Actuarial Society of South Africa

WRITTEN EXAMINATION

10 OCTOBER 2019

Subject A213 — Contingencies

Time allowed: Two hours and 15 minutes reading time
Maximum: 100 marks

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes at the start of the examination to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 2 hours to complete the paper.*

4. *The use of calculators is not permitted during the reading time.*

5. *Mark allocations are shown in brackets.*

6. *Attempt all questions, beginning your answer to each question on a new page.*

7. *You should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION:

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

QUESTION 1

A select mortality table with a select period of five years is being prepared based on the ultimate q_x-rates and l_x-values in the ELT15 (Males) table. The following information has been provided:

$$q_{[x]} = 0.5q_x$$
$$q_{[x]+1} = 0.6q_{x+1}$$
$$q_{[x]+2} = 0.7q_{x+2}$$
$$q_{[x]+3} = 0.8q_{x+3}$$
$$q_{[x]+4} = 0.9q_{x+4}$$

Find the value of $l_{[50]}$. [Total 7]

QUESTION 2

A 2-year term assurance policy is issued to a life currently aged x. The benefit amounts are R10 000 if the life dies in the first year and R20 000 if the life dies in the second year.

Benefits are payable at the end of year of death.

i. Write down an expression representing the present value random variable for this benefit.
[3]

ii. Calculate the variance of the present value random variable assuming that $q_x = 0.015$ and $q_{x+1} = 0.020$ with an interest of 8% per annum effective. [6]

[Total 9]

QUESTION 3

An insurer sells single premium deferred annuity policies to female lives aged 45 exactly. Each policy provides an annuity income of R180 000 per annum payable annually in advance, commencing at age 60. The policy also provides for a death benefit that is equal to the single premium payable under the contract. The death benefit is payable immediately on death if death occurs after age 60.

Basis:
Mortality	AM92 Ultimate
Interest	6% per annum
Initial expenses	1% of the single premium
Renewal expenses	R100 per annum payable from the 2nd policy year onwards (assume that all expenses are paid at the beginning of the year)

i. Given that the single premium is R987 559.31, calculate the reserve at the end of the 20th policy year [3]

ii. There are 1 300 policies in force at the end of the 19th policy year. It is also known that 20 lives died during the 20th policy year. Calculate the mortality profit during the 20th policy year. [5]

iii. Explain why it is reasonable that the insurer has incurred a mortality profit or loss in the 20th policy year. [3]

[Total 11]

QUESTION 4

i. Describe the four different methods of allocating bonuses to conventional with-profits policies. [4]

ARK Life has recently launched a 20-year with-profits endowment assurance policy. The policy offers a sum assured of R500 000 to a life aged 45 exactly. The sum assured plus declared bonuses are payable at the end of year of death or on maturity of the policy, if earlier. Level premiums are payable monthly in advance.

ii. A simple bonus, expressed as a level percentage of the sum assured, vests at the beginning of each year. Calculate the level simple bonus rate that can be supported each year if the monthly premium is R2 730.

Basis:
Mortality	AM92 Ultimate
Rate of interest	4% per annum
Initial expenses	15% of the first year's annual premium
Renewal expenses	5% of each premium payable and
	$R50*(1.04)^{k}$ on each policy anniversary (where $k \geq 1$ is the exact duration of the policy on that anniversary)
Claim expenses	R250 at termination of the contract [15]

[Total 19]

PLEASE TURN OVER

QUESTION 5

i.   Show that $\bar{A}_x = \bar{A}^1_{xy} + \bar{A}^2_{xy}$.   [2]

ii.   Show that $\bar{A}_{xy} = \bar{A}^1_{xy} + \bar{A}^1_{xy}$.   [2]

iii.   Consequently, show that $\bar{A}_{\overline{xy}} = \bar{A}^2_{xy} + \bar{A}^2_{xy}$.   [2]

[Total 6]

QUESTION 6

A life aged 40 exactly purchases a special term assurance policy with a term of 20 years from a leading South African life insurer. The benefit paid on death is as follow:

o   On death between exact ages 40 and 45 the benefit is equal to the total premiums paid to date of death, without interest.
o   From age 45 exactly onwards, an amount of R500 000 is paid on death.

All payments on death are made at the end of the year of death. Level premiums are paid annually in advance for the full 20-year term or until earlier death. Claim expenses are only incurred from exact age 45 and onwards.

i.   Calculate the annual premium payable.

Basis:
Mortality   AM92 Ultimate
Interest   6% per annum
Initial expenses   50% of the first year's premium, incurred at outset
Renewal expenses   7.5% of the second and each subsequent year's premium, incurred
                    at the beginning of the respective policy years
Claims expense   5% of the sum assured payable at the end of the year of death
Initial Commission R500, paid at outset   [10]

A consulting actuary has highlighted that the current pricing basis does not allow for any inflation of future expenses.

ii.   Briefly describe what the impact on the premium will be due to expense inflation, if the inflation rate is assumed to be positive.   [1]

[Total 11]

PLEASE TURN OVER

QUESTION 7

You are an actuary working in the valuation office of a small life insurer operating in an African country. Your company sells without-profit whole life assurance polices with sums assured of R75 000 which are payable at the end of the year of death. Level premiums of R1 000 are payable annually in advance.

i.   Describe the conditions under which the retrospective reserves of a policy will be equal to the prospective reserves.   [3]

One of the policyholders wishes to surrender his policy on the policy's 4th anniversary, immediately before the premium then due. The insurance company calculates a surrender value equal to the gross retrospective policy reserve as at the date of surrender. The policyholder was 55 years old exactly when he purchased the policy.

ii.   Calculate the surrender value.

       Basis:
       Mortality   AM92 Select
       Interest   6% pa
       Expenses   Initial Expenses of R2 000, at outset
                          Administration fee on surrender of R200   [7]

The insurance regulator requires that statutory reserves are calculated on a prospective basis and that companies must include the following margins on top of their internal reserving basis:

Claim expenses: 10% of the sum assured
Renewal expenses margin: Renewal expense assumption increased with 15%
Interest: The company's reserving basis interest assumption reduced, in
                            absolute terms, by 2%

iii.   Calculate the reserve for a policy exactly five years old, which was sold to a policyholder aged 50 exactly at the time of the sale.

       Basis without margins:
       Mortality   AM92 Ultimate
       Renewal Expenses   R500 pa
       Initial Expenses   R2 000
       Interest   6% pa   [4]

iv.   Explain how each of the retrospective and prospective reserves would be affected if the actuary's expectations of future mortality have changed so that it is now believed to be 10% lower than the current mortality basis.   [4]
                                                                                   [Total 18]

PLEASE TURN OVER

QUESTION 8

i. Show that $\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t}$, where $\mu_{x+t:y+t}$ is the force of failure of the joint life status. Also state the assumption(s) necessary for this relationship to hold.   [3]

Two male lives are both aged 40 exactly.

ii. Calculate the expected present value of a 15-year term assurance with a benefit of R500 000 payable immediately on the second death.

    Basis:

    Mortality $\mu_x = 0.02$ for all x for male life 1

                      $\mu_y = 0.03$ for all y for male life 2

    Interest   5% per annum   [7]
                                              [Total 10]

QUESTION 9

A special joint life annuity of R100 per day is payable in respect of a male life aged 65 exactly and female life aged 68 exactly. Payments commence on the first death and continue for three years after the second death.

Calculate the expected present value of this annuity.

Basis:
Mortality   PMA92C20 (male), PFA92C20 (female)
Rate of interest   4% per annum
Expenses   Nil

                                               [Total 9]

                                       [GRAND TOTAL 100]

<center>**END OF EXAMINATION**</center>