Actuarial Society of South Africa

EXAMINERS REPORT

September 2025

Subject A211

Question 1

i. $d^{(4)}$

ii. $\ddot{s}_{\overline{n|}}^{(12)}$

iii.
The numerators of all four expressions are $(1+i)^n - 1$ and thus the highest numerical value is the one with the lowest denominator, $d^{(12)}$.

iv. $(X-Y)a_{\overline{n|}} + Y(Ia)_{\overline{n-1|}} + Ynv^n$

v. Fixed interest security – small coupon payments and one large redemption amount.

[Total 8]

Question 2

i.
PV(Liabilities) = $10{,}000v^{15}$

PV(Assets) $= 4{,}450v^{15-m} + 5{,}618v^{15+m}$

First condition:
$10{,}000v^{15} \ = \ 4{,}450v^{15-m} + 5{,}618v^{15+m}$

$\Rightarrow 10{,}000 \ = \ 4{,}450v^{-m} + 5{,}618v^{+m}$

DMT(Liabilities) = $\frac{15\times 10{,}000v^{15}}{10{,}000v^{15}} = 15$

DMT(Assets) $= \frac{(15-m)\times 4{,}450v^{15-m} + (15+m)\times 5{,}618v^{15+m}}{4{,}450v^{15-m}+5{,}618v^{15+m}}$

Second condition:
$(15-m)4{,}450v^{15-m} + (15+m)5{,}618v^{15+m} = 15(4{,}450v^{15-m} + 5{,}618v^{15+m})$
$\Rightarrow 5{,}618mv^{15+m} = 4{,}450mv^{15-m}$
$\Rightarrow v^{2m} = \frac{4{,}450}{5{,}618}$
$\Rightarrow m = 1.99997 \approx 2$

[8]

ii.

PV(Liabilities) = $10{,}000 \times 1.04^{-15} = 5{,}552.65$

PV(Assets) $= 4{,}450 \times 1.04^{-13} + 5{,}618 \times 1.04^{-17} = 5{,}556.69$

Profit $= 5{,}556.69 - 5{,}552.65 = 4.04$

[3]
[Total 11]

Even though the solving of m in part (i) was "non-standard", half of the marks were available for setting up the two standard equations for immunization. Some candidates tried to solve for m using only the present value equation, without verifying that the value obtained for m also satisfies the Second Condition.

The common error in part (i) was writing down an equation for the volatility but labelling it as the discounted mean term or vice versa.

Question 3

i.

$$\delta(t) = \frac{\frac{d}{dt}V_t}{V_t}$$

$$\Rightarrow \delta(t) = \frac{d}{dt}\ln(V_t)$$

$$\Rightarrow \int_{t_1}^{t_2} \delta(t)dt = [\ln(V_t)]_{t_1}^{t_2}$$

$$\Rightarrow \int_{t_1}^{t_2} \delta(t)dt = \ln\left(\frac{V_{t_2}}{V_{t_1}}\right)$$

$$\Rightarrow \exp\left(\int_{t_1}^{t_2} \delta(t)dt\right) = \frac{V_{t_2}}{V_{t_1}}$$

$$= A(t_1, t_2)$$

[4]

ii.

$$8{,}600 \times \exp\left(\int_0^5 0.02dt\right) \times \exp\left(\int_5^8 rdt\right)$$

$$= 8{,}600 \times \exp[0.02m]_0^5 \times \exp[rm]_5^8 = 8{,}600 \times \exp[0.1] \times \exp[3r] = 10{,}492$$

$$8{,}600 \times e^{0.1+3r} = 10{,}492$$

$$0.1+3r=\ln\left(\frac{10,492}{8,600}\right)\Rightarrow r=0.03295$$

[4]

iii.

$8{,}600 \times (1+0.055\times n)=10{,}492$

$\Rightarrow n=4$

[3]

iv.

The effective annual yield on Investment B is greatest, since with the same deposit it delivers in four years what Investment A delivers in eight years.

[2]

[Total 13]

Part (i) was the worst answered question in the paper. Candidates did not follow the instruction in the question to start with the definition of $\delta(t)$ and then to derive the required equation. Candidates often started with the equation and tried to work backward. This approach gained no credit.

Parts (ii) and (iii) were answered well, with part (iv) providing a challenge to less well-prepared candidates.

Question 4

i.

$130{,}000 = 10{,}000 a_{\overline{5}|11\%} + Y v^5 a_{\overline{24-5}|11\%}$

$$\Rightarrow Y=\frac{130{,}000-10{,}000\times 3.6959}{0.5935\times 7.8393}$$

$\Rightarrow Y \approx 19{,}999.19$ (Rounded to the nearest Rand)

[4]

ii.

$L_3 = 130{,}000(1+i)^3 - 10{,}000 s_{\overline{3}|11\%}$

$= 177{,}792.03 - 33{,}421$

$= 144{,}371.03$

$C_4 = 10{,}000 - i\times L_3 =$ -5,880.81 $\Rightarrow$ No capital is repaid.

[5]

iii.

Since the interest on the outstanding loan amount is greater than the instalment, no capital is repaid and the outstanding loan amount grows as interest not paid is added to the loan.

[3]

iv.

The capital element of the eighth instalment is positive since the larger instalments exceed the interest on the loan amount.

[1]

[Total 13]

Part (i) was answered well.

Part (ii) could be challenging depending on which method (prospective or retrospective) was used to calculate the loan outstanding. If the method for calculating the loan outstanding is not specified, candidates should look critically at the cashflow pattern to decide which method might be easier. In this case, the retrospective method for calculating loan outstanding was the easier method since the payments were constant.

Part (iii) asked for an explanation in words of the numbers calculated in part (ii). This created difficulty for many candidates, which is an indication that the calculation can be done (mechanically) without a clear understanding of what the numbers imply.

Question 5

i.

Convert all cashflows to 2021 values:

$$D_1 = 10{,}000 \times 4.80 \times \frac{92.7}{96.5} = 46{,}109.84$$

$$D_2 = 10{,}000 \times 5.25 \times \frac{92.7}{99.8} = 48{,}765.03$$

$$D_3 = 10{,}000 \times 5.33 \times \frac{92.7}{102.1} = 48{,}392.85$$

$$R = 10{,}000 \times 88.00 \times \frac{92.7}{102.1} = 798{,}981.39$$

The equation of value, at the real rate of return r, is

$$720{,}000 = 46{,}109.84v_r + 48{,}765.03v_r^2 + (48{,}392.85 + 798{,}981.39)v_r^3$$

Try 10% $\Rightarrow$ RHS $= 718{,}864.53$

Try 9% $\Rightarrow$ RHS $= 737{,}675.55$

$$\Rightarrow r = 9\% + \frac{720{,}000 - 737{,}675.55}{718.864.53 - 737{,}675.55} \times (10\% - 9\%)$$

$$\Rightarrow r = 9.940\%$$

[10]

ii.

The nominal yield would be greater, since it includes positive inflation.

In this scenario the inflation rate is positive, and therefore the real yield would be less than the nominal yield.

[3]
[Total 13]

Part (i) was done well.

Part (ii) was done poorly, with many candidates not knowing the correct relationship between the nominal and real yield when inflation is positive. Many candidates referred to increasing inflation, rather than positive inflation, as a key part of the answer.

Question 6

i.

Since $g(1-t) = \frac{0.085}{1.05}(1-0.4) = 4.86\% > \ 4.43\% \ = i^{(4)}$ there is a capital loss and the worst case scenario is redemption at the earliest date (i.e., $n = 10$.)

$P = 1{,}000 \times 0.085 \times (1-0.4) \times a^{(4)}_{\overline{10}|4.5\%} + 1.05 \times 1{,}000 \times 1.045^{-10}$
$= 1{,}086.42$
$a^{(4)}_{\overline{10}|4.5\%} = 8.04502087$

[5]

ii.

The running yield is the ratio of the coupon rate to the price, hence $\frac{85}{1{,}086.42} = \ 7.82\%$

[2]

iii.

The maximum yield is earned if the security is redeemed as late as possible, since the minimum yield is earned if redeemed as early as possible. If the maximum yield was 5%, the purchase price would therefore be *(n=15)*

$P = 1{,}000 \times 0.085 \times (1-0.4) \times a^{(4)}_{\overline{15}|5\%} + 1.05 \times 1{,}000 \times 1.05^{-15}$
$= 1{,}044.26$

Since this is lower than the price calculated in (i), it indicates that the maximum net yield that the investor can earn is less than 5%.

OR calculate max yield

$$1086.42 = 1{,}000 \times 0.085 \times (1 - 0.4) \times a^{(4)}_{\overline{15}|i\%} + 1.05 \times 1{,}000 \times (1 + i)^{-15}$$

$$1086.42 = 51a^{(4)}_{\overline{15}|i\%} + 1050(1 + i)^{-15}$$

If solve for *i=4.6149%* which is lower than 5%

[4]
[Total 11]

Part (i) was answered well.

Candidates struggled with part (iii) as this question was in terms of the maximum yield and candidates are more used to answering questions on the minimum yield.

Question 7

i.
Liquidity preference

Expectation theory

Market segmentation

[2]

ii.

$$DMT(\delta) = \frac{\sum tc_t e^{-\delta t}}{\sum c_t e^{-\delta t}}$$

[1]

iii.

$$DMT = \frac{800(v + 2v^2 + \cdots + 12v^{12})}{800(v + v^2 + \cdots + v^{12})}$$
$$= \frac{(Ia)_{\overline{12}|4.5\%}}{a_{\overline{12}|4.5\%}}$$
$$= \frac{54.5100}{9.1186} = 5.9779 \text{ years}$$

[4]

iv.

$$DMT = \frac{800\left(\frac{1}{1.045} + 2 \times \frac{1.045}{1.045^2} + \cdots + 12 \times \frac{1.045^{11}}{1.045^{12}}\right)}{800\left(\frac{1}{1.045} + \frac{1.045}{1.045^2} + \cdots + \frac{1.045^{11}}{1.045^{12}}\right)}$$

$$= \frac{800 \times \frac{1}{1.045} \times \left(1 + 2 \times \frac{1.045}{1.045} + \cdots + 12 \times \frac{1.045^{11}}{1.045^{11}}\right)}{800 \times \frac{1}{1.045} \times \left(1 + \frac{1.045}{1.045} + \cdots + \frac{1.045^{11}}{1.045^{11}}\right)}$$

$$= \frac{1 + 2 + \cdots + 12}{1 + 1 + \cdots + 1}$$

$$= \frac{78}{12}$$

$= 6.5$ years

[4]

[Total 11]

Part (iv) was poorly answered, with a common error being the application of the geometric series, which leads to a division by zero and an answer that does not exist.

Candidates must write down the equations from first principles and then inspect the equations before applying any type of formula. In this case candidates should have noticed that the growth rate and the interest rate are the same and realized that this simplifies the calculation significantly.

Question 8

i.

First, we need to determine when the loan will be repaid.

$$800{,}000 + 10{,}000 \times 12\ddot{a}^{(12)}_{\overline{1}|7.5\%} \times \left(1 + \frac{1.06}{1.075} + \frac{1.06^2}{1.075^2}\right) = (450{,}000 - 250{,}000)\bar{a}_{\overline{t}|7.5\%}$$

$$\Rightarrow 800{,}000 + 10{,}000 \times 11.6113 \times 2.9583 = 200{,}000\frac{1 - 1.075^{-t}}{\ln(1.075)}$$

$$\Rightarrow 800{,}000 + 343{,}500.14 = 200{,}000\frac{1 - 1.075^{-t}}{\ln(1.075)}$$

$$-t = \frac{\ln(0.58651)}{\ln(1.075)}$$

$\Rightarrow t = 7.377856$ years

[6]

ii

After the loan has been repaid, all proceeds will be invested at 6.25%

$A_{10} = 200{,}000\bar{s}_{\overline{10-t}|6.25\%}$

$\Rightarrow A_{10} = 200{,}000\dfrac{1.0625^{10-7.377855} - 1}{\ln(1.0625)}$

$\Rightarrow A_{10} = 568{,}411.53$

[4]

iii.

The annual rate of the continuous cashflow is the revenue less the running costs, which has an accumulated value of @ 6.25%

$(450{,}000 - 250{,}000)\bar{s}_{\overline{10}|6.25\%} = 2{,}749{,}825.88$

the accumulated value of the interest payments is

$800{,}000 \times 0.075 \times s_{\overline{10}|6.25\%} = 60{,}000 \times s_{\overline{10}|6.25\%}$ =800,194.34

and the accumulated value of the marketing costs is

$10{,}000 \times 12\ddot{a}^{(12)}_{\overline{1}|6.25\%} \times \left(1 + \frac{1.06}{1.0625} + \frac{1.06^2}{1.0625^2}\right) \times 1.0625^{10} = 640{,}572.412995$

The loan needs to be repaid at $t = 10$, the accumulated value of the project is therefore
$A_{10} = 2{,}749{,}825.88 - 800{,}194.34 - 640{,}572.412995 - 800{,}000$
$= 509{,}059.1270048$

[7]

iv.
Since the interest on the loan is greater than the rate earned on excess funds, the venture is more profitable when the loan is repaid as soon as possible. The accumulated value of the venture is therefore less when the loan is for a fixed term.

[3]

In part (i) interpolation is not an acceptable method for calculating the discounted payback period (DPP). Interpolation should only be used when a mathematical solution does not exist, which is not the case here.

In part (ii) many candidates demonstrated a lack of knowledge as to the meaning of the DPP. At the DPP the accumulated profit, using the interest rate at which you borrowed the money, is zero. So, in this part only the cashflows after the DPP should be used with the new interest rate of 6.25% per annum.

In part (iii) a clear lack of knowledge on interest-only loans was demonstrated.

[Total 20]
[GRAND TOTAL 100]