 217	R2 683 589	R2 686 172	R2 680 994	7.03%	7.14%	6.93%	
67.5%	R2 573 963	R2 771 317	R2 787 035	R2 793 877	7.67%	8.28%	8.54%	
77.5%	R2 636 798	R2 815 022	R2 843 862	R2 850 280	6.76%	7.85%	8.10%	
87.5%	R2 663 993	R2 910 027	R2 942 769	R2 961 872	9.24%	10.46%	11.18%	
100.0%	R2 725 034	R2 950 774	R2 973 359	R2 986 665	8.28%	9.11%	9.60%	
Average	R2 019 977	R2 130 762	R2 140 989	R2 141 471	5.02%	5.45%	5.45%	

TABLE 8. Average months to ruin and minimum months to ruin at 1% level for 3% drawdown rate (Source: Sygnia)

Low returns		Average months to ruin				Minimum months to ruin at 1%			
3% drawdown									
Equity weight	Static	No inflation	5% cut	10% cut		Static	No inflation	5% cut	10% cut
0.0%	360	360	360	360		360	360	360	360
2.5%	360	360	360	360		360	360	360	360
5.0%	360	360	360	360		360	360	360	360
7.5%	360	360	360	360		360	360	360	360
12.5%	360	360	360	360		360	360	360	360
15.0%	360	360	360	360		360	360	360	360
17.5%	360	360	360	360		360	360	360	360
20.0%	360	360	360	360		360	360	360	360
22.5%	360	360	360	360		360	360	360	360
27.5%	360	360	360	360		360	360	360	360
30.0%	360	360	360	360		360	360	360	360
32.5%	360	360	360	360		360	360	360	360
35.0%	360	360	360	360		360	360	360	360
37.5%	360	360	360	360		360	360	360	360
47.5%	360	360	360	360		360	360	360	360
57.5%	359	360	360	360		346	360	360	360
67.5%	359	360	360	360		328	360	360	360
77.5%	358	360	360	360		299	360	360	360
87.5%	357	360	360	360		277	360	360	360
100.0%	356	360	360	360		251	360	360	360

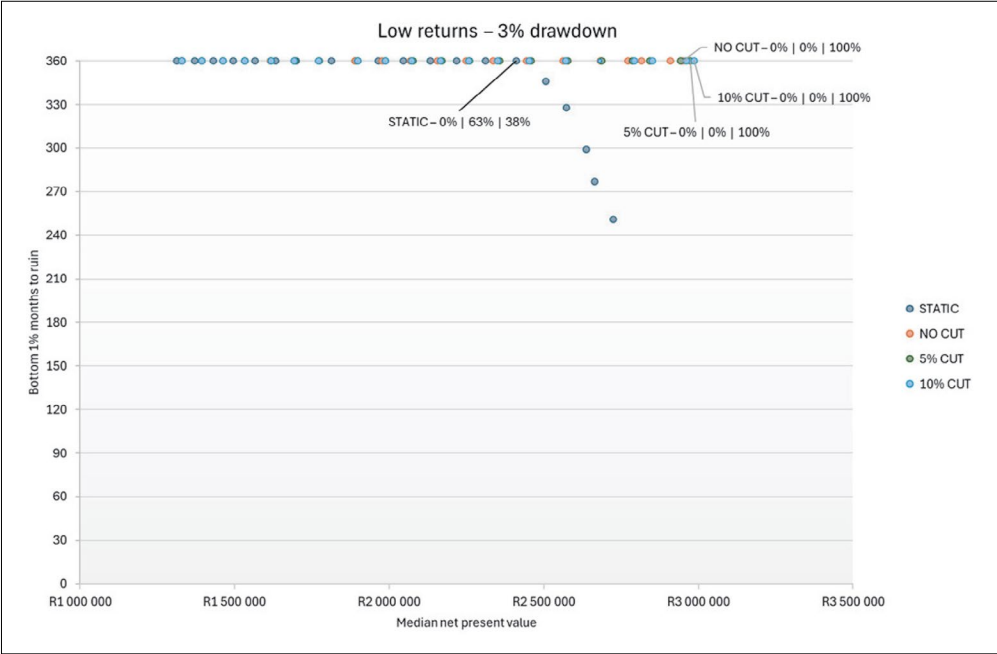


FIGURE 7. Scatter plot for low returns—3% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

5.6 Low return analysis: 5% drawdown

5.6.1 For the 5% drawdown, the NPV median improves on average between 9.62% and 10.38% per drawdown threshold, with the increase coming with higher equity allocation and higher drawdown thresholds.

5.6.2 AMTR improves too, with average months rising across the board per portfolio. The greatest number of months in the AMTR are in all three dynamic drawdown methods across the 15% to 67.5% equity range. The 360-month average can be achieved with only the first trigger, the no inflationary increase method.

5.6.3 With a static drawdown, the best MMTR is achieved with the optimal portfolio holding 2.5% equity, which lasts for 291 months. Adding a 5% cut option to the dynamic drawdown allows 77.5% of equity to be held and extends the MMTR to 360 months, an increase of 69 months with an increase in NPV of 93.0%. Moving to the 10% cut drawdown threshold does not allow more equity to be added and only increases the NPV marginally, so the 5% cut is likely to be the preferred dynamic option.

5.6.4 For the high return scenario, the MMTR can be increased by 57 months, and the NPV is 147% higher using the 67.5% equity asset allocation with the 5% haircut.

TABLE 9. Median NPVs of the dynamic drawdown methods relative to static method for 5% drawdown (Source: Sygnia)

Low returns								
5% drawdown		% difference from static						
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 186 210	R1 206 264	R1 206 221	R1 206 221	1.69%	1.69%	1.69%	
2.5%	R1 225 790	R1 249 300	R1 248 704	R1 248 704	1.92%	1.87%	1.87%	
5.0%	R1 261 387	R1 300 356	R1 298 609	R1 298 609	3.09%	2.95%	2.95%	
7.5%	R1 300 793	R1 350 780	R1 350 292	R1 350 292	3.84%	3.81%	3.81%	
12.5%	R1 348 080	R1 409 882	R1 408 451	R1 408 451	4.58%	4.48%	4.48%	
15.0%	R1 391 399	R1 466 142	R1 465 673	R1 463 784	5.37%	5.34%	5.20%	
17.5%	R1 431 220	R1 524 406	R1 525 609	R1 522 079	6.51%	6.59%	6.35%	
20.0%	R1 499 581	R1 611 918	R1 619 671	R1 614 841	7.49%	8.01%	7.69%	
22.5%	R1 553 208	R1 675 646	R1 681 924	R1 678 634	7.88%	8.29%	8.08%	
27.5%	R1 602 637	R1 748 805	R1 763 986	R1 754 811	9.12%	10.07%	9.50%	
30.0%	R1 657 604	R1 815 951	R1 831 892	R1 827 067	9.55%	10.51%	10.22%	
32.5%	R1 708 000	R1 885 343	R1 903 784	R1 899 462	10.38%	11.46%	11.21%	
35.0%	R1 764 185	R1 958 473	R1 985 675	R1 977 076	11.01%	12.55%	12.07%	
37.5%	R1 808 157	R2 033 841	R2 053 742	R2 053 658	12.48%	13.58%	13.58%	
47.5%	R1 861 377	R2 119 404	R2 131 148	R2 141 355	13.86%	14.49%	15.04%	
57.5%	R1 927 427	R2 210 343	R2 224 212	R2 213 815	14.68%	15.40%	14.86%	
67.5%	R1 970 867	R2 259 924	R2 285 339	R2 292 027	14.67%	15.96%	16.30%	
77.5%	R2 001 497	R2 332 434	R2 366 435	R2 377 560	16.53%	18.23%	18.79%	
87.5%	R2 020 585	R2 368 555	R2 411 262	R2 445 394	17.22%	19.33%	21.02%	
100.0%	R2 022 046	R2 438 134	R2 471 347	R2 484 217	20.58%	22.22%	22.86%	
Average	R1 627 103	R1 798 295	R1 811 699	R1 812 903	9.62%	10.34%	10.38%	

TABLE 10. Average months to ruin and minimum months to ruin at 1% level for 5% drawdown rate (Source: Sygnia)

Low returns		Average months to ruin				Minimum months to ruin at 1%			
5% drawdown									
Equity weight	Static	No inflation	5% cut	10% cut		Static	No inflation	5% cut	10% cut
0.0%	301	331	332	332		288	302	302	302
2.5%	310	341	342	342		291	309	309	309
5.0%	320	351	352	352		285	317	317	317
7.5%	329	357	357	357		278	326	326	326
12.5%	335	359	359	359		274	335	335	335
15.0%	338	360	360	360		267	346	346	346
17.5%	340	360	360	360		260	357	358	358
20.0%	342	360	360	360		249	360	360	360
22.5%	342	360	360	360		243	360	360	360
27.5%	342	360	360	360		235	360	360	360
30.0%	342	360	360	360		229	360	360	360
32.5%	342	360	360	360		223	360	360	360
35.0%	341	360	360	360		217	360	360	360
37.5%	341	360	360	360		212	360	360	360
47.5%	340	360	360	360		204	360	360	360
57.5%	339	360	360	360		196	360	360	360
67.5%	337	359	360	360		187	360	360	360
77.5%	335	359	359	360		172	339	360	360
87.5%	332	358	359	359		164	284	318	345
100.0%	328	357	358	358		153	247	270	298

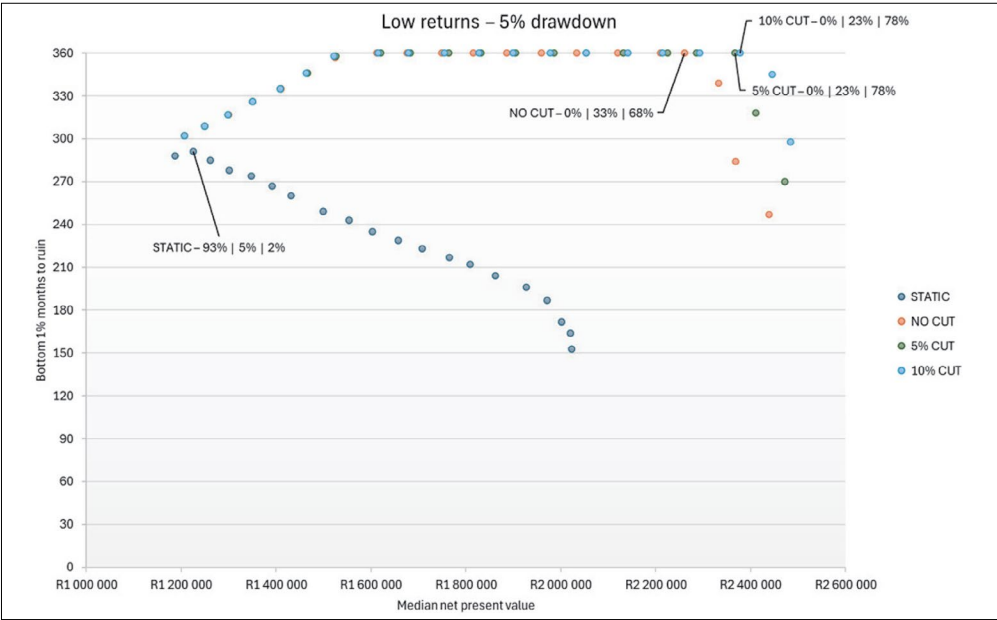


FIGURE 8. Scatter plot for low returns—5% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

5.7 Low return analysis: 7% drawdown

5.7.1 The NPV median similarly improves by between 7.53% and 8.99%, but with far more reliance on the second and third dynamic triggers at the higher equity levels.

5.7.2 The AMTR is significantly improved using higher equity allocations and higher threshold triggers. On average, the highest trigger and highest equity offer the best outcome.

5.7.3 Looking at the worst-case MMTR scenario, the 2.5% equity optimal portfolio results in the best static drawdown, but this can be improved from 199 months to 256 months by allocating 37.5% equity and enacting the third trigger (10% cut). Given the higher asset–liability mismatch (higher drawdown requirement a higher allocation to volatile risky assets, which results in capital depletion in bad times), it is notable that the optimal solution is constrained to less equity than the 5% drawdown. The best-case increase in the number of months is 57 months with the NPV increasing 41.3% at the 10% haircut trigger.

5.7.4 Figure 9 shows the peak in MMTR at 37.5% equity with a 10% haircut. This allows for an increase of 57 months for the worst-case scenario while increasing the NPV by 41%.

5.7.5 For the high return scenario, the MMTR can be increased by 89 months, and the NPV is 72% higher using the 30% equity asset allocation with the 10% cut rule. It

TABLE 11. Median NPVs of the dynamic drawdown methods relative to static method for 7% drawdown (Source: Sygnia)

Low returns								
7% drawdown		% difference from static						
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 132 431	R1 137 985	R1 137 985	R1 137 985	0.49%	0.49%	0.49%	
2.5%	R1 154 499	R1 162 930	R1 163 489	R1 163 489	0.73%	0.78%	0.78%	
5.0%	R1 176 178	R1 192 476	R1 192 445	R1 192 445	1.39%	1.38%	1.38%	
7.5%	R1 203 610	R1 222 774	R1 222 709	R1 222 709	1.59%	1.59%	1.59%	
12.5%	R1 225 580	R1 255 286	R1 255 791	R1 255 090	2.42%	2.47%	2.41%	
15.0%	R1 252 497	R1 286 946	R1 289 096	R1 287 071	2.75%	2.92%	2.76%	
17.5%	R1 274 764	R1 319 583	R1 320 681	R1 319 274	3.52%	3.60%	3.49%	
20.0%	R1 312 786	R1 367 668	R1 372 785	R1 370 458	4.18%	4.57%	4.39%	
22.5%	R1 340 388	R1 406 013	R1 413 607	R1 409 292	4.90%	5.46%	5.14%	
27.5%	R1 367 668	R1 442 407	R1 458 348	R1 450 247	5.46%	6.63%	6.04%	
30.0%	R1 397 706	R1 482 410	R1 496 023	R1 494 090	6.06%	7.03%	6.90%	
32.5%	R1 427 885	R1 524 328	R1 542 977	R1 542 500	6.75%	8.06%	8.03%	
35.0%	R1 449 371	R1 567 594	R1 593 722	R1 595 918	8.16%	9.96%	10.11%	
37.5%	R1 471 638	R1 605 307	R1 632 533	R1 631 304	9.08%	10.93%	10.85%	
47.5%	R1 496 214	R1 663 126	R1 694 607	R1 704 460	11.16%	13.26%	13.92%	
57.5%	R1 517 798	R1 712 912	R1 758 578	R1 769 124	12.86%	15.86%	16.56%	
67.5%	R1 537 262	R1 773 394	R1 825 086	R1 824 544	15.36%	18.72%	18.69%	
77.5%	R1 548 247	R1 811 346	R1 840 498	R1 856 616	16.99%	18.88%	19.92%	
87.5%	R1 564 530	R1 852 111	R1 902 301	R1 913 248	18.38%	21.59%	22.29%	
100.0%	R1 569 925	R1 856 836	R1 926 186	R1 949 205	18.28%	22.69%	24.16%	
Average	R1 371 049	R1 482 172	R1 501 972	R1 504 453	7.53%	8.84%	8.99%	

TABLE 12. Average months to ruin and minimum months to ruin at 1% level for 7% drawdown rate (Source: Sygnia)

Low returns		Average months to ruin				Minimum months to ruin at 1%			
7% drawdown									
Equity weight	Static	No inflation	5% cut	10% cut		Static	No inflation	5% cut	10% cut
0.0%	206	221	221	221		199	206	206	206
2.5%	210	228	229	229		199	209	209	209
5.0%	214	239	241	241		196	213	213	213
7.5%	219	249	254	254		191	217	217	217
12.5%	225	260	266	265		188	222	222	222
15.0%	231	271	280	279		183	225	225	225
17.5%	237	282	294	293		179	229	229	229
20.0%	246	298	309	307		172	236	235	235
22.5%	252	307	319	317		168	239	240	239
27.5%	259	316	328	327		165	243	244	243
30.0%	263	322	332	331		161	246	247	247
32.5%	268	327	336	337		157	246	250	250
35.0%	271	331	340	340		154	239	253	253
37.5%	274	334	341	342		151	229	251	256
47.5%	277	337	344	345		147	225	238	255
57.5%	279	338	344	345		139	206	222	236
67.5%	279	338	344	346		132	192	206	218
77.5%	279	337	342	345		127	175	191	206
87.5%	278	336	341	344		121	157	167	184
100.0%	275	333	338	342		113	145	153	163

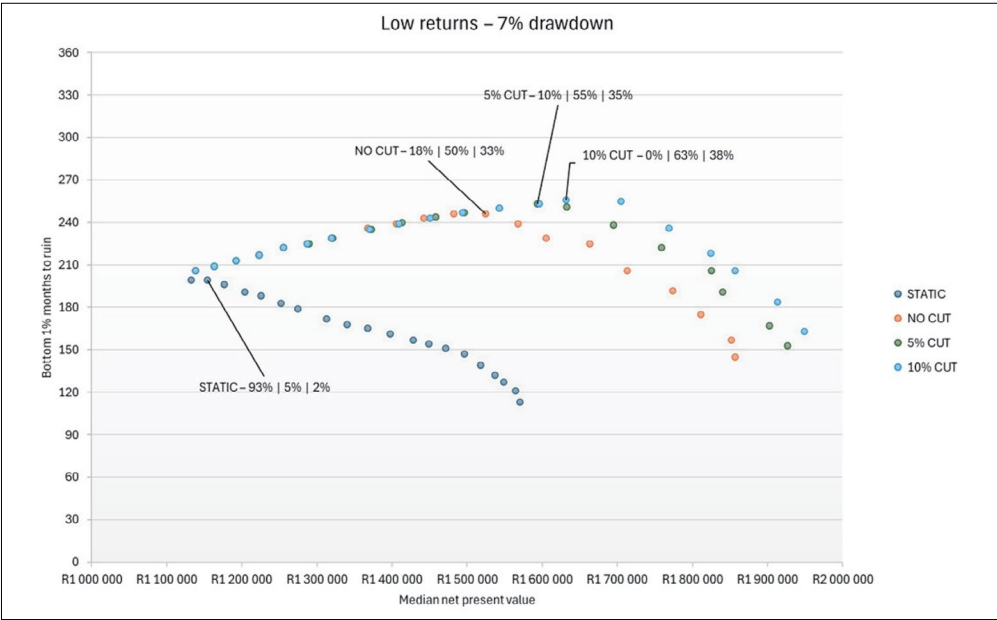


FIGURE 9. Scatter plot for low returns—7% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

is notable that the optimal equity allocation is lower (30%) under the high return scenario than under the low return scenario (37.5%), which follows the same pattern as the 5% drawdown.

5.8 Low return analysis: 9% drawdown

5.8.1 For the 9% drawdown, less value is added by the dynamic drawdown. The median NPVs increase on average by between 3.42% and 4.29%. There is a higher reliance on the second and third thresholds and triggers (the 5% cut and 10% cut).

5.8.2 The AMTR indicator is maximised at the maximum amount of equity and maximum haircut—but when combined with the high drawdown requirements, the MMTR can only be improved by 28 months, using a portfolio of 32.5% equity. This is a smaller improvement than for the 3%, 5% and 7% drawdowns. Figure 10 shows the narrow spread between the static and dynamic drawdown options. While dynamic drawdowns add value, the dynamic drawdown adds less to a living annuity beyond a certain drawdown level.

5.8.3 For the high return scenario, the MMTR can only be improved by 37 months, using a portfolio of 25% equity and maximum haircut. The NPV is increased by 31.7%. Like the 7% drawdown, the 9% drawdown returns a lower equity allocation in the higher return scenario.

TABLE 13. Median NPVs of the dynamic drawdown methods relative to static method for 9% drawdown (Source: Sygnia)

Low returns								
9% drawdown						% difference from static		
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 101 789	R1 106 532	R1 108 097	R1 108 097	0.43%	0.57%	0.57%	
2.5%	R1 115 980	R1 124 311	R1 123 879	R1 123 879	0.75%	0.71%	0.71%	
5.0%	R1 137 031	R1 143 992	R1 145 133	R1 145 133	0.61%	0.71%	0.71%	
7.5%	R1 150 909	R1 164 664	R1 164 664	R1 164 664	1.20%	1.20%	1.20%	
12.5%	R1 171 496	R1 185 440	R1 185 068	R1 185 068	1.19%	1.16%	1.16%	
15.0%	R1 185 440	R1 206 680	R1 208 062	R1 207 205	1.79%	1.91%	1.84%	
17.5%	R1 206 680	R1 227 798	R1 229 840	R1 226 908	1.75%	1.92%	1.68%	
20.0%	R1 234 561	R1 260 822	R1 263 593	R1 261 948	2.13%	2.35%	2.22%	
22.5%	R1 255 147	R1 283 283	R1 285 890	R1 283 413	2.24%	2.45%	2.25%	
27.5%	R1 276 203	R1 308 586	R1 311 998	R1 308 869	2.54%	2.80%	2.56%	
30.0%	R1 297 348	R1 331 165	R1 335 384	R1 330 947	2.61%	2.93%	2.59%	
32.5%	R1 311 289	R1 353 622	R1 360 671	R1 360 407	3.23%	3.77%	3.75%	
35.0%	R1 328 536	R1 374 977	R1 384 071	R1 383 418	3.50%	4.18%	4.13%	
37.5%	R1 338 799	R1 391 157	R1 402 778	R1 399 127	3.91%	4.78%	4.51%	
47.5%	R1 345 600	R1 410 710	R1 432 315	R1 432 650	4.84%	6.44%	6.47%	
57.5%	R1 363 395	R1 436 395	R1 456 863	R1 456 696	5.35%	6.86%	6.84%	
67.5%	R1 366 935	R1 466 432	R1 477 545	R1 494 193	7.28%	8.09%	9.31%	
77.5%	R1 370 459	R1 459 864	R1 489 116	R1 507 670	6.52%	8.66%	10.01%	
87.5%	R1 373 983	R1 475 369	R1 509 702	R1 523 705	7.38%	9.88%	10.90%	
100.0%	R1 366 935	R1 491 226	R1 510 551	R1 536 287	9.09%	10.51%	12.39%	
Average	R1 264 926	R1 310 151	R1 319 261	R1 322 014	3.42%	4.09%	4.29%	

TABLE 14. Average months to ruin and minimum months to ruin at 1% level for 9% drawdown rate (Source: Sygnia)

Low returns									
9% drawdown		Average months to ruin				Minimum months to ruin at 1%			
Equity weight	Static	No inflation	5% cut	10% cut	Static	No inflation	5% cut	10% cut	
0.0%	157	167	166	166	153	157	157	157	
2.5%	159	171	171	171	152	159	159	159	
5.0%	162	176	177	177	149	162	162	162	
7.5%	165	182	183	183	146	163	163	163	
12.5%	168	187	190	189	144	166	166	166	
15.0%	171	193	196	196	141	168	168	168	
17.5%	174	198	204	203	139	170	170	170	
20.0%	180	206	213	212	133	173	173	174	
22.5%	183	212	222	220	130	175	175	175	
27.5%	188	219	231	230	128	176	178	178	
30.0%	192	225	240	239	125	172	179	179	
32.5%	197	232	248	248	122	167	176	181	
35.0%	201	237	252	252	119	162	170	181	
37.5%	204	242	258	259	116	157	166	176	
47.5%	210	249	266	268	114	153	159	171	
57.5%	214	255	273	275	111	141	151	164	
67.5%	218	259	277	281	106	135	144	149	
77.5%	221	262	277	282	100	126	135	141	
87.5%	222	264	279	284	96	118	127	133	
100.0%	223	264	279	284	91	111	118	122	

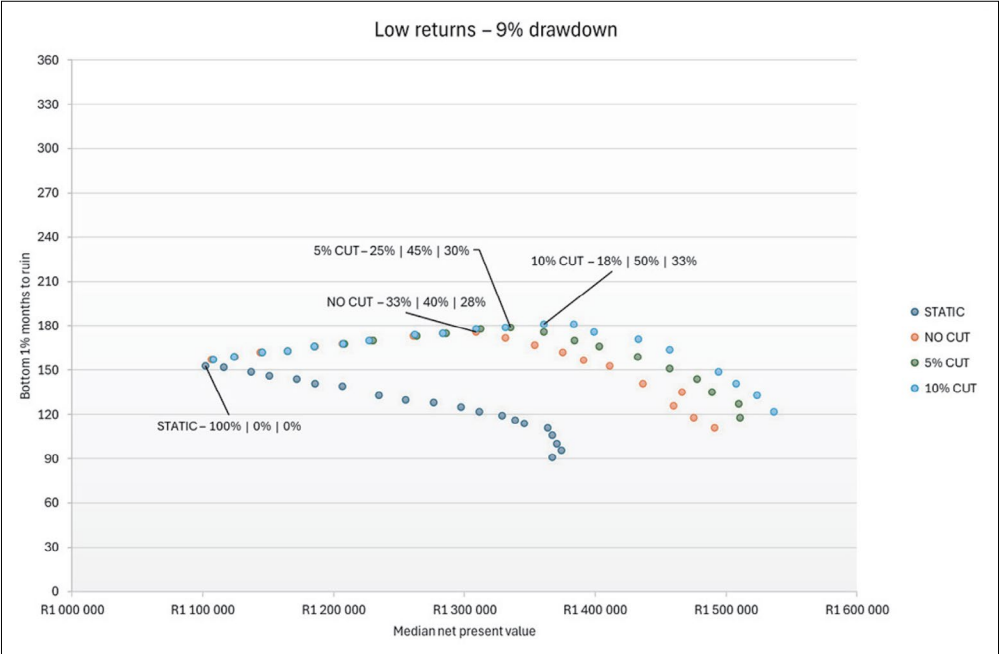


FIGURE 10. Scatter plot for low returns—9% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

TABLE 15. Results summary: Improvements relative to static drawdown method (Source: Sygnia)

Low return				
Drawdown	Months improvement	NPV improvement	Equity increase	Method
3%	Remained 360	27.7%	37.5% to 100%	No inflation
5%	291 to 360	93.0%	2.5% to 77.5%	5% cut
7%	199 to 253	41.3%	2.5% to 37.5%	10% cut
9%	153 to 181	23.5%	0% to 32.5%	10% cut
High return				
Drawdown	Months improvement	NPV improvement	Equity increase	Method
3%	Remained 360	31.0%	45% to 100%	No inflation
5%	303 to 360	146.6%	2.5% to 67.5%	5% cut
7%	203 to 287	71.8%	2.5% to 30%	10% cut
9%	154 to 191	31.7%	2.5% to 25%	10% cut

6. CONCLUSION

6.1 Analysis findings

6.1.1 A dynamic drawdown using the annual annuity adjustment method does not significantly improve the average payout for the client (relative to the static method). While the payout may last longer, it does not result in a meaningful financial outcome for the client.

6.1.2 The worst-case 3% drawdown scenario is not significantly improved by any dynamic method, and the static drawdown is sufficient to last 360 months. This makes intuitive sense, as there is little selling of capital in market downturns and hence less permanent impact. However, the median NPV can be vastly improved by taking on higher levels of risk while not sacrificing the MMTR relative to the 360-month benchmark.

6.1.3 Dynamic drawdown using the capital method dramatically improves the optimal MMTR and NPV for the 5%, 7% and 9% drawdowns, as it allows higher allocation to risky assets.

6.1.4 That said, the optimal portfolio asset mix under dynamic drawdown still becomes less risky, and equity allocations drop from 5% to 7% to 9% despite higher returns being required, as the volatility outweighs the benefit. In addition, more aggressive haircuts are required as the drawdown is increased—while higher returns are required to meet higher drawdowns, the higher volatility erodes capital.

6.1.5 The higher return scenario always improves the NPV relative to the low return scenario. The high return scenario sees a lower allocation to equities than the low return scenario in the optimal portfolio for the 5%, 7% and 9% drawdowns.

6.1.6 This analysis shows that the dynamic drawdown method can improve the client's total retirement payout by allowing exposure to a higher return/risk asset allocation mix, and that it can be used to create a low-cost, self-insuring with-profit annuity alternative. The success of the method will depend on the return assumptions, threshold and trigger levels used. Importantly, however, the process creates a simple tool that allows clients to systematically achieve a better outcome by changing their drawdown once a year. The choice of taking no inflationary increase, a 5% haircut or a 10% haircut is down to the client's risk preference, as is the choice of the 1% or 5% MMTR. In some years, dynamic drawdowns may result in flat or decreased payouts to preserve capital, so the client must understand that they will have to take on more lifestyle risk (reducing spending in some years) to achieve better long-term returns and a better retirement lifestyle. This is a key consideration that can lead to enhanced understanding by the client and allow them a better outcome.

6.2 Practical application

6.2.1 While low drawdown rates such as 3% are well suited for static living annuities, especially when one considers estate bequeathal, we confirm the analysis of both Maré (2016) and Van Appel et al. (2021) that drawdown levels of 5% or higher will not

last over a 30-year period. This analysis suggests that a 5% drawdown could last 28 years on average and that a 7% drawdown could last 23 years under static drawdown. Many investors would thus most likely look at traditional guaranteed life annuities.

6.2.2 However, dynamic drawdowns do affect the result. The 5% drawdown becomes sustainable and lasts the full 360-month term, with the worst case extended by 69 months. At the 7% drawdown level, the living annuity does not last the full term but can be extended by 57 months.

6.2.3 Even if annuitants do not switch to living annuities based on this research, financial advisors can use this tool and concept from a planning and asset allocation perspective to improve the outcomes of existing investors.

6.2.4 An open topic for future research would be to compare living annuities with dynamic drawdown rules to traditional guaranteed annuities and with-profit annuities. Given the better performance of dynamic versus static, dynamic living annuities may fare better. At that point, the impact of advice fees on the relevant products would be important.

ACKNOWLEDGEMENTS

We would like to thank Emlyn Flint, Ph.D. for his invaluable guidance, advice and meticulous review of our work. We would also like to thank Samantha Jones, Ph.D. for helping us solve some of the complex arithmetical problems we encountered. We are grateful to Steven Empedocles for his significant contribution to calculating the optimal asset allocations. Thank you to our reviewers, David Rodda, Ph.D. and Mayur Lodhia. Finally, we extend our appreciation to Sygnia Asset Management for providing access to historical data and supporting the production of this paper.

REFERENCES

- Association for Savings and Investment South Africa (ASISA) (2023). Living annuity model proves robust in volatile times. Media release, 29 November
- Berezin, P (2019). Population aging and the stock market: On a collision course? BCA Research, 12 April. https://www.bcaresearch.com/reports/view_report/27425/gis
- Bester, T (2016). Comparing five annuity options offered at retirement in South Africa. Master's thesis. Stellenbosch University
- Bloomberg (2024). Chart of the day, 22 July. Public debt climbed to record highs around the world (data from BIS, IMF)
- Butler, M, Hu, B & Kloppers, D (2013). A comparison of probability of ruin and expected discounted utility as objective functions for choosing a post-retirement investment strategy. *South African Actuarial Journal* 13(1): 185–219

- Butler, A, Philbrick, M, Gordillo, R & Guan, M (2015). Sensitivity of safe withdrawal rates to longevity, market and failure risk preferences with implications for asset allocation. *SSRN Electronic Journal*, 8 March. https://www.investresolve.com/inc/uploads/pdf/ReSolve_Sensitivity_of_Safe-Withdrawal_Rates_Whitepaper_2015.pdf
- Chartered Financial Analyst Society of South Africa/Actuarial Society of South Africa (CFASSA) (2024). Webinar on annuities in South Africa on 5 June 2024. [Quoting ASISA and Momentum Investments.] https://presentations.corpcam.com/webcast16x9_dc.aspx?id=CFA05062024
- Congressional Budget Office (CBO) (2024). The long-term budget outlook under alternative scenarios for the economy and the budget. <https://www.cbo.gov/system/files/2024-05/60169-scenarios.pdf>
- De Villiers-Strijdom, J (2013). Comparing annuity options at retirement. Master's thesis, Stellenbosch University
- Goemans, M & Ncube, M (2008). Optimal annuity strategies after retirement. Paper delivered at the Actuarial Society of South Africa's 2008 Convention. 23–24 October, Cape Town
- Guyton, JT & Klinger, WJ (2006). Decision rules and maximum initial withdrawal rates. *Journal of Financial Planning* 19(3): 50–58
- Jones, S (2019). Modelling credit spreads in the illiquid South African debt market. PhD thesis, University of Cape Town
- Leviton, S, Dolya, Y & Rusconi, R (2009). Evaluating post-retirement investment strategies. Unpublished research paper
- Lodhia, M & Swanepoel, J (2012). Living versus guaranteed annuities: In search for a sustainable retirement income. Paper delivered at the Actuarial Society of South Africa's 2012 Convention. 16–17 October, Cape Town
- Loeys, J & Wise, A (2024). JP Morgan long-term strategy, 26 July. Are annuities a good retirement investment?
- Maré, E (2016). Safe spending rates for South African retirees. *South African Journal of Science* 112 (Jan/Feb): 1–4
- Meucci, A (2001). Common pitfalls in mean variance asset allocation. Wilmott Technical Article, October
- Myerson, R (n.d.). Simtools.xlam. [Microsoft Excel add-in]. <https://home.uchicago.edu/~rmyerson/addins.htm>
- Peta, D (2024). The US fiscal outlook is unsightly, but a debt crisis is not inevitable. BCA Research US Bond Strategy, 19 August
- Schwab, C (2022). Market corrections are more common than you think. 22 February. <https://www.schwab.com/learn/story/market-corrections-are-more-common-than-you-think#:~:text=Pullbacks%20of%2010%25%20or%20more,days%20within%20a%20calendar%20year>
- Scott, JS, Sharpe, WF & Watson, JG (2009). The 4% rule—at what price? *Journal of Investment Management* 7(3): 31–48

- Theron, E (2013). Living annuities: Can the wealthy survive? Unpublished Honours research essay, University of Pretoria
- Van Appel, V, Maré, E & Van Niekerk, A (2021). Quantitative guidelines for retiring (more safely) in South Africa. *South African Actuarial Journal* 21: 75–91
- Waring, MB & Siegal, LB (2015). The only spending rule article you will ever need. *Financial Analysts Journal* 71(1): 91–107
- Wise, A (2023). Lowering our long-run US bond yield forecast. JPMorgan Research Global Long-term Strategy

APPENDIX A

The tables and figures below summarise the results for the same analysis done using the high return scenario defined in section 2 of the paper.

TABLE A1. Average months to ruin and minimum months to ruin at 1% level for each simulated drawdown rate (Source: Sygnia)

High returns		Average months to ruin			
Equity weight		3%	5%	7%	9%
0.0%	360	323	215	162	
2.5%	360	338	222	166	
5.0%	360	347	229	170	
7.5%	360	350	238	175	
10.0%	360	351	247	179	
12.5%	360	352	259	186	
15.0%	360	351	269	192	
17.5%	360	351	277	198	
20.0%	360	351	284	205	
22.5%	360	350	289	212	
25.0%	360	349	294	221	
27.5%	360	348	296	227	
30.0%	360	347	298	232	
35.0%	360	347	300	236	
45.0%	360	345	301	243	
57.5%	359	343	301	248	
67.5%	359	340	300	249	
77.5%	358	337	298	250	
87.5%	356	334	295	249	
100.0%	355	330	291	248	

High returns		Minimum months to ruin at 1%			
Equity weight		3%	5%	7%	9%
0.0%	360	295	201	153	
2.5%	360	303	203	154	
5.0%	360	301	202	153	
7.5%	360	298	198	150	
10.0%	360	292	194	147	
12.5%	360	281	188	143	
15.0%	360	272	183	139	
17.5%	360	262	178	136	
20.0%	360	250	173	132	
22.5%	360	240	168	129	
25.0%	360	229	160	124	
27.5%	360	221	155	121	
30.0%	360	212	152	117	
35.0%	360	207	147	115	
45.0%	360	201	143	113	
57.5%	344	193	138	107	
67.5%	304	181	129	104	
77.5%	272	166	123	97	
87.5%	237	156	114	94	
100.0%	220	144	107	89	

TABLE A2. Net present value median comparison of annually corrected method versus static drawdown across all simulated portfolios and drawdowns (Source: Sygnia)

High returns		Net present value median														
	Annually corrected		Static drawdown		Annually corrected		Static drawdown		Annually corrected		Static drawdown					
Equity weight	3% drawdown				5% drawdown				7% drawdown				9% drawdown			
0.0%	R1 458 604	R1 458 604	R1 273 076	R1 273 076	R1 181 537	R1 181 537	R1 137 031	R1 137 031								
2.5%	R1 552 972	R1 552 972	R1 333 854	R1 336 127	R1 220 124	R1 220 124	R1 164 664	R1 164 664								
5.0%	R1 649 156	R1 649 156	R1 397 881	R1 399 315	R1 252 497	R1 252 497	R1 192 468	R1 192 552								
7.5%	R1 754 805	R1 754 805	R1 470 810	R1 470 810	R1 298 841	R1 291 205	R1 220 682	R1 220 682								
10.0%	R1 861 869	R1 861 869	R1 550 819	R1 550 819	R1 335 039	R1 334 859	R1 247 454	R1 244 885								
12.5%	R2 005 179	R2 005 179	R1 655 827	R1 655 827	R1 406 898	R1 389 350	R1 287 168	R1 283 283								
15.0%	R2 136 083	R2 136 083	R1 740 278	R1 740 278	R1 449 993	R1 438 676	R1 336 545	R1 311 289								
17.5%	R2 262 806	R2 262 806	R1 835 991	R1 835 991	R1 521 081	R1 485 351	R1 338 799	R1 338 799								
20.0%	R2 397 721	R2 397 721	R1 936 188	R1 936 188	R1 563 366	R1 537 262	R1 377 940	R1 370 459								
22.5%	R2 535 636	R2 535 636	R2 033 465	R2 033 465	R1 609 088	R1 591 803	R1 430 638	R1 401 864								
25.0%	R2 690 270	R2 690 270	R2 164 785	R2 164 785	R1 697 629	R1 657 428	R1 493 450	R1 436 395								
27.5%	R2 846 856	R2 846 856	R2 277 493	R2 277 493	R1 762 154	R1 725 830	R1 482 999	R1 468 144								
30.0%	R2 995 928	R2 995 928	R2 384 103	R2 384 103	R1 806 886	R1 796 996	R1 526 944	R1 488 962								
35.0%	R3 155 253	R3 155 253	R2 478 986	R2 478 986	R1 867 742	R1 845 921	R1 591 200	R1 527 158								
45.0%	R3 358 203	R3 358 203	R2 637 389	R2 637 389	R1 966 404	R1 939 255	R1 629 971	R1 589 753								
57.5%	R3 636 143	R3 636 143	R2 812 634	R2 812 634	R2 044 410	R2 030 600	R1 707 946	R1 638 940								
67.5%	R3 715 761	R3 715 761	R2 864 143	R2 864 143	R2 128 107	R2 113 530	R1 717 857	R1 652 818								
77.5%	R3 852 375	R3 852 375	R2 951 431	R2 951 431	R2 236 615	R2 177 037	R1 753 707	R1 687 349								
87.5%	R3 990 170	R3 990 170	R2 993 371	R2 993 371	R2 278 776	R2 234 876	R1 771 199	R1 715 606								
100.0%	R4 016 967	R4 016 967	R3 000 760	R3 000 760	R2 263 494	R2 189 746	R1 801 139	R1 708 589								

% difference between annually corrected and static					
Equity weight	3% drawdown	5% drawdown	7% drawdown	9% drawdown	
0.0%	0.00%	0.00%	0.00%	0.00%	0.00%
2.5%	0.00%	-0.17%	0.00%	0.00%	0.00%
5.0%	0.00%	-0.10%	0.00%	-0.01%	
7.5%	0.00%	0.00%	0.59%	0.00%	
10.0%	0.00%	0.00%	0.01%	0.21%	
12.5%	0.00%	0.00%	1.26%	0.30%	
15.0%	0.00%	0.00%	0.79%	1.93%	
17.5%	0.00%	0.00%	2.41%	0.00%	
20.0%	0.00%	0.00%	1.70%	0.55%	
22.5%	0.00%	0.00%	1.09%	2.05%	
25.0%	0.00%	0.00%	2.43%	3.97%	
27.5%	0.00%	0.00%	2.10%	1.01%	
30.0%	0.00%	0.00%	0.55%	2.55%	
35.0%	0.00%	0.00%	1.18%	4.19%	
45.0%	0.00%	0.00%	1.40%	2.53%	
57.5%	0.00%	0.00%	0.68%	4.21%	
67.5%	0.00%	0.00%	0.69%	3.94%	
77.5%	0.00%	0.00%	2.74%	3.93%	
87.5%	0.00%	0.00%	1.96%	3.24%	
100.0%	0.00%	0.00%	3.37%	5.42%	
Average	0.00%	-0.01%	1.25%	2.00%	

TABLE A3. Median NPVs of the dynamic drawdown methods relative to static method for 3% drawdown (Source: Sygnia)

High returns								
3% drawdown		% difference from static						
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 458 604	R1 491 020	R1 490 205	R1 490 205	2.22%	2.17%	2.17%	
2.5%	R1 552 972	R1 585 686	R1 584 544	R1 584 544	2.11%	2.03%	2.03%	
5.0%	R1 649 156	R1 694 519	R1 690 513	R1 690 513	2.75%	2.51%	2.51%	
7.5%	R1 754 805	R1 808 541	R1 811 658	R1 811 369	3.06%	3.24%	3.22%	
10.0%	R1 861 869	R1 926 090	R1 926 733	R1 925 260	3.45%	3.48%	3.40%	
12.5%	R2 005 179	R2 084 116	R2 085 277	R2 082 870	3.94%	3.99%	3.87%	
15.0%	R2 136 083	R2 226 133	R2 226 133	R2 224 748	4.22%	4.22%	4.15%	
17.5%	R2 262 806	R2 366 990	R2 372 898	R2 365 622	4.60%	4.87%	4.54%	
20.0%	R2 397 721	R2 509 911	R2 518 090	R2 508 685	4.68%	5.02%	4.63%	
22.5%	R2 535 636	R2 650 405	R2 667 617	R2 663 915	4.53%	5.21%	5.06%	
25.0%	R2 690 270	R2 831 412	R2 843 079	R2 840 357	5.25%	5.68%	5.58%	
27.5%	R2 846 856	R2 991 555	R3 011 422	R3 003 943	5.08%	5.78%	5.52%	
30.0%	R2 995 928	R3 166 268	R3 183 101	R3 172 154	5.69%	6.25%	5.88%	
35.0%	R3 155 253	R3 348 840	R3 378 987	R3 375 934	6.14%	7.09%	6.99%	
45.0%	R3 358 203	R3 562 639	R3 575 983	R3 582 195	6.09%	6.48%	6.67%	
57.5%	R3 636 143	R3 825 482	R3 845 165	R3 864 763	5.21%	5.75%	6.29%	
67.5%	R3 715 761	R4 047 830	R4 061 205	R4 081 279	8.94%	9.30%	9.84%	
77.5%	R3 852 375	R4 165 073	R4 195 684	R4 223 604	8.12%	8.91%	9.64%	
87.5%	R3 990 170	R4 314 866	R4 347 690	R4 359 529	8.14%	8.96%	9.26%	
100.0%	R4 016 967	R4 400 212	R4 431 259	R4 459 277	9.54%	10.31%	11.01%	
Average	R2 693 638	R2 849 879	R2 862 362	R2 865 538	5.19%	5.56%	5.61%	

TABLE A4. Average months to ruin and minimum months to ruin at 1% level for 3% drawdown rate (Source: Sygnia)

High returns		Average months to ruin				Minimum months to ruin at 1%			
3% drawdown									
Equity weight	Static	No inflation	5% cut	10% cut		Static	No inflation	5% cut	10% cut
0.0%	360	360	360	360		360	360	360	360
2.5%	360	360	360	360		360	360	360	360
5.0%	360	360	360	360		360	360	360	360
7.5%	360	360	360	360		360	360	360	360
10.0%	360	360	360	360		360	360	360	360
12.5%	360	360	360	360		360	360	360	360
15.0%	360	360	360	360		360	360	360	360
17.5%	360	360	360	360		360	360	360	360
20.0%	360	360	360	360		360	360	360	360
22.5%	360	360	360	360		360	360	360	360
25.0%	360	360	360	360		360	360	360	360
27.5%	360	360	360	360		360	360	360	360
30.0%	360	360	360	360		360	360	360	360
35.0%	360	360	360	360		360	360	360	360
45.0%	360	360	360	360		360	360	360	360
57.5%	359	360	360	360		344	360	360	360
67.5%	359	360	360	360		304	360	360	360
77.5%	358	360	360	360		272	360	360	360
87.5%	356	360	360	360		237	360	360	360
100.0%	355	360	360	360		220	360	360	360

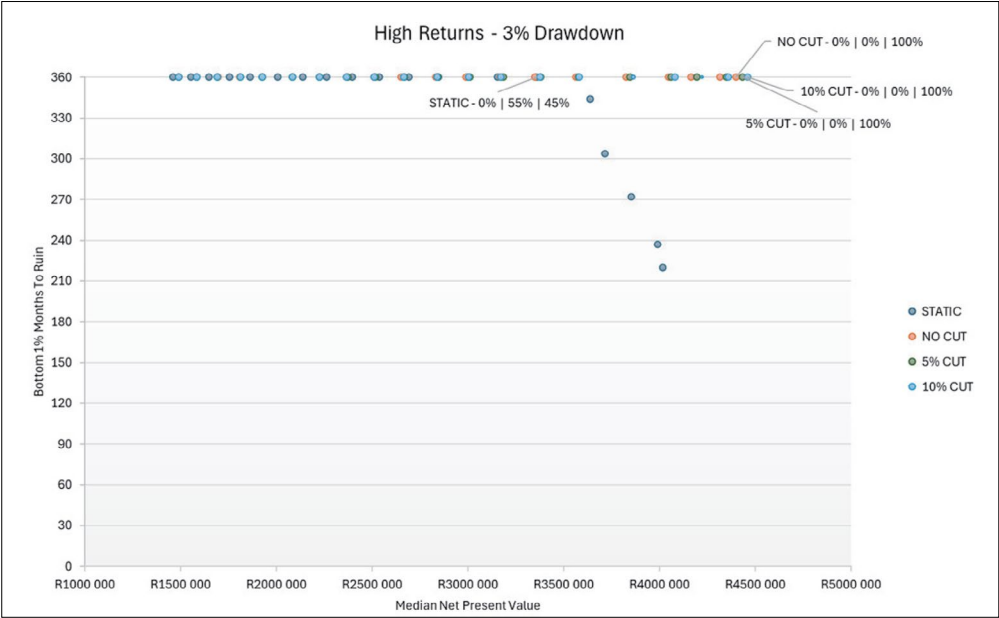


FIGURE A1. Scatter plot for low returns—3% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

TABLE A5: Median NPVs of the dynamic drawdown methods relative to static method for 5% drawdown (Source: Sygnia)

High returns							
5% drawdown		% difference from static					
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut
0.0%	R1 273 076	R1 313 175	R1 312 573	R1 312 573	3.15%	3.10%	3.10%
2.5%	R1 336 127	R1 379 211	R1 378 390	R1 378 390	3.22%	3.16%	3.16%
5.0%	R1 399 315	R1 460 464	R1 460 366	R1 460 366	4.37%	4.36%	4.36%
7.5%	R1 470 810	R1 549 544	R1 549 868	R1 549 223	5.35%	5.38%	5.33%
10.0%	R1 550 819	R1 642 173	R1 644 222	R1 641 115	5.89%	6.02%	5.82%
12.5%	R1 655 827	R1 759 525	R1 765 568	R1 757 696	6.26%	6.63%	6.15%
15.0%	R1 740 278	R1 866 681	R1 872 814	R1 866 314	7.26%	7.62%	7.24%
17.5%	R1 835 991	R1 981 711	R1 988 326	R1 984 110	7.94%	8.30%	8.07%
20.0%	R1 936 188	R2 092 508	R2 115 132	R2 100 885	8.07%	9.24%	8.51%
22.5%	R2 033 465	R2 215 077	R2 232 765	R2 227 714	8.93%	9.80%	9.55%
25.0%	R2 164 785	R2 367 503	R2 376 099	R2 376 099	9.36%	9.76%	9.78%
27.5%	R2 277 493	R2 496 525	R2 501 663	R2 502 028	9.62%	9.84%	9.86%
30.0%	R2 384 103	R2 629 571	R2 631 352	R2 630 507	10.30%	10.37%	10.34%
35.0%	R2 478 986	R2 735 633	R2 760 420	R2 776 929	10.35%	11.35%	12.02%
45.0%	R2 637 389	R2 900 394	R2 945 491	R2 965 380	9.97%	11.68%	12.44%
57.5%	R2 812 634	R3 151 874	R3 181 638	R3 182 552	12.06%	13.12%	13.15%
67.5%	R2 864 143	R3 263 698	R3 295 283	R3 293 297	13.95%	15.05%	14.98%
77.5%	R2 951 431	R3 369 154	R3 390 866	R3 464 347	14.15%	14.89%	17.38%
87.5%	R2 993 371	R3 501 208	R3 542 607	R3 588 965	16.97%	18.35%	19.90%
100.0%	R3 000 760	R3 525 391	R3 572 746	R3 604 071	17.48%	19.06%	20.11%
Average	R2 139 849	R2 360 051	R2 375 909	R2 383 128	9.23%	9.85%	10.06%

TABLE A6: Average months to ruin and minimum months to ruin at 1% level for 5% drawdown rate (Source: Sygnia)

High returns		Average months to ruin				Minimum months to ruin at 1%			
5% drawdown									
Equity weight	Static	No inflation	5% cut	10% cut	Static	No inflation	5% cut	10% cut	Static
0.0%	323	353	354	354	295	325	326	326	295
2.5%	338	358	358	358	303	337	336	336	303
5.0%	347	360	360	360	301	351	351	351	301
7.5%	350	360	360	360	298	360	360	360	298
10.0%	351	360	360	360	292	360	360	360	292
12.5%	352	360	360	360	281	360	360	360	281
15.0%	351	360	360	360	272	360	360	360	272
17.5%	351	360	360	360	262	360	360	360	262
20.0%	351	360	360	360	250	360	360	360	250
22.5%	350	360	360	360	240	360	360	360	240
25.0%	349	360	360	360	229	360	360	360	229
27.5%	348	360	360	360	221	360	360	360	221
30.0%	347	360	360	360	212	360	360	360	212
35.0%	347	360	360	360	207	360	360	360	207
45.0%	345	360	360	360	201	360	360	360	201
57.5%	343	360	360	360	193	360	360	360	193
67.5%	340	359	359	360	181	360	360	360	181
77.5%	337	358	359	359	166	286	335	360	166
87.5%	334	357	358	358	156	239	256	282	156
100.0%	330	355	356	357	144	196	218	234	144

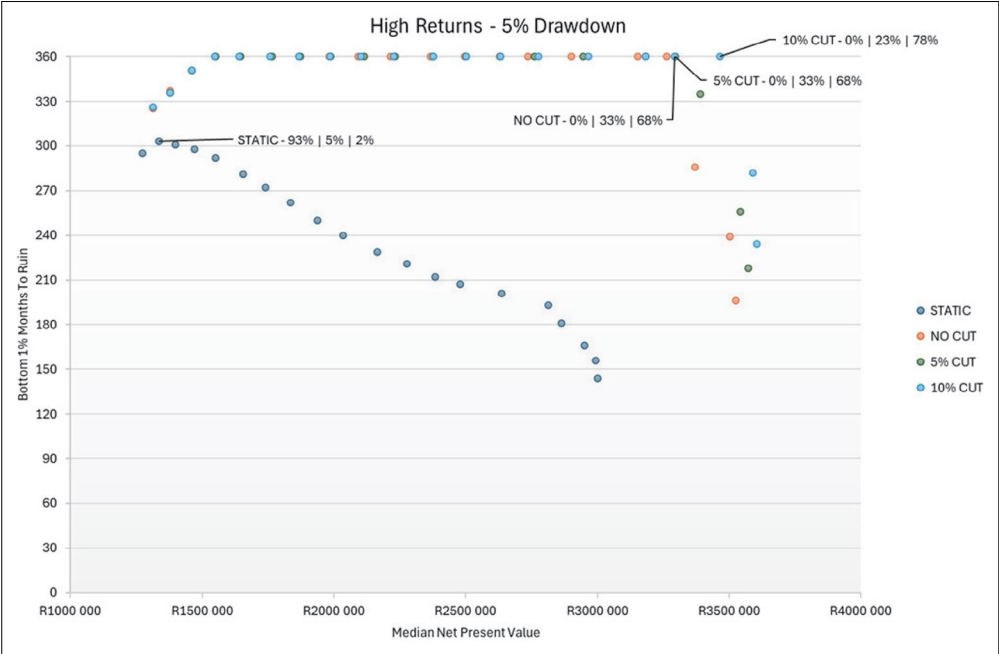


FIGURE A2. Scatter plot for low returns—5% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

TABLE A7. Median NPVs of the dynamic drawdown methods relative to static method for 7% drawdown (Source: Sygnia)

High returns								
7% drawdown						% difference from static		
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 181 537	R1 198 055	R1 197 472	R1 197 472	1.40%	1.35%	1.35%	
2.5%	R1 220 124	R1 237 454	R1 236 419	R1 236 419	1.42%	1.34%	1.34%	
5.0%	R1 252 497	R1 283 584	R1 282 840	R1 282 840	2.48%	2.42%	2.42%	
7.5%	R1 291 205	R1 334 491	R1 333 986	R1 333 972	3.35%	3.31%	3.31%	
10.0%	R1 334 859	R1 387 897	R1 389 984	R1 386 063	3.97%	4.13%	3.84%	
12.5%	R1 389 350	R1 460 554	R1 465 878	R1 464 109	5.12%	5.51%	5.38%	
15.0%	R1 438 676	R1 527 750	R1 540 007	R1 533 547	6.19%	7.04%	6.59%	
17.5%	R1 485 351	R1 607 121	R1 612 730	R1 609 886	8.20%	8.58%	8.38%	
20.0%	R1 537 262	R1 688 658	R1 695 640	R1 693 122	9.85%	10.30%	10.14%	
22.5%	R1 591 803	R1 777 494	R1 781 541	R1 779 935	11.67%	11.92%	11.82%	
25.0%	R1 657 428	R1 882 155	R1 891 403	R1 893 168	13.56%	14.12%	14.22%	
27.5%	R1 725 830	R1 973 103	R2 004 533	R1 993 720	14.33%	16.15%	15.52%	
30.0%	R1 796 996	R2 065 866	R2 099 968	R2 095 871	14.96%	16.86%	16.63%	
35.0%	R1 845 921	R2 173 234	R2 199 971	R2 203 282	17.73%	19.18%	19.36%	
45.0%	R1 939 255	R2 300 695	R2 333 668	R2 332 273	18.64%	20.34%	20.27%	
57.5%	R2 030 600	R2 421 614	R2 478 938	R2 505 161	19.26%	22.08%	23.37%	
67.5%	R2 113 530	R2 534 403	R2 608 630	R2 639 563	19.91%	23.43%	24.89%	
77.5%	R2 177 037	R2 595 696	R2 666 990	R2 717 838	19.23%	22.51%	24.84%	
87.5%	R2 234 876	R2 680 391	R2 742 345	R2 750 559	19.93%	22.71%	23.07%	
100.0%	R2 189 746	R2 743 001	R2 780 050	R2 843 871	25.27%	26.96%	29.87%	
Average	R1 671 694	R1 893 661	R1 917 150	R1 924 634	11.82%	13.01%	13.33%	

TABLE A8. Average months to ruin and minimum months to ruin at 1% level for 7% drawdown rate (Source: Sygnia)

High returns		Average months to ruin				Minimum months to ruin at 1%			
7% drawdown									
Equity weight	Static	No inflation	5% cut	10% cut	Static	No inflation	5% cut	10% cut	
0.0%	215	240	242	242	201	216	216	216	
2.5%	222	247	249	249	203	221	221	221	
5.0%	229	260	264	264	202	226	226	226	
7.5%	238	275	281	281	198	233	232	232	
10.0%	247	292	301	300	194	238	239	239	
12.5%	259	311	319	318	188	247	247	247	
15.0%	269	324	332	332	183	252	253	253	
17.5%	277	334	340	339	178	259	261	260	
20.0%	284	341	346	346	173	265	266	268	
22.5%	289	346	350	349	168	270	272	272	
25.0%	294	350	353	353	160	278	279	281	
27.5%	296	352	354	355	155	269	287	289	
30.0%	298	352	355	356	152	246	277	292	
35.0%	300	353	355	356	147	239	266	287	
45.0%	301	353	355	357	143	227	252	284	
57.5%	301	351	354	356	138	210	227	245	
67.5%	300	348	351	354	129	194	200	210	
77.5%	298	345	348	351	123	169	181	188	
87.5%	295	341	345	348	114	151	157	165	
100.0%	291	336	340	344	107	142	148	154	

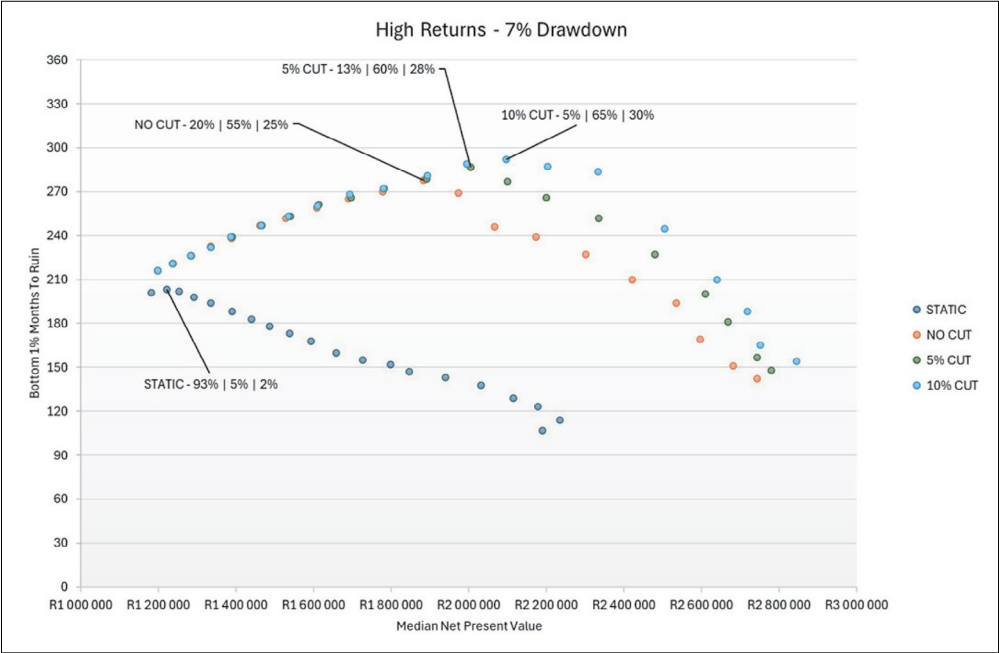


FIGURE A3. Scatter plot for low returns—7% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)

TABLE A9. Median NPVs of the dynamic drawdown methods relative to static method for 9% drawdown (Source: Sygnia)

High returns						% difference from static		
9% drawdown								
Equity weight	Static	No inflation	5% cut	10% cut	No inflation	5% cut	10% cut	
0.0%	R1 137 031	R1 148 353	R1 147 434	R1 147 434	1.00%	0.91%	0.91%	
2.5%	R1 164 664	R1 174 935	R1 175 559	R1 175 559	0.88%	0.94%	0.94%	
5.0%	R1 192 552	R1 204 758	R1 204 513	R1 204 513	1.02%	1.00%	1.00%	
7.5%	R1 220 682	R1 234 752	R1 235 117	R1 235 117	1.15%	1.18%	1.18%	
10.0%	R1 244 885	R1 267 466	R1 267 026	R1 266 719	1.81%	1.78%	1.75%	
12.5%	R1 283 283	R1 311 289	R1 309 235	R1 308 716	2.18%	2.02%	1.98%	
15.0%	R1 311 289	R1 349 255	R1 351 388	R1 347 657	2.90%	3.06%	2.77%	
17.5%	R1 338 799	R1 388 203	R1 388 180	R1 387 878	3.69%	3.69%	3.67%	
20.0%	R1 370 459	R1 432 265	R1 433 882	R1 432 547	4.51%	4.63%	4.53%	
22.5%	R1 401 864	R1 477 660	R1 487 318	R1 482 221	5.41%	6.10%	5.73%	
25.0%	R1 436 395	R1 527 158	R1 543 671	R1 534 043	6.32%	7.47%	6.80%	
27.5%	R1 468 144	R1 569 209	R1 601 868	R1 590 676	6.88%	9.11%	8.35%	
30.0%	R1 488 962	R1 610 922	R1 658 281	R1 643 638	8.19%	11.37%	10.39%	
35.0%	R1 527 158	R1 666 859	R1 703 169	R1 700 355	9.15%	11.53%	11.34%	
45.0%	R1 589 753	R1 725 845	R1 777 628	R1 791 565	8.56%	11.82%	12.69%	
57.5%	R1 638 940	R1 797 548	R1 863 789	R1 895 793	9.68%	13.72%	15.67%	
67.5%	R1 652 818	R1 863 931	R1 941 175	R1 951 461	12.77%	17.45%	18.07%	
77.5%	R1 687 349	R1 935 460	R2 005 246	R2 023 554	14.70%	18.84%	19.93%	
87.5%	R1 715 606	R1 985 066	R2 038 246	R2 060 673	15.71%	18.81%	20.11%	
100.0%	R1 708 589	R1 967 367	R2 047 760	R2 087 511	15.15%	19.85%	22.18%	
Average	R1 428 961	R1 531 915	R1 559 024	R1 563 381	6.58%	8.26%	8.50%	

TABLE A10. Average months to ruin and minimum months to ruin at 1% level for 9% drawdown rate (Source: Sygnia)

High returns									
9% drawdown		Average months to ruin				Minimum months to ruin at 1%			
Equity weight	Static	No inflation	5% cut	10% cut	Static	No inflation	5% cut	10% cut	
0.0%	162	176	177	177	153	163	163	163	
2.5%	166	180	181	181	154	165	166	166	
5.0%	170	187	188	188	153	169	169	169	
7.5%	175	194	196	195	150	172	172	172	
10.0%	179	202	205	204	147	175	176	176	
12.5%	186	213	219	218	143	178	179	179	
15.0%	192	221	232	230	139	181	182	182	
17.5%	198	231	242	241	136	184	185	185	
20.0%	205	241	256	255	132	184	186	187	
22.5%	212	251	268	268	129	184	188	190	
25.0%	221	262	278	279	124	178	187	191	
27.5%	227	270	287	288	121	167	182	190	
30.0%	232	277	294	296	117	161	172	184	
35.0%	236	282	300	302	115	156	168	179	
45.0%	243	288	304	306	113	151	161	169	
57.5%	248	293	307	311	107	141	153	160	
67.5%	249	294	306	311	104	133	140	149	
77.5%	250	294	305	310	97	124	132	138	
87.5%	249	293	302	307	94	114	119	125	
100.0%	248	291	300	305	89	104	112	119	

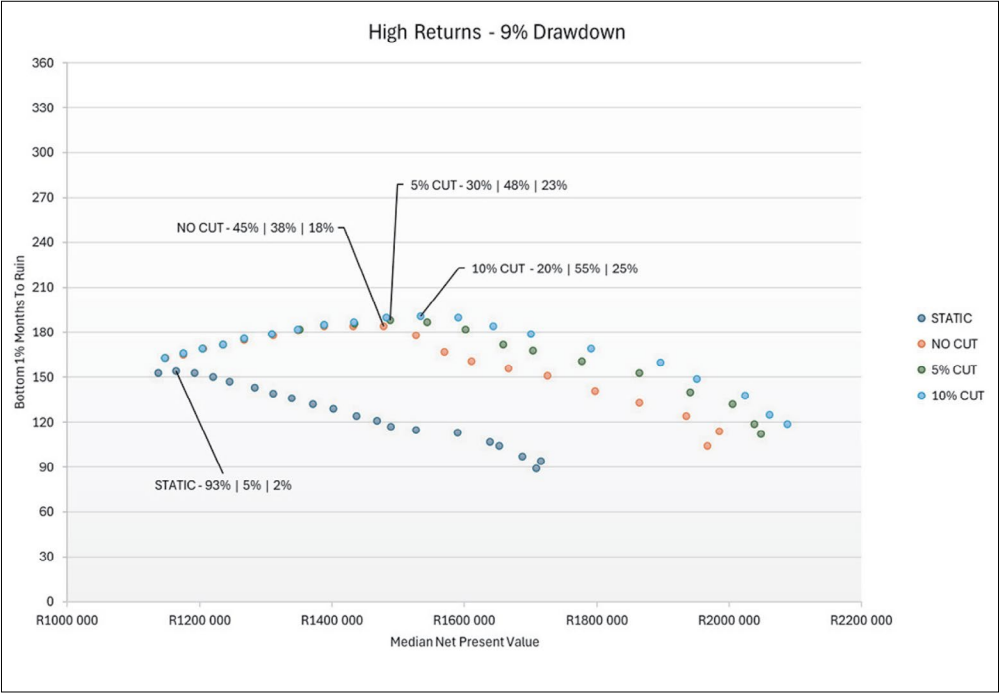


FIGURE A4. Scatter plot for low returns—9% drawdown scenario, measuring minimum months to ruin at 1% against median net present value for all optimal portfolios and drawdown methods (Source: Sygnia)