Subject A213

Contingencies

Intermediate Technical Syllabus

For the 2019 Examinations

October 2018

Aim

The aim of the Contingencies subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value cashflows that are dependent on death, survival, or other uncertain risks.

Competencies

On the successful completion of this subject, the candidate will be able to describe, interpret and discuss mathematical techniques used to model and value cashflows which are contingent on mortality and morbidity risks.

Links to other subjects

Concepts are introduced in:
A111 – Actuarial Statistics
A211 – Financial Mathematics

Topics in this subject are further built upon in:
A214 – Financial Engineering and Loss Reserving
A311 – Actuarial Risk Management
NA311 – Core Actuarial Professional Practice
F101 – Health and Care Principles
F102 – Life Insurance Principles
F104 – Retirement and Related benefits

Syllabus Topics

1   Single decrement models (20%)

2   Multiple decrement models (20%)

3   Pricing and reserving (60%)

The weightings are indicative of the approximate balance of the assessment of this subject between the main syllabus topics, averaged over a number of examination sessions.

The weightings also have a correspondence with the amount of learning material underlying each syllabus topic. However, this will also reflect aspects such as:

- The relative complexity of each topic, and hence the amount of explanation and support required for it
- The need to provide thorough foundation understanding on which to build the other objectives
- The extent of prior knowledge which is expected
- The degree to which each topic area is more knowledge or application based

Skills Level

The use of a specific command verb within a syllabus objective does not indicate that this is the only form of question which can be asked on the topic covered by that objective. The Examiners may ask a question on any syllabus topic using any of the agreed command verbs, as are defined in the document "Command verbs used in the Associate and Fellowship written examinations".

Questions may be set at any skill level: Knowledge (demonstration of a detailed knowledge and understanding of the topic), Application (demonstration of an ability to apply the principles underlying the topic within a given context) and Higher Order (demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations).

In the Contingencies subject, the approximate split of assessment across the three skill types is 20% Knowledge, 65% Application and 15% Higher Order skills.

Detailed Syllabus Objectives

1. Single decrement models

1.1. Define various assurance and annuity contracts.
    1.1.1. Define the following terms:
- Whole life assurance
- Term assurance
- Pure endowment
- Endowment assurance
- Whole life level annuity
- Temporary level annuity
- Guaranteed level annuity
- Premium
- Benefit

including assurance and annuity contracts where benefits are deferred.
    1.1.2. Describe the operation of conventional with-profits contracts, in which profits are distributed by the use of regular reversionary bonuses, and by terminal bonuses. Describe the benefits payable under the above assurance-type contracts.
    1.1.3. Describe the operation of conventional unit-linked contracts, in which death benefits can be expressed as a combination of an absolute amount and the value of a unit fund.
    1.1.4. Describe the operation of accumulating with-profits contracts, in which benefits take the form of an accumulating fund of premiums, where either:
- the fund is defined in monetary terms, has no explicit charges, and is increased by the addition of regular guaranteed and bonus interest payments plus a terminal bonus; or
- the fund is defined in terms of the value of a unit fund, is subject to explicit charges, and is increased by regular bonus additions plus a terminal bonus (Unitised with-profits).

In the case of unitised with-profits, the regular additions can take the form of (a) unit price increases (guaranteed and/or discretionary), or (b) allocations of additional units.
In either case a guaranteed minimum monetary death benefit may be applied.

1.2. Develop formulae for the means and variances of the payments under various assurance and annuity contracts, assuming a constant deterministic interest rate.

1.2.1. Describe the life table functions l_x and d_x and their select equivalents $l_{[x]+r}$ and $d_{[x]+r}$.

1.2.2. Define the following probabilities: $_np_x$, $_nq_x$, $_{n|m}q_x$, $_{n|}q_x$ and their select equivalents $_np_{[x]+r}$, $_nq_{[x]+r}$, $_{n|m}q_{[x]+r}$, $_{n|}q_{[x]+r}$.

1.2.3. Express the probabilities defined in 1.2.2 in terms of life table functions defined in 1.2.1.

1.2.4. Define the assurance and annuity factors and their select and continuous equivalents. Extend the annuity factors to allow for the possibility that payments are more frequent than annual but less frequent than continuous.

1.2.5. Understand and use the relations between annuities payable in advance and in arrear, and between temporary, deferred and whole life annuities.

1.2.6. Understand and use the relations between assurance and annuity factors using equation of value, and their select and continuous equivalents

1.2.7. Obtain expressions in the form of sums/integrals for the mean and variance of the present value of benefit payments under each contract defined in 1.1.1, in terms of the (curtate) random future lifetime, assuming:
   - contingent benefits (constant, increasing or decreasing) are payable at the middle or end of the year of the contingent event or continuously.
   - annuities are paid in advance, in arrear or continuously, and the amount is constant, or increases or decreases by a constant monetary amount or by a fixed or time-dependent variable rate.
   - premiums are payable in advance, in arrear or continuously; and for the full policy term or for a limited period.
   Where appropriate, simplify the above expressions into a form suitable for evaluation by table look-up or other means.

1.2.8. Define and evaluate the expected accumulations in terms of expected values for the contracts described in 1.1.1 and contract structures described in 1.2.7.

2. Multiple decrement and multiple life models

2.1. Define and use assurance and annuity functions involving two lives.

2.1.1. Extend the techniques of objectives 1.2 to deal with cashflows dependent upon the death or survival of either or both of two lives.

2.1.2. Extend the technique of 2.1.1 to deal with functions dependent upon a fixed term as well as age.

2.2. Describe and illustrate methods of valuing cashflows that are contingent upon multiple transition events.

2.2.1. Define health insurance, and describe simple health insurance premium and benefit structures.

2.2.2. Explain how a cashflow, contingent upon multiple transition events, may be valued using a multiple-state Markov Model, in terms of the forces and probabilities of transition.

2.2.3. Construct formulae for the expected present values of cashflows that are contingent upon multiple transition events, including simple health insurance premiums and benefits, and calculate these in simple cases. Regular premiums and sickness benefits are payable continuously and assurance benefits are payable immediately on transition.

2.3. Describe and use methods of projecting and valuing expected cashflows that are contingent upon multiple decrement events.

2.3.1. Describe the construction and use of multiple decrement tables.

2.3.2. Define a multiple decrement model as a special case of a multiple-state Markov model.

2.3.3. Derive dependent probabilities for a multiple decrement model in terms of given forces of transition, assuming forces of transition are constant over single years of age.

2.3.4. Derive forces of transition from given dependent probabilities, assuming forces of transition are constant over single years of age.

3. Pricing and reserving

3.1. Define the gross random future loss under an insurance contract, and state the principle of equivalence.

3.2. Describe and calculate gross premiums and reserves for assurance and annuity contracts.

3.2.1. Define and calculate gross premiums for the insurance contract benefits as defined in objective 1.1 under various scenarios, using the equivalence principle or otherwise. This includes scenarios where:
- Contracts may accept only a single premium
- Regular premiums and annuity benefits may be payable annually, more frequently than annually, or continuously.
- Death benefits (which increase or decrease by a constant compound rate or by a constant monetary amount) may be payable at the end of the year of death, or immediately on death.
- Survival benefits (other than annuities) may be payable at defined intervals other than at maturity.

3.2.2. State why an insurance company will set up reserves.

3.2.3. Define and calculate gross prospective and retrospective reserves.

3.2.4. State the conditions under which in general the prospective reserve is equal to the retrospective reserve allowing for expenses.

3.2.5. Prove that, under the appropriate conditions, the prospective reserve is equal to the retrospective reserve, with or without allowance for expenses, for all fixed benefit and increasing/decreasing benefit contracts.

3.2.6. Obtain recursive relationships between successive periodic gross premium reserves, and use this relationship to calculate the profit earned from a contract during the period.

3.2.7. Outline the concepts of net premiums and net premium valuation and how they relate to gross premiums and gross premium valuation, respectively.

3.3. Define and calculate, for a single policy or a portfolio of policies (as appropriate):
- death strain at risk
- expected death strain
- actual death strain
- mortality profit

for policies with death benefits payable immediately on death or at the end of the year of death; for policies paying annuity benefits at the start of the year or on survival to the end of the year; and for policies where single or non-single premiums are payable.

3.4. Project expected future cashflows for whole life, endowment and term assurances, annuities, unit-linked contracts, and conventional/unitised with-profits contracts, incorporating multiple decrement models as appropriate.

3.4.1. Profit test life insurance contracts of the types listed above and determine the profit vector, the profit signature, the net present value, and the profit margin

3.4.2. Show how a profit test may be used to price a product, and use a profit test to calculate a premium for life insurance contracts of the types listed above.

3.4.3. Show how gross premium reserves can be computed, using the above cashflow projection model, and included as part of profit testing.

3.5. Show how, for unit-linked contracts, non-unit reserves can be established to eliminate ("zeroise") future negative cashflows, using a profit test model.

Assessment

Combination of a one hour 45 minute computer based modelling assignment and a two hour written examination.

End of Syllabus