Actuarial Society of South Africa

WRITTEN EXAMINATION MEMORANDUM

10 OCTOBER 2019

Subject A213 — Contingencies

Time allowed: Two hours and 15 minutes reading time
Maximum: 100 marks

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes at the start of the examination to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 2 hours to complete the paper.*

4. *The use of calculators is not permitted during the reading time.*

5. *Mark allocations are shown in brackets.*

6. *Attempt all questions, beginning your answer to each question on a new page.*

7. *You should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION:

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

GENERAL COMMENTS

The paper rewarded prepared students well. Many students ran out of time with this paper possibly due to using more than 1.2 minutes per mark. Marks in A213 are more generally awarded than other actuarial examinations due to only being a two-hour paper for 100 marks.

Many students lost out on many marks because of poor layout of answers. It is highly advisable to always show the values calculated for actuarial notation separately to make it clear to the examiners where a student might have gone wrong. Examiners are making through with errors but where it is not clear where an error was or how it affected the rest of a solution, limited marks are awarded.

Generally speaking, the paper was straight forwarded and students who practiced many past papers scored very well.

QUESTION 1

$q_{[50]} = 0.5 * q_{50}$

= 0.5*0.00464

=0.00232

$p_{[50]} = 1 - q_{[50]}$

=0.99768

$q_{[50]+1} = 0.6 * q_{51}$

= 0.6* 0.00519

=0.003114

$p_{[50]+1} = 1 - q_{[50]+1}$

=0.996886

$q_{[50]+2} = 0.7 * q_{52}$

=0.7*0.00577

= 0.004039

$p_{[50]+2} = 1 - q_{[50]+2}$

= 0.995961

$q_{[50]+3} = 0.8 * q_{53}$

= 0.8*0.00642

=0.005136

$p_{[50]+3} = 1 - q_{[50]+3}$

=0.994864

$q_{[50]+4} = 0.9 * q_{54}$

= 0.9*0.00714

= 0.006426

$p_{[50]+4} = 1 - q_{[50]+4}$

=0.993574

$$\frac{l_{55}}{l_{[50]}} = p_{[50]} * p_{[50]+1} * p_{[50]+2} * p_{[50]+3} * p_{[50]+4}$$

$$l_{[50]} = \frac{91217}{90.994864 * 0.99357 * 0.99768 * 0.99689 * 0.99596)}$$

=93 160.7

Examiners' Comments

This question was relatively well answered by students who were able to follow the recursive relationships and to link the basic q_x -rates and l_x -values formulaically.

QUESTION 2

(i)The present value random variable is

$$PV = \begin{cases} 10\,000v & \text{if } K_x = 0 \\ 20\,000v^2 & \text{if } K_x = 1 \\ 0 & \text{if } K_x \geq 2 \end{cases}$$

(ii) $E(PV) = 10000v_{8\%}q_x + 20000v_{8\%}^2 p_x q_{x+1}$

$$= \frac{10000}{(1.08)} * 0.015 + \frac{20000}{(1.08)^2} * (1 - 0.015) * (0.02)$$

=476.68

$$E(PV^2) = (10000v_{8\%})^2 q_x + (20000v_{8\%}^2)^2 p_x q_{x+1}$$

$$= (\frac{10000}{1.08})^2 * (0.015) + (\frac{20000}{(1.08)^2})^2 * (1 - 0.015) * (0.02)$$

=7 078 043.47

$$Var(PV) = 7078043.47 - (476.68)^2$$

=6 850 819.28

Examiners' Comments

In part i, many students did not provide a random variable and only an equation for the EPV. This was not awarded marks.

In part ii, students who recalled how to calculate a variance of a random variable scored well.

QUESTION 3

(i) ${}_{20}V = (180000 + 100)\ddot{a}_{65} + P\bar{A}_{65}$

$= (180100 * 10.569) + 987559.31 * (1.06)^{0.5} * (0.40177)$

$=$ <u>2 311 978.37</u>

(ii) DSAR = $P(1+i)^{0.5} - {}_{20}V$

$= 987\ 559.31 * (1.06)^{0.5} - 2\ 311\ 978.37$

= - 1 295 223.83

ADS = 20*-1 295 223.83

=-25 904 476.69

EDS = 1 300* q_{64} *DSAR

= 1 300* 0.012716*(-1 295 223.83)

= -21 411 086.16

Mortality Profit = EDS-ADS

=-21 411 086.16-(-25 904 476.69)

= <u>4 493 390.53</u>

(iii) The insurer has incurred a mortality profit because

- The death strain at risk is negative in the 20^{th} year
- Actual number of deaths (20) is greater than the expected number of deaths (16.5)
- With negative death strain at risk, more reserves are releases on each death, hence the profit

Examiners' Comments

A relatively straight forward question done well by many. Students scored particularly poorly in part iii which asks for a reasonability of the result. Students should generally spend more time thinking about their answers and its meaning rather than merely regurgitating a method that they studied.

QUESTION 4

(i) Simple reversionary bonus-the rate of bonus each year is a percentage of initial sum assured
Compound Reversionary bonus - the rate of bonus each year is a percentage of the initial sum assured plus reversionary bonuses previously declared
Super compound bonus – there are two rates of bonus. One is applied to the basic sum assured, the other is applied to the previously declared bonus.
Terminal bonus- Sum assured is increased at maturity or on earlier claim. The bonus is normally a percentage of the final sum assured.

(ii) Let b denote the simple bonus rate (expressed as a percentage of sum assured)

$$12P\ddot{a}^{(12)}_{45:\overline{20}|} = (500000+250)A_{45:\overline{20}|} + 5000b[(IA)_{45:\overline{20}|} + 20*v^{20}*\frac{l_{65}}{l_{45}}] + 0.15(12P) + 0.05*12P\ddot{a}^{(12)}_{45:\overline{20}|} + 50(\ddot{a}^{0\%}_{45:\overline{20}|} - 1)$$

Where

$$A_{45:\overline{20}|} = 0.46998$$

$$\ddot{a}^{(12)}_{45:\overline{20}|} = \ddot{a}_{45:\overline{20}|} - \frac{11}{24}\left(1 - v^{20}*\frac{l_{65}}{l_{45}}\right)$$

$$=13.780 - \frac{11}{24}\left(1 - v^{20}*\frac{8821.2612}{9801.3123}\right)$$

=13.50993

$$\ddot{a}^{0\%}_{45:\overline{20}|} = (1+e_{45}) - {}_{20}p_{45}*(1+e_{65})$$

$$= (1+34.271) - \frac{8821.2612}{9801.3123}*(1+16.645)$$

=19.3906

$$(IA)_{45:\overline{20}|} = (IA)_{45} - v^{20}*\frac{l_{65}}{l_{45}}(20A_{65} + (IA)_{65}$$

=8.33628-(0.45639)*(0.900)*(20*0.52786+7.89442)

=0.757241

$$20*v^{20}*\frac{l_{65}}{l_{45}} = 20*0.45639*0.900)$$

=8.2150397

Hence

0.95*2730*13.50993=500250*0.46998+1.8*2730+50*(19.3906-1)+5000b(8.2150397+0.757241)

$$b = \frac{179476.78}{44861.40}$$

B = 4.0006948

i.e. a simple bonus rate of 4% per annum.

Examiners' Comments

Part i was basic bookwork but concerning that some students did not score full marks for this part.

Part ii was done well by those who noted that an increasing assurance function is required to deal with the simply (not compound) bonus rate. Only a few students managed to get a reasonable result.

QUESTION 5

i.

$$\bar{A}^{1}_{x\,y} + \bar{A}^{\;\;2}_{x\,y} = \int_0^\infty v^t\,{}_tp_x\mu_{x+t}\,{}_tp_y dt + \int_0^\infty v^t\,{}_tp_x\mu_{x+t}\,{}_tq_y dt$$

$$= \int_0^\infty v^t\,{}_tp_x\mu_{x+t}({}_tp_y + {}_tq_y)dt$$

$$= \int_0^\infty v^t\,{}_tp_x\mu_{x+t}dt$$

$$= \bar{A}_x$$

ii.

$$\bar{A}^{1}_{x\,y} + \bar{A}^{\;\;1}_{x\,y} = \int_0^\infty v^t\,{}_tp_{xy}\mu_{x+t}dt + \int_0^\infty v^t\,{}_tp_{xy}\mu_{y+t}dt$$

$$= \int_0^\infty v^t\,{}_tp_{xy}(\mu_{x+t} + \mu_{y+t})dt$$

$$= \int_0^\infty v^t\,{}_tp_{xy}\mu_{x+t:y+t}dt$$

$$= \bar{A}_{xy}$$

ii.

$$\bar{A}^{\;\;2}_{x\,y} = \bar{A}_y - \bar{A}^{\;\;1}_{x\,y}$$

Similary :

$$\bar{A}^{2}_{x\,y} = \bar{A}_x - \bar{A}^{1}_{x\,y}$$

So :

$$\bar{A}^{\;\;2}_{x\,y} + \bar{A}^{2}_{x\,y} = \bar{A}_y - \bar{A}^{\;\;1}_{x\,y} + \bar{A}_x - \bar{A}^{1}_{x\,y}$$

$$= \bar{A}_y + \bar{A}_x - \left(\bar{A}^{\;\;1}_{x\,y} + \bar{A}^{1}_{x\,y}\right)$$

$$= \bar{A}_y + \bar{A}_x - \bar{A}_{x\,y}$$

$$= \bar{A}_{\overline{x\,y}}$$

Examiners' Comments

This question contained an unfortunate typing error in part ii. As a result, part ii and iii was disregarded and the marks was adjusted. Students who noted the error received bonus marks.

Part ii should have read:

$$\bar{A}_{xy} = \bar{A}^{1}_{xy} + \bar{A}_{x\overset{1}{y}}$$

As the first death can either be life x or life y to die first.

The published exam paper is the corrected version.

QUESTION 6

i.

EPV of benefits:

$$P(IA)^{1}_{40:\overline{5}|} + 500,000v^{5}\,{}_{5}p_{40}A^{1}_{45:\overline{15}|}$$

Now:

$$(IA)^{1}_{40:\overline{5}|} = (IA)_{40} - v^{5}\,{}_{5}p_{40}\left[(IA)_{45} + 5A_{45}\right]$$

$${}_{5}p_{40} = \frac{l_{45}}{l_{40}} = \frac{9,801.3123}{9,856.2863} = 0.9944$$

So:

$$(IA)^{1}_{40:\overline{5}|} = 3.85435 - (1.06)^{-5} \times 0.9944\left[4.37062 + 5 \times 0.15943\right] = 0.01423$$

and:

$$A^{1}_{45:\overline{15}|} = A_{45} - v^{15}\,{}_{15}p_{45}A_{60}$$

$$= 0.15943 - (1.06)^{-15} \times \frac{9,287.2164}{9,801.3123} \times 0.32692 = 0.030172$$

EPV of Premiums =

$$P\ddot{a}_{40:\overline{20}|} = 11.998P$$

EPV of Expenses =

$$0.5P + 0.075P(\ddot{a}_{40:\overline{20}|} - 1) + 0.05 \times 500,000 v^5 {}_5p_{40} A^1_{45:\overline{15}|} + 500$$
$$= 0.5P + 0.075P(10.998) + (18,576.84)(0.030172) + 500$$
$$= 1.32485P + 1,060.503$$
$now:$

$$EPV\ of\ Premiums = EPV\ of\ Benefits + EPV\ of\ Expenses$$

$$11.998P = 1.32485P + 0.01423P + 1,2270.82$$
$Thus:$
$$P = R1,151.23$$

ii. Premium will increase, as future expenses will be higher.

Examiners' Comments

In part i, many students missed the part that claim expenses are only incurred from exact age 45 and onwards.

Part ii showed whether a student had good exam technique or not. This was a very simple question and could have been answered without doing part i.

QUESTION 7

i.

1. The retrospective and prospective reserves are calculated on the same basis, and
2. This basis is the same as the basis used to calculate the premiums used in the reserve calculation.

ii.

[EPV of Premiums – EPV of Benefits – EPV of Expenses] accumulated to time of surrender

$$= \left[1,000\ddot{a}_{[55]:\overline{4}|} - 75,000A^{1}_{[55]:\overline{4}|} - 2,000\right](1.06)^4 \times \frac{l_{[55]}}{l_{59}} - 200$$

$$Now:$$

$$\ddot{a}_{[55]:\overline{4}|} = \ddot{a}_{[55]} - v^4 \times {}_4p_{[55]}\ddot{a}_{59}$$

$$with:$$

$${}_4p_{[55]} = \frac{l_{59}}{l_{[55]}} = \frac{9,354.0040}{9,545.9229} = 0.979895$$

$$Thus:$$

$$\ddot{a}_{[55]:\overline{4}|} = 13.072 - (1.06)^{-4} \times 0.9799 \times 12.138 = 3.650817$$

$$A^{1}_{[55]:\overline{4}|} = A_{[55]} - v^4 \times {}_4p_{[55]}A_{59}$$

$$= 0.26008 - (1.06)^{-4} \times 0.9799 \times 0.31294 = 0.017185$$

So, the surrender value = R 266.36

iii.

Prospective reserve = EPV of Benefits + EPV of Expense – EPV of Premiums

$$So:$$

$${}_5V_{50} = 75,000 \times 1.1A_{55} + 500 \times 1.15(\ddot{a}_{55}) - 1,000\ddot{a}_{55}$$

$$= 75,000(1.1)(0.38950) + 500(1.15)(15.873) - 1,000(15.873)$$

$$= R25,387.73$$

iv.

Retrospective reserve – No change, as the reserve is only backwards looking i.e. only consider past experience.

Prospective reserve – Decrease, as lower future mortality will result in claims being paid later than currently expected, therefore a reduction in the expected present value of benefits.
In addition, the insurer will expect to receive more premiums, also reducing the prospective reserve.

Examiners' Comments

Part i was simple bookwork and hence done well by many. Some only gave vague descriptions and therefore lost out on easy marks here.

In part ii, students did fairly well but many incorrectly allowed for the surrender administration fee of R200.

Part iii was generally well done.

Part iv was done worse than expected. Many students incorrectly noted that the retrospective reserve will also change.

QUESTION 8

i.

$$\mu_{x+t:y+t} = -\frac{d}{dt}\ln l_{x+t:y+t}$$
$$= -\frac{d}{dt}\ln l_{x+t} l_{y+t}$$
$$= -\frac{d}{dt}\left\{\ln l_{x+t} + \ln l_{y+t}\right\}$$
$$= \mu_{x+t} + \mu_{y+t}$$

This relationship holds under the assumption that life *x* and life *y* are independent.

ii.

EPV of benefit =

$$= 500,000\bar{A}^{\;1}_{40:40:\overline{15}|}$$
$$= 500,000\int_0^{15} v^t\left({}_t p_{40}^{m1}\mu_{40+t}^{m1}\,{}_t q_{40}^{m2} + {}_t p_{40}^{m2}\mu_{40+t}^{m2}\,{}_t q_{40}^{m1}\right)dt$$
$$= 500,000\int_0^{15} (1.05)^{-t}[e^{-0.03t}(0.03)(1-e^{-0.02t}) + e^{-0.02t}(0.02)(1-e^{-0.03t})]dt$$
$$= 500,000\int_0^{15} e^{-\ln(1.05)t}[0.03e^{-0.03t} - 0.03e^{-0.05t} + 0.02e^{-0.02t} - 0.02e^{-0.05t})]dt$$
$$= 500,000\int_0^{15} [0.03e^{-0.07879t} + 0.02e^{-0.06879t} - 0.05e^{-0.09879t}]dt$$
$$= 500,000\left[\frac{-0.03e^{-0.07879t}}{0.07879} - \frac{0.02e^{-0.06879t}}{0.06879} + \frac{0.05e^{-0.09879t}}{0.09879}\right]_0^{15}$$
$$= 500,000(-0.10539 + 0.165374)$$
$$= 29,993.74$$

Examiners' Comments

At this stage of the paper, time pressure started to show for some students.

Part i was standard bookwork, but as before, an area where students lose out on easy marks.

Many scored full marks for part ii but many made no attempt.

QUESTION 9

The EPV of benefit can be approximated with a continuous annuity.

EPV of annual benefit of 1 =

$$\left[\bar{a}_{\overline{65:68}} - \bar{a}_{65:68}\right] + \bar{a}_{\overline{3}|}\bar{A}_{\overline{65:68}}$$

$now:$

$$\bar{a}_{\overline{65:68}} = \bar{a}^{m}_{65} + \bar{a}^{f}_{68} - \bar{a}_{65:68}$$

$$= (13.666 - 0.5) + (13.723 - 0.5) - (11.382 - 0.5) = 15.507$$

$$\bar{A}_{\overline{65:68}} = 1 - \delta\bar{a}_{\overline{65:68}}$$

$$= 1 - \ln(1.04) \times 15.507 = 0.3918$$

$$\bar{a}_{\overline{3}|} = \frac{(1 - v^3)}{\delta} = \frac{[1 - (1.04)^{-3}]}{\ln(1.04)} = 2.83023$$

EPV of 1 = 5.7339

Annual benefit = $365.25 \times 100 = 36,525$ *(or 36500)*

Thus EPV $= 36,525 \times 5.7339 = 209,430.57$

Examiners' Comments

This question scored the lowest of the paper. This is a typical question on joint lives.