ACTUARIAL SOCIETY OF SOUTH AFRICA

# Subject A211

# Financial Mathematics

# Intermediate Technical Syllabus

For the 2019 Examinations

October 2018

## Aim

The aim of the Financial Mathematics subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value known cashflows.

## Competencies

On the successful completion of this subject, the candidate will be able to

1. Describe the basic principles of actuarial modelling.
2. Describe, interpret and discuss the theories of interest rates.
3. Describe, interpret and discuss mathematical techniques used to model and value known cashflows.

## Links to other subjects

Concepts are introduced in:
A111 – Actuarial Statistics

Topics in this subject are further built upon in:
A113 – Business Finance
A213 – Contingencies
A214 – Financial Engineering and Loss Reserving
A311 – Actuarial Risk Management
NA311 – Core Actuarial Professional Practice
F105 – Finance and Investment Principles

## Syllabus Topics

1 Data and basics of modelling (15%)

2 Theory of interest rates (40%)

3 Equation of value and its applications (45%)

The weightings are indicative of the approximate balance of the assessment of this subject between the main syllabus topics, averaged over a number of examination sessions.

The weightings also have a correspondence with the amount of learning material underlying each syllabus topic. However, this will also reflect aspects such as:

- The relative complexity of each topic, and hence the amount of explanation and support required for it
- The need to provide thorough foundation understanding on which to build the other objectives
- The extent of prior knowledge which is expected
- The degree to which each topic area is more knowledge or application based

## Skills Level

The use of a specific command verb within a syllabus objective does not indicate that this is the only form of question which can be asked on the topic covered by that objective. The Examiners may ask a question on any syllabus topic using any of the agreed command verbs, as are defined in the document "Command verbs used in the Associate and Fellowship written examinations".

Questions may be set at any skill level: Knowledge (demonstration of a detailed knowledge and understanding of the topic), Application (demonstration of an ability to apply the principles underlying the topic within a given context) and Higher Order (demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations).

In the Financial Mathematic subject, the approximate split of assessment across the three skill types is 25% Knowledge, 65% Application and 10% Higher Order skills.

## Detailed Syllabus Objectives

### 1. Data and basics of modelling

1.1. Data analysis
- 1.1.1. Describe the possible aims of a data analysis (e.g. descriptive, inferential, and predictive).
- 1.1.2. Describe the stages of conducting a data analysis to solve real-world problems in a scientific manner and describe tools suitable for each stage.
- 1.1.3. Describe sources of data and explain the characteristics of different data sources, including extremely large data sets.
- 1.1.4. Explain the meaning and value of reproducible research and describe the elements required to ensure a data analysis is reproducible.

1.2. Describe the principles of actuarial modelling.
- 1.2.1. Describe why and how models are used.
- 1.2.2. Explain the benefits and limitations of modelling.
- 1.2.3. Explain the difference between a stochastic and a deterministic model, and identify the advantages/disadvantages of each.
- 1.2.4. Describe the characteristics of, and explain the use of, scenario-based and proxy models.
- 1.2.5. Describe, in general terms, how to decide whether a model is suitable for any particular application.
- 1.2.6. Explain the difference between the short-run and long-run properties of a model, and how this may be relevant in deciding whether a model is suitable for any particular application.
- 1.2.7. Describe, in general terms, how to analyse the potential output from a model, and explain why this is relevant to the choice of model.

1.2.8. Describe the process of sensitivity testing of assumptions and explain why this forms an important part of the modelling process.
1.2.9. Explain the factors that must be considered when communicating the results following the application of a model.

1.3. Describe how to use a generalised cashflow model to describe financial transactions.
1.3.1. State the inflows and outflows in each future time period and discuss whether the amount or the timing (or both) is fixed or uncertain for a given cashflow process.
1.3.2. Describe in the form of a cashflow model the operation of financial instruments like zero coupon bonds, fixed interest securities, index- linked securities, cash on deposit, equities, interest only loans, repayment loans and annuities certain; and insurance contracts like endowments, term assurances, contingent annuities, car insurance policies and health cash plans.

## 2. Theory of interest rates

2.1. Show how interest rates may be expressed in different time periods.
2.1.1. Describe the relationship between the rates of interest and discount over one effective period, arithmetically and by general reasoning.
2.1.2. Derive the relationships between the rate of interest payable once per measurement period (effective rate of interest) and the rate of interest payable p (> 1) times per measurement period (nominal rate of interest) and the force of interest.
2.1.3. Calculate the equivalent annual rate of interest implied by the accumulation of a sum of money over a specified period where the force of interest is a function of time.

2.2. Demonstrate a knowledge and understanding of real and money interest rates.

2.3. Describe how to take into account time value of money using the concept of compound interest and discounting.
2.3.1. Accumulate a single investment at a constant rate of interest under the operation of simple and compound interest.
2.3.2. Define the present value of a future payment.
2.3.3. Discount a single investment under the operation of a simple (commercial) discount at a constant rate of discount.

2.4. Calculate the present value and accumulated value for a given stream of cashflows under the following individual or combination of scenarios:
2.4.1. Cashflows are equal at each time period.
2.4.2. Cashflows vary with time, and may or may not be a continuous function of time.
2.4.3. Some of the cashflows are deferred for a period of time.
2.4.4. The rate of interest or discount is constant.
2.4.5. The rate of interest or discount varies with time, and may or may not be a continuous function of time.

2.5. Define and derive the following compound interest functions (where payments can be in advance or in arrears) in terms of $i$, $v$, $n$, $d$, $\delta$, $i^{(p)}$ and $d^{(p)}$:

2.5.1. $a_{\overline{n}|}$, $s_{\overline{n}|}$, $a^{(p)}_{\overline{n}|}$, $s^{(p)}_{\overline{n}|}$, $\ddot{a}_{\overline{n}|}$, $\ddot{s}_{\overline{n}|}$, $\ddot{a}^{(p)}_{\overline{n}|}$, $\ddot{s}^{(p)}_{\overline{n}|}$, $\bar{a}_{\overline{n}|}$ and $\bar{s}_{\overline{n}|}$

2.5.2. ${}_{m|}a_{\overline{n}|}$, ${}_{m|}a^{(p)}_{\overline{n}|}$, ${}_{m|}\ddot{a}_{\overline{n}|}$, ${}_{m|}\ddot{a}^{(p)}_{\overline{n}|}$ and ${}_{m|}\bar{a}_{\overline{n}|}$

2.5.3. $(Ia)_{\overline{n}|}$, $(I\ddot{a})_{\overline{n}|}$, $(I\bar{a})_{\overline{n}|}$ and $(\overline{Ia})_{\overline{n}|}$ and the respective deferred annuities

2.6. Show an understanding of the term structure of interest rates.

2.6.1. Describe the main factors influencing the term structure of interest rates.

2.6.2. Explain what is meant by, derive the relationships between and evaluate:

- Discrete spot rates and forward rates.
- Continuous spot rates and forward rates.

2.6.3. Explain what is meant by the par yield and yield to maturity.

2.7. Understanding duration, convexity and immunisation of cashflows:

2.7.1. Define the duration and convexity of a cashflow sequence, and illustrate how these may be used to estimate the sensitivity of the value of the cashflow sequence to a shift in interest rates.

2.7.2. Evaluate the duration and convexity of a cashflow sequence.

2.7.3. Explain how duration and convexity are used in the (Redington) immunisation of a portfolio of liabilities.

## 3. Equation of value and its applications

3.1. Define an equation of value.

3.1.1. Define an equation of value, where payment or receipt is certain.

3.1.2. Describe how an equation of value can be adjusted to allow for uncertain receipts or payments.

3.1.3. Understand the two conditions required for there to be an exact solution to an equation of value.

3.2. Use the concept of equation of value to solve various practical problems.

3.2.1. Apply the equation of value to loans repaid by regular instalments of interest and capital. Obtain repayments, interest and capital components, the effective interest rate (APR) and construct a schedule of repayments.

3.2.2. Calculate the price of, or yield (nominal or real allowing for inflation) from, a bond (fixed-interest or index-linked) where the investor is subject to deduction of income tax on coupon payments and redemption payments are subject to deduction of capital gains tax.

3.2.3. Calculate the running yield and the redemption yield for the financial instrument as described in 3.2.2.

3.2.4. Calculate the upper and lower bounds for the present value of the financial instrument as described in 3.2.2 when the redemption date can be a single date within a given range at the option of the borrower.

3.2.5. Calculate the present value or yield (nominal or real allowing for inflation) from an ordinary share or property, given constant or variable rate of growth of dividends or rents.

3.3. Show how discounted cashflows and equation of value techniques can be used in project appraisals.
  3.3.1. Calculate the net present value and accumulated profit of the receipts and payments from an investment project at given rates of interest.
  3.3.2. Calculate the internal rate of return, payback period and discounted payback period and discuss their suitability for assessing the suitability of an investment project.

## Assessment

Two hour written examination.

## End of Syllabus