

Abstracts of recent postgraduate theses and dissertations at South African universities

Essays on statistical economics with applications to financial market instability, limit distribution of loss aversion, and harmonic probability weighting functions

By G Charles-Cadogan for PhD (Economics) at the University of Cape Town, 2015

This dissertation is comprised of four essays. It develops statistical models of decision-making in the presence of risk with applications to economics and finance. The methodology draws upon economics, finance, psychology, mathematics and statistics. Each essay contributes to the literature by either introducing new theories and empirical predictions or extending old ones with novel approaches.

The first essay (Chapter II) includes, to the best of our knowledge, the first known limit distribution of the myopic loss aversion (MLA) index derived from micro-foundations of behavioural economics. That discovery predicts several new results. We prove that the MLA index is in the class of α -stable distributions. This striking prediction is upheld empirically with data from a published meta-study on loss aversion; published data on cross-country loss aversion indexes; and macroeconomic loss aversion index data for US and South Africa. The latter results provide contrast to Hofstede's cross-cultural uncertainty avoidance index for risk perception. We apply the theory to information-based asset pricing and show how the MLA index mimics information flows in credit risk models. We embed the MLA index in the pricing kernel of a behavioural consumption-based capital asset pricing model (B-CCAPM) and resolve the equity premium puzzle. Our theory predicts: (1) stochastic dominance of good states in the B-CCAPM Markov matrix induce excess volatility; and (2) a countercyclical fourfold pattern of risk attitudes.

The second essay (Chapter III) introduces a probability model of 'irrational exuberance' and financial market instability implied by index option prices. It is based on a behavioural empirical local Lyapunov exponent (BELLE) process we construct from micro-foundations of behavioural finance. It characterises stochastic stability of financial markets, with risk attitude factors, in fixed point neighbourhoods of the probability weighting functions implied by index option prices. It provides a robust early warning system for market crash across different credit risk sources. We show how the model would have predicted the Great Recession of 2008. The BELLE process characterises Minsky's financial instability

hypothesis that financial markets transit from financial relations that make them stable to those that make them unstable.

The third essay (Chapter IV) introduces an outcome-dependent harmonic probability weighting function (HPWF) based on an information theory of stochastic choice. We use the HPWF to resolve the preference reversal (PR) puzzle—which is observed in economics and psychology experiments when a decision maker's (DMs) preferences over the same items change depending upon how she is subsequently asked to construct a preference. We use the principle of maximum entropy to synthesise information processing, probabilistic choice, and momentary fluctuation hypotheses proposed by various researchers to explain intransitivity implied by PR phenomenon. The HPWF theory is illustrated via simulation. Additionally, we show how the HPWF decomposes regret theory, and rank dependent utility (RDU), into core expected utility theory (EUT) plus functionally equivalent stochastic error addends. This theoretical prediction finds support in Hey and Orme's (1994) seminal experiments on the difference between generalised EUT and core EUT models. We also prove that experimenter interference with the probability cycle of DMs' HPWF causes them to observe preference reversal in stochastic choice experiments even though the true state is transitive and there is no violation of procedure invariance.

The fourth essay (Chapter V) shows that Bernoulli's (1738) original utility function is alive and well. For example, several papers reexamine Bernoulli's (1738) expected utility resolution of the St Petersburg Paradox in the context of cumulative prospect theory (CPT). We go a step further. We re-examine the geometry of Bernoulli's original sketch of his utility function. We prove that, contrary to received literature, which alleges that Bernoulli's utility function is unable to generate a loss aversion index (ULA), the geometry of Bernoulli's original sketch accommodates a ULA index with smooth reference dependent utility functions. In fact, it provides a solution to the open problem of closed form global ULA index formula in prospect theory. Like in the first essay, the ULA index predicted by Bernoulli's utility function is a-stable. Under fairly mild assumptions, we show how it supports a Fisher z-transform statistical test for the loss aversion index and we show how the test can be applied.

An assessment of the feasibility of using administrative data in producing mid-year population estimates for South Africa

By Mbongeni C Hlabano for MPhil (Actuarial Science) at the University of Cape Town, 2015

The production of mid-year population estimates is an important undertaking which informs various stakeholders in policy formation and decision-making. For instance, national governments use mid-year estimates to allocate seats in parliament to various constituents and public health sectors use them to monitor and improve service delivery. Mid-year population estimates undoubtedly serve very important purposes that affect lives of many people. As such, national statistical offices in various countries are given the mandate to produce annual mid-year population estimates.

Statistics South Africa (Stats SA) assumes the function of producing and publishing official mid-year estimates of the population in South Africa. Stats SA produces its mid-year

estimates using DemProj, population projection software which is part of the SPECTRUM suite of policy models developed by the Futures Institute. However, Stats SA does not publish full details of its adaptation of Demproj when producing its mid-year estimates as it regards this as proprietary. Concerns have been raised about the accuracy of the official mid-year estimates in terms of age distribution, particularly for ages below 40 last birthday in 2011 (e.g. Dorrington, 2013). As such, this research critically analyses the method used by Stats SA to produce mid-year estimates and assesses the feasibility of using administrative data to produce mid-year estimates for South Africa.

The base population is adapted from the 2001 census population. Birth and death registration data are used in a cohort component approach to produce alternative mid-year estimates for South Africa for the years 2002–2011. Prior to using these data, they are adjusted for incompleteness of registration. Levels of completeness of birth and death registration are estimated by extrapolating earlier estimates of completeness from various researchers. The mid-year estimates obtained are compared with those published by Stats SA in order to assess the relative quality of the two series of mid-year estimates. The mid-year estimates for 2011 are also compared with the mid-year population estimated from the 2011 census. These comparisons help identify the mismatches to the census and their possible causes and as such, these may lead to improved population estimates in the future, and a viable alternative method to that currently being used by Stats SA.

Compound Lévy random bridges and credit risky asset pricing

By Dennis Chimemerem Ikpe for PhD (Mathematics and Applied Mathematics) at the University of Cape Town, 2016

In this thesis, we study random bridges of a certain class of Lévy processes and their application to credit risky asset pricing. In the first part, we construct the compound random bridges (CLRBs) and analyse some tools and properties that make them suitable models for information processes. We focus on the Markov property, dynamic consistency, measure changes and increment distributions. Thereafter, we study their applications in credit risky asset pricing. We generalise the information-based credit risky asset pricing framework to incorporate prematurity default possibilities. Lastly we derive closed-form expressions for default trend and intensities for a randomly timed cash flow with a CLRB as the background partial information process. We obtain analytical expressions for specific CLRBs. The second part looks at application of stochastic filtering in the current information-based asset pricing framework. First, we formulate our information-based framework as a filtering problem under incomplete information. We derive the Kalman–Bucy filter in one dimension for bridges of Lévy processes with a given finite variance.

Enhanced minimum variance optimisation: a pragmatic approach

By Lakhoo Lala Bernisha Janti for MSc (Statistics) at the University of Cape Town, 2016

Numerous studies have been carried out over the past decade that have found that low volatility portfolios have outperformed market capitalisation-based portfolios. The minimum variance portfolio (MVP) is one such portfolio and has attracted the attention of academics and practitioners alike. This thesis endeavours to extend on this, by means of adding factors to the minimum variance framework via the objective function, which are intended to increase the likelihood of outperforming both the market and the minimum variance portfolio (MVP). An analysis of the impact of these factor tilts on the MVP is carried out in the South African environment, represented by the FTSE-JSE Shareholder weighted Index as the benchmark portfolio. The main objective is to examine if the systematic and robust methods employed, which involve the incorporation of factor tilts into the multi-criteria problem, together with covariance shrinkage—improve the performance of the MVP. The factor tilts examined include Active Distance, Concentration and Volume. Additionally, the constant correlation model is employed in the estimation of the shrinkage intensity, structured covariance target and shrinkage estimator. The results of this study showed that with specific levels of factor tilting, one can generally improve both absolute and risk-adjusted performance and lower concentration levels in comparison to both the MVP and benchmark. Additionally, lower turnover levels were observed across all tilted portfolios, relative to the MVP. Furthermore, covariance shrinkage enhanced all portfolio statistics examined, but significant improvement was noted on drawdown levels, capture ratios and risk. This is in contrast to the results obtained when the standard sample covariance matrix was employed.

Analysing the structure and nature of medical scheme benefit design in South Africa

By Josh Tana Kaplan for MPhil (Actuarial Science) at the University of Cape Town, 2015

This dissertation intends to shed light on open-membership medical scheme benefit design in South Africa. This will be done by analysing the benefit design of 118 benefit options, so as to provide an overview of the structure and nature of the benefit offerings available in the market in 2014. In addition, affordability of these benefit options was analysed in order to identify whether or not there exist connections between the benefits on offer and the price of cover. This paper will argue that at present, the large number of benefit options available in the market, the lack of standardisation between benefit options, together with the mosaic of confusing terminology employed in scheme brochures, creates a highly complex environment that hampers consumer decision-making. However, this implicit complexity was found to be necessary owing to the incomplete regulatory environment surrounding medical schemes. The findings of this investigation show that benefit design requires significant attention in order to facilitate equitable access to cover in South Africa.

Understanding the low volatility anomaly in the South African equity market

By Bhekinkosi Khuzwayo for MCom (Statistics) at the University of Cape Town, 2015

The Capital Asset Pricing Model (CAPM) advocates that expected return has a linear proportional relationship with beta (and subsequently volatility). As such, the higher the systematic risk of a security the higher the CAPM expected return. However, empirical results have hardly supported this view as argued as early as Black (1972). Instead, an anomaly has been evidenced across a multitude of developed and emerging markets, where portfolios constructed to have lower volatility have outperformed their higher volatility counterparts as found by Baker and Haugen (2012). This result has been found to exist in most equity markets globally. In the South African market the studies of Khuzwayo (2011), Panulo (2014) and Oladele (2014) focused on establishing whether low volatility portfolios had outperformed market-cap weighted portfolios in the South African market. While they found this to be the case, it is important to understand if this is truly an anomaly or just a result of prevailing market conditions that have rewarded lower volatility stocks over the back-test period. As such, those conditions might not exist in the future and low volatility portfolios might then underperform. This research does not aim to show, yet again, the existence of this ‘anomaly’; instead the aim is to dissect if there is any theoretical backing for low volatility portfolios to outperform high volatility portfolios. If this can be uncovered, then it should help one understand if the ‘anomaly’ truly exists and also if it can be expected to continue into the future.

A framework for regime identification and asset allocation

By Mpumelelo Kondlo for MSc (Statistics) at the University of Cape Town, 2016

The purpose of this thesis was to examine a regime-based asset allocation strategy and to establish the potential advantages of accounting for economic regimes in tactical asset allocation. The setting is South African-centric. The relevance of such a study is supported by the increased number of practitioner research reports devoted to this area of research. This includes unpublished research by Silberman and Munro (2008) from Cadiz Securities, Sheikh and Sun (2011) from J.P. Morgan Asset Management, Davis, Aliaga-Díaz and Patterson, (2011), and Briere and Signori (2012) from Amundi Asset Management.

A broad range of asset classes are considered in this thesis including local and foreign cash, local and foreign bonds, local and foreign equity, property, gold, commodities and inflation-linked bonds. It is worth mentioning that while other studies considered a subset of these asset classes, no study was found that considered inflation-linked bonds as a separate asset classes. The following process was followed in the thesis:

1. Performance of a PCA decomposition on the economic data for dimension reduction and the establishment of the weighting factors of the economic variables.
2. Use of a time series of the PCA-reduced data to perform a Fuzzy Cluster Analysis on the time points for different economic regime identification.
3. Evaluation and analysis of the performance of the individual asset classes relative to specified benchmarks under the different regimes.

4. The implementation of asset allocation optimisation for each spliced regime using an approach which uses ordinal returns rather than the actual returns.
5. Assessment of the performance of a portfolio constructed using tactical asset allocation based on the identified regimes.
6. Comparison of the regime-balanced portfolio with a range of benchmarks in-sample, including proprietary data of the Alexander Forbes Large Manager Watch.
7. The thesis highlights the potential that regime-based tactical asset allocation can add if accurate regime-based forecasting is implemented.

Low-volatility alternative equity indices

By Oladele Oluwatosin Seun for MSc (Statistics) at the University of Cape Town, 2015

The objective of the thesis is to assess the suitability of a range of low-volatility index construction methodologies in the South African market environment using indices as building blocks. The thesis is conducted as follows:

- It constructs low-volatility portfolios using a variety of low-volatility construction techniques with the underlying components being the nine second-tier indices on the Johannesburg Stock Exchange (JSE). The performance of the various portfolios is assessed in an out-of-period setting. This work follows the study by Leclerc et al. (2013) in the international literature.
- An additional technique implemented involves the use of a covariance shrinkage estimator (proposed by Ledoit and Wolf (2004)) to improve estimation errors in the covariance matrix.
- Prior work by Khuzwayo (2011) and Panulo (2014) on low-volatility portfolios in South Africa is extended by the inclusion of several more low-volatility strategies found in the literature. In total, nine different low-volatility strategies were considered including a recent strategy based on Principle Component Analysis. These strategies are summarised in the accompanying table.
- The covariance bi-plot was also used as a graphical way of interpreting the risks of the low volatility portfolios in an out-of-period setting.
- The thesis finds that there were two low-volatility strategies that seemed to perform best in the South African environment.
- The entire study was also repeated using individual stocks as building blocks for the low-volatility portfolios to establish if the results using the indices were consistent with the results for stocks.

Optimal investment, consumption and life insurance in a Lévy market

By Calisto Justino Guambe for MSc (Mathematics of Finance) at the University of Pretoria, 2015

The purpose of this dissertation is to solve an optimal investment, consumption and life insurance problem described by jump-diffusion processes in two settings.

First, we consider a problem with random parameters of a wage earner who wants to save to his beneficiary for his death. Using one risk-free asset and one risky asset price given by a geometric jump-diffusion process, we obtain the optimal strategy via the dynamic programming approach, combining the Hamilton–Jacobi–Bellman equation with a backward stochastic differential equations with jumps.

Secondly, we discuss the optimal investment, consumption and life insurance problem with capital constraints. The problem consists of one risk-free asset and two risky asset prices defined in an independent Brownian motion and Poisson process. We derive the optimal strategy of the unconstrained problem via martingale approach, from which the problem with capital constraint is solved applying the option-based portfolio insurance method.

Evaluation of the South African equity markets in a value-at-risk framework

By Lesedi Mabitsele for MSc (Financial Engineering) at the University of Pretoria, 2015

The statistical distribution of financial returns plays a key role in evaluating Value-at-Risk using parametric methods. Traditionally, when evaluating parametric Value-at-Risk, the statistical distribution of the financial returns is assumed to be normally distributed. However, though simple to implement, the Normal distribution underestimates the kurtosis and skewness of the observed financial returns. This dissertation focuses on the evaluation of the South African equity markets in a Value-at-Risk framework. Value-at-Risk is estimated on five equity stocks listed on the Johannesburg Stock Exchange, including the FTSE/JSE TOP40 index and the S&P 500 index. The statistical distribution of the financial returns is modelled using the Normal Inverse Gaussian and is compared to the financial returns modelled using the Normal, Skew t -distribution and Student t -distribution. We then estimate Value-at-Risk under the assumption that financial returns follow the Normal Inverse Gaussian, Normal, Skew t -distribution, Student t -distribution and Extreme Value Theory and backtesting was performed under each distribution assumption. The results of these distributions are compared and discussed.

Variable annuity guarantees pricing under the variance gamma framework

By Alvin Macharia Ngugi for MSc (Financial Engineering) at the University of Pretoria, 2015

The purpose of this study is a consideration of the pricing of embedded derivatives in life insurance variable annuity products in a Lévy process setting. This is one of the practical issues that continue to face life insurers in the management of derivatives embedded within these products. It also addresses how such providers can protect themselves against adverse scenarios through a hedging framework built from the pricing framework.

The objective is to comparatively consider the price differentials of a life insurer that prices its variable annuity guarantees under the more actuarially accepted regime-switching framework versus the use of a Lévy framework. The framework should address the inadequacies of conventional deterministic pricing approaches used by life insurers given the increasing complexity of the option-like products sold. The study seeks to apply finance

models in the insurance context given the similarities in payoff structure of the products offered while taking into consideration the differences that may exist.

The underlying Lévy process used in this study is the variance gamma process. This process is useful in option pricing given its ability to model higher moments, skewness and kurtosis, and also incorporate stochastic volatility.

The results of the research compare well with the regime-switching framework with the added merit of the use of a more refined model for the underlying that captures most of the observed market dynamics.

Conditional large deviations for losses on portfolios with heterogeneity between constituent groups

By Nadine Mari Walters for PhD (Actuarial Science) at the University of Pretoria, 2015

The research provides an estimate of the distribution of losses between homogeneous classes in a large financial portfolio, given that a large total loss was experienced. For example, in the case of a corporate bond portfolio, individual constituents could be grouped into classes according to their credit rating. In order to calculate the amount of capital required for supporting risky portfolios, financial institutions often use scenario analysis, particularly stress testing, as part of the risk management procedure. When simulating stressed scenarios it is useful to have this additional information in order to allocate appropriate proportions of the total loss between the constituents of the portfolio. Institutions can therefore obtain additional information to aid in capital allocation and the setting of exposure limits to the different classes. The approach taken to derive the probabilities makes use of large deviations theory with techniques from measure- and probability theory. The assumption of independent and identically distributed random variables is relaxed in that constituents in different portfolio groups are not assumed to be identically distributed. Furthermore, the results can easily be applied to the case where constituents are dependent on some external factor, thereby relaxing the independence assumption as well. The asymptotic joint conditional distribution of the individual portfolio constituents given a specified large loss was found to take the form of a Gibbs measure. This approximation performed well even for small or almost completely heterogeneous portfolios. Furthermore we provide an illustration of how to use these results in order to allocate large losses among portfolio groups.

An application of exponential smoothing methods to weather-related data

By Double-Hugh Marera for MSc (Statistics) at the University of the Witwatersrand, 2016

Exponential smoothing is a recursive time series technique whereby forecasts are updated for each new incoming data values. The technique has been widely used in forecasting, particularly in business and inventory modelling. Up until the early 2000s, exponential smoothing methods were often criticised by statisticians for lacking an objective statistical basis for model selection and modelling errors. Despite this, exponential smoothing methods appealed to forecasters due to their forecasting performance and relative ease of use. In

this research report, we apply three commonly used exponential smoothing methods to two datasets which exhibit both trend and seasonality. We apply the method directly on the data without de-seasonalising the data first. We also apply a seasonal naive method for benchmarking the performance of exponential smoothing methods. We compare both *in-sample* and *out-of-sample* forecasting performance of the methods. The performance of the methods is assessed using forecast accuracy measures. Results show that the Holt–Winters exponential smoothing method with additive seasonality performed best for forecasting monthly rainfall data. The simple exponential smoothing method outperformed the Holt’s and Holt–Winter’s methods for forecasting daily temperature data.

Survival analysis of bank loans and credit risk prognosis

By Mercy Marimo for MSc (Statistics) at the University of the Witwatersrand, 2015

Standard survival analysis methods model lifetime data where cohorts are tracked from the point of origin, until the occurrence of an event. If more than one event occurs, a special model is chosen to handle competing risks. Moreover, if the events are defined such that most subjects are not susceptible to the event(s) of interest, standard survival methods may not be appropriate. This project is an application of survival analysis in a consumer credit context. The data used in this study were obtained from a major South African financial institution covering a five-year observation period from April 2009 to March 2014. The aim of the project was to follow up on cohorts from the point where vehicle finance loans originated to either default or early settlement events and compare survival and logistic modeling methodologies. As evidenced by the empirical Kaplan–Meier survival curve, the data typically had long term survivors with heavy censoring as at March 2014. Cause-specific Cox regression models were fitted and an adjustment was made for each model, to accommodate a proportion p of long-term survivors. The corresponding Cumulative Incidence Curves were calculated per model, to determine probabilities at a fixed horizon of 48 months. Given the complexity of the consumer credit lifetime data at hand, we investigated how logistic regression methods would compare. Logistic regression models were fitted per event type. The models were assessed for goodness of fit. Their ability to differentiate risk was determined using the model Gini Statistics. Model assessment results were satisfactory. Methodologies were compared for each event type using Receiver Operating Characteristic curves and area under the curves. The Results show that survival methods perform better than logistic regression methods when modelling lifetime data in the presence of competing risks and long-term survivors.

Improved confidence intervals for a small area mean under the Fay–Herriot model

By Yegnanew Alem Shiferaw for PhD (Demography) at the University of the Witwatersrand, 2016

There is a growing demand for small area estimates for policy- and decision-making, local planning and fund distribution. Surveys are generally designed to give representative estimates at national or regional level, but estimates of variables of interest are often also

needed at the small area levels. These cannot be reliably obtained from the survey data as the sample sizes at these levels are too small. This problem is addressed by using small area estimation techniques. The main aim of this thesis is to develop confidence intervals (CIs) which are accurate to terms $O(m^{-3/2})$ under the FH model using the Taylor series expansion. Rao (2003a), among others, notes that there is a situation in mixed model estimation that the estimates of the variance component of the random effect, A , can take negative values. In this case, Prasad and Rao (1990) consider $\hat{A} = 0$. Under this situation, the contribution of the mean squared error (MSE) estimate, assuming all parameters are known, becomes zero. As a solution, Rao (2003a) among others proposed a weighted estimator with fixed weights (i.e., $w_1 = 1/2$). In addition, if the MSE estimate is negative, we cannot construct CIs based on the empirical best linear unbiased predictor (EBLUP) estimates. Datta, Kubokawa, Molina and Rao (2011) derived the MSE estimator for the weighted estimator with fixed weights which is always positive. We use their MSE estimator to derive CIs based on this estimator to overcome the above difficulties. The other criticism of the MSE estimator is that it is not area-specific since it does not involve the direct estimator in its expression. Following Rao (2001), we propose area specific MSE estimators and use them to construct CIs. The performance of the proposed CIs are investigated via simulation studies and compared with the Cox (1975) and Prasad and Rao (1990) methods. Our simulation results show that the proposed CIs have higher coverage probabilities. These methods are applied to standard poverty and percentage of food expenditure measures estimated from the 2010/11 Household Consumption Expenditure survey and the 2007 census datasets.