

EXAMINATION

26 September 2025 (am)

Subject A211 — Financial Mathematics Intermediate Technical

Examination writing time: Two hours and fifteen minutes (includes reading time)

Scan and upload time: Twenty minutes (at the end of the examination writing time)

[Total time on examination countdown timer: Two hours and thirty-five minutes]

Total marks: 100

INSTRUCTIONS TO THE CANDIDATE

Failure to adhere to any of these instructions could incur examination sanctions including disqualification from the examination.

1. *The question paper is only available on the ASSA Exam Platform as a PDF download and may not be printed.*
2. *Your computer must be placed, and camera angled, so that your head and shoulders as well as your writing area on your desk are visible to the invigilator. Readjust your camera if you bump or move your computer by accident.*
3. *Ensure that you write in a quiet room with a closed door visible behind you and in view of the proctor.*
4. *You are required to write your answers on clean A4 examination sheets. Write only on one side of the paper and number your pages.*
5. *Write your **full Candidate Number**, which starts with “A211-exa-2025-...” at the top of each page **during the examination time only**. **Do not use your name and/or member number anywhere on your answer script**. Your candidate number is reflected on your exam permit.*
6. *The cell phone to be used to scan your final answer script must be switched **OFF** during the two hours and 15 minutes examination time. Place the cell phone out of reach.*
7. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may start answering the paper whenever you are ready.*
8. *Attempt all questions, beginning your answer to each question on a new page.*

9. *Write in black or dark blue pen.*
10. *Show calculations where appropriate. You may use blank paper to carry out rough work calculations. This rough work paper must not be scanned as part of your answer script. You may use a calculator from the ASSA-approved list only.*
11. *You may not access any file from your computer, use any other computer software (e.g., Email or Excel), or open or use any other browser, collaboration tools or AI tools, including those embedded in MS Word and Adobe AI assistant (e.g. Copilot, ChatGPT etc.) during the examination. Nor may you use Grammarly, Grammarly Premium or similar built-in software tools including language and grammar tools which must be disabled. Microsoft Word spellcheck is allowed.*
12. *You are not permitted to bring any other material into the examination (e.g. a Formulae and Tables book). Any such information/material that may be required will be provided to you in the examination.*
13. *Mark allocations are shown in brackets.*
14. *Assume that months are all equal length, unless otherwise stated.*
15. *Your examination countdown timer reflects your TOTAL examination time. Writing time and reading time is 2 hours and fifteen minutes. Scan and upload time is 20 minutes at the end of the examination writing time. Audio prompts will remind you of the remaining writing time and when you should stop writing to start scanning to upload. **You may NOT continue to write during the 20 minutes of scanning and upload time as this could lead to disqualification from the examination.***
16. *Scan ALL your answer pages only to PDF so that your candidate number is clear at the top of each page. Rough notes must not be uploaded.*
17. *Save your PDF scanned file using your **full candidate number** as file name. **Do not name your answer script with your name or member number.***
18. *Transfer your scanned script file to your computer and upload it into the ASSA Exam Platform.*
19. *Click on the **Upload Answers** link below the examination paper link. Ensure you click on **Finish Attempt** below the upload box and again on **Finish All and Submit**, **before** the 20-minute scanning and upload time is up. (After submission the number of files successfully submitted will be reflected.)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above. The onus remains on you to read and implement all examination instructions.

END OF INSTRUCTIONS

QUESTION 1

An annual effective interest rate of i is used in the following calculations.

An actuarial student has calculated the following interest rates:

i	$i^{(12)}$	δ	$d^{(4)}$
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- i. Which of these interest rates will have the lowest numerical value?

[1]

An actuarial student has calculated the annuity values:

$s_{\overline{n} }$	$\ddot{s}_{\overline{n} }^{(12)}$	$\bar{s}_{\overline{n} }$	$s_{\overline{n} }^{(4)}$
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- ii. Which of these annuity values will have the highest numerical value?

[1]

- iii. Explain, with reason(s), your choice in (ii).

[2]

A life insurance company wishes to calculate the present value of an n -year increasing annuity, payable annually in arrears. The first annuity payment is of amount X and each subsequent payment increases by amount Y .

The following expressions have been proposed to calculate the present value of the annuity.

A. $Xa_{\overline{n} } + Y(Ia)_{\overline{n} }$	B. $(X - Y)a_{\overline{n} } + Y(Ia)_{\overline{n-1} }$
C. $(X + Y)(Ia)_{\overline{n} }$	D. $(X - Y)a_{\overline{n} } + Y(Ia)_{\overline{n-1} } + Ynv^n$

- iv. Write down the letter that corresponds to the expression that would correctly provide the present value of the described annuity.

[2]

A financial product has the following cashflow model:

The lender makes an initial deposit in exchange for a regular series of positive cashflows of equal amounts followed by a larger positive cashflow at the end of the term.

Consider the following financial products:

Zero-coupon bond	Annuity-certain	Fixed interest bond	Repayment loan
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- v. Which of the financial products listed is most likely to have the cashflow described above? Explain your answer briefly.

[2]

[Total 8]

PLEASE TURN OVER

QUESTION 2

An investor has a single liability of R10,000 due in 15 years' time.

The investor fully immunizes the liability by investing R4,450 in a zero-coupon bond with a term of $(15 - m)$ and R5,618 in a zero-coupon bond with a term of $(15 + m)$, where all terms are measured in years.

The yield on zero-coupon bonds of all terms is currently 6% per annum effective.

- i. Calculate the value of m .

[8]

The yield on zero-coupon bonds of all terms immediately decreases to 4% per annum effective.

- ii. Calculate the investor's profit or loss.

[3]

[Total 11]

QUESTION 3

The force of interest at time t , $\delta(t)$, is the instantaneous change in the fund value, expressed as an annualized percentage of the current fund value, V_t . This implies that

$$\delta(t) = \frac{\frac{d}{dt} V_t}{V_t}$$

$A(t_1, t_2)$ is defined as the accumulation at time t_2 of an investment of 1 at time t_1 .

- i. Derive an expression for $A(t_1, t_2)$, in terms of $\delta(t)$, by starting with the definition of $\delta(t)$ as given above.

[4]

The annual force of interest, $\delta(t)$, is a function of time t (measured in years) and is given by:

$$\delta(t) = \begin{cases} 0.02 & \text{for } 0 \leq t < 5 \\ r & \text{for } t \geq 5 \end{cases}$$

PLEASE TURN OVER

An investor is considering investing in Investment A.

Investment A:

An eight-year zero-coupon bond with a redemption value of R10,492.

Using $\delta(t)$, Investment A has a present value of R8,600.

- ii. Calculate the numerical value of r .

[4]

Alternatively, the investor could invest R8,600 in Investment B.

Investment B:

A savings account which earns simple interest at a rate of 5.5% per annum, will provide an amount of R10,492 after n years.

- iii. Calculate the numerical value of n .

[3]

The effective annual yields on Investment A and Investment B have been calculated.

- iv. Explain, with reason(s), which investment will have the highest effective annual yield.
No further calculations are required.

[2]

[Total 13]

QUESTION 4

An entrepreneur takes out a loan of R130,000 that is to be repaid by annual installments in arrears over 24 years. The bank agreed that the first five installments only need to be R10,000 each. From the sixth installment each installment is of amount RY.

An effective annual interest rate of 11% applies to the loan.

- i. Calculate Y.

[4]

- ii. Calculate the capital portion of the fourth installment.

[5]

- iii. Comment on the numerical answer of (ii).

[3]

The capital portion of the eighth installment is R 3,392.52

- iv. Explain the relative capital portions of the fourth and eighth installments.

[1]

[Total 13]

PLEASE TURN OVER

QUESTION 5

An investor bought 10,000 shares at R72 per share on 1 January 2022. The shares paid annual dividends according to the following table:

Date	Dividend per share	Consumer Price Index (CPI)
31 December 2021		92.7
31 December 2022	R4.80	96.5
31 December 2023	R5.25	99.8
31 December 2024	R5.33	102.1

On 31 December 2024, the investor sold all their shares at R88 per share after receiving the dividends on that date.

- i. Calculate, using linear interpolation, the annual real yield earned by the investor. [10]
 - ii. State, with reason(s), whether the real or nominal yield would be greater for this investment. [3]
- [Total 13]

QUESTION 6

A fixed interest bond pays coupons at 8.5% per annum, quarterly in arrears. The bond is redeemable at 105% per R100 nominal on any coupon date between 10 and 15 years after the date of issue.

A particular investor is subject to income tax (only) at 40% and wishes to invest in this fixed interest bond, at issue, to earn a net yield of at least 4.5% per annum effective.

- i. Calculate the price, per R1,000 nominal, for this fixed interest bond. [5]

The fixed interest bond is purchased for the price calculated in (i).

- ii. Calculate the running yield for this fixed interest bond. [2]

The investor wishes to know whether the maximum net annual effective yield that he can earn on this fixed interest bond is more than or less than 5%.

- iii. Explain, by performing appropriate calculations, whether the maximum yield is more than or less than 5%. [4]
- [Total 11]

PLEASE TURN OVER

QUESTION 7

The term structure of interest rates refers to the fact that interest rates vary according to the term of the investment.

- i. State the three most popular explanations for this phenomenon.

[2]

Consider a project that produces a series of cashflows, c_t at time t , which can be discounted at the force of interest, δ per annum, to produce the net present value

$$NPV(\delta) = \sum c_t e^{-\delta t}$$

It is given that $NPV(\delta) \neq 0$.

- ii. Write down an expression for the discounted mean term (DMT) of this project, in terms c_t and δ .

[1]

An investor purchases a 12-year level annuity, payable annually in arrears. The annuity payments are R800 per annum and the interest rate is 4.5% per annum effective.

- iii. Calculate the discounted mean term (DMT) of this level annuity.

[4]

The payments of this annuity are altered, such that the first payment is R800, and each subsequent payment increases at a rate of 4.5% per annum compound. The interest rate applicable is still 4.5% per annum effective.

- iv. Calculate the discounted mean term (DMT) of this new annuity.

[4]

[Total 11]

QUESTION 8

An investor decides to purchase a takeaway restaurant for R800,000. The restaurant is expected to produce an income of R450,000 per annum, payable continuously, for the next ten years.

The running costs will be R250,000 per annum, payable continuously. In addition, the investor will incur marketing costs for the first three years of the project. The marketing costs are R10,000 per month, payable in advance, and increase by 6% at the start of each subsequent year.

The investor currently has insufficient funds to finance this venture, but can borrow the initial outlay of R800,000 from a bank, which will charge an annual effective rate of 7.5%.

The investor wishes to use any excess income to repay the loan as soon as possible. Thereafter, excess funds can be invested and earn 6.25% per annum effective.

PLEASE TURN OVER

- i. Calculate the discounted payback period (DPP) of the loan.
(Hint: The discounted payback period (DPP) is longer than three years.) [6]
- ii. Calculate the accumulated profit of this venture after ten years (at $t = 10$). [4]

Alternatively, the investor has the option of borrowing the R800,000 by taking out an interest-only loan for a fixed term of ten years at an effective rate of interest of 7.5% per annum. The loan is to be repaid in full after ten years (with no early repayment option) and interest on the loan is paid at the end of each year.

- iii. Calculate the accumulated profit of this option after the loan has been repaid at $t = 10$. [7]
- iv. Explain, with reasons, the relative numerical values of (ii) and (iii). [3]

[Total 20]

[GRAND TOTAL 100]

END OF EXAMINATION