Actuarial Society of South Africa

EXAMINATION

SEMESTER 1 2021
EXAMINER COMMENTS

Subject A213 — Contingencies

WRITTEN EXAMINATION

Generally, students would have scored better marks where the following exam techniques have been applied.

- Label each question clearly. Some students marked their page numbers as well as question numbers next to each other which caused confusion at times.
- Start each question on a new page.
- Use more space rather than less and make it clear how you break up key parts of a question. This makes it easier to see your workings and to understand what was done to give principal marks for incorrect solutions.
- Show the results of your intermediate steps.
- Some students lost marks because the electronic scans were not clear, although we have observed an improvement in this regard during this session.
- Better planning of your time.

Overall, the results from this semester's exams were somewhat disappointing. Well prepared students continued to score well with a large number of clear passes, however, we have seen a material increase in the number of students in the FC and FD categories who failed to grasp the basic concepts of the syllabus.

QUESTION 1

This question had a range of extreme results with many scoring full marks and many little to no marks.

Average mark: 50-55%

Given that $l_x = 115 - x$

we know that $d_x = l_{x+1} - l_x$ for all values of x

$\rightarrow d_x = (115 - x) - (115 - (x+1)) = 1$

we want to get $A^{\,1}_{45:\overline{20|}}$

$$\text{where } A^{\,1}_{45:\overline{20|}} = \sum_{k=0}^{19} v^{k+1} \frac{d_{45+t}}{l_{45}}$$

$$= \sum_{k=0}^{19} v^{k+1} \frac{1}{l_{45}}$$

$$= \frac{a_{\overline{20|}}}{(115-45)}$$

$$\text{where } a_{\overline{20|}} = \frac{(1-v^{20})}{0.075} = 10.19499$$

$$\therefore\ A^{\,1}_{45:\overline{20|}} = \frac{10.19499}{70} = 0.145636$$

Value of benefit = 100*0.145636 = 145.64 (2 d.p.)

QUESTION 2

This was s standard bookwork question. Some students might not have expected this part of the syllabus to be examined in the written paper. Students are reminded that the full syllabus can be examined in either the written paper or the computer-based paper.

Average mark: 60-65%

i) Unit Fund

-belongs to the policyholder

-this fund keeps track of the premiums allocated to units and the benefits payable from this fund to policyholders are denominated in these units

-this fund is normally subject to unit fund charges

Non-unit Fund

-belongs to the company

-this fund keeps track of premiums paid by the policyholder which are not allocated to units together with unit fund charges from the unit fund

-company expenses will be charged to this fund together with other benefits above the available unit fund that are payable to policyholders

ii) The profit vector is a array of the expected year-end profits for policies which are still in force at the start of each year.

The profit signature is an array of the expected year end profits allowing for survivorship from the start of the contract.

QUESTION 3

Many students lost out on easy marks where they have defined the "loss" random variable rather than the "profit" random variable. Some lost marks for giving an expression for the expected present value and not a random variable. Many students also showed a poor understanding of deriving a variance expression for the given random variable.

Average mark: 45-55%

(i) Profit=present value of premiums less present value of benefits

If the curtate lifetime of a policyhoder is K_x then the present value is

of the premiums is $P\ddot{a}_{\overline{K_x+1}|}$ and the present value of the benefits is SV^{Kx+1}

since $\ddot{a}_{\overline{K_x+1}|} = \dfrac{1-V^{Kx+1}}{d}$ it follows that

$$\text{Profit} = P\left(\frac{1-V^{K_{[35]}+1}}{d}\right) - SV^{K_{[35]}+1}$$

$$\text{Profit} = \frac{P}{d} - \left(\frac{P}{d}+S\right)V^{K_{[35]}+1}$$

$$(ii) Var(\text{Profit}) = Var\left(\frac{P}{d} - \left(\frac{P}{d}+S\right)V^{K_{[35]}+1}\right)$$

$$= \left(\frac{P}{d}+S\right)^2 Var\left(V^{K_{[35]}+1}\right)$$

$$= \left(\frac{P}{d}+S\right)^2 \left({}^2A_{[35]} - \left(A_{[35]}\right)^2\right)$$

where P=92; S=10 000; ${}^2A_{[35]} = 0.04861$

$A_{[35]} = 0.19207$; d = $\dfrac{0.04}{1.04} = 0.038461535$

$$Var(\text{Profit}) = \$^2\left(\frac{92}{0.038461535}+10000\right)^2 * \left(0.04861-(0.19207)^2\right)$$

$=\$^2\ (12\ 392)^2 * 0.011719115$

$=\$^2\ 1\ 799\ 606.82$

Standard deviation = $\sqrt{1\ 799\ 606.82}$ = \$1 341.49

QUESTION 4

This question was one of the higher order questions in the paper which tested the application of the basic theory of how ultimate and select mortality works. The result was disappointing in the context that students were given the answer so generally had the opportunity to rework their calculations until they find the correct solution.

Average mark: 30-40%

$$\ddot{a}_{40:\overline{20}|} = 1 + p_{40}v + {}_2p_{40}v^2\ddot{a}_{42:\overline{18}|}$$

$$\text{now } \ddot{a}_{42:\overline{18}|} = \frac{\left(\ddot{a}_{40:\overline{20}|} - p_{40}v - 1\right)}{{}_2p_{40}v^2}$$

where $\ddot{a}_{40:\overline{20}|} = 17.598;\ v = 0.990099010\ \textit{and}\ v^2 = 0.980296049$

$p_{40} = 1 - q_{40} = 1 - 0.000937 = 0.999063$

$p_{41} = 1 - q_{41} = 1 - 0.001014 = 0.998986$

${}_2p_{40} = p_{40} * p_{41} = 0.99804995$

$$\therefore \ddot{a}_{42:\overline{18}|} = \left(\frac{17.598 - 1 - 0.990099010 * 0.999063}{0.99804995 * 0.980296049}\right) = 15.95367664$$

expressing the select annuity in expanded form gives the following

$$\ddot{a}_{[40]:\overline{20}|} = 1 + p_{[40]}v + {}_2p_{[40]}v^2\ddot{a}_{42:\overline{18}|}$$

where

$p_{[40]} = 1 - q_{[40]} = 1 - 0.000788 = 0.999212$

$p_{[40]+1} = 1 - q_{[40]+1} = 1 - 0.000962 = 0.999038$

${}_2p_{[40]} = p_{[40]} * p_{[40]+1} = 0.999212 * 0.999038 = 0.998250758$

$$\ddot{a}_{[40]:\overline{20}|} = 1 + (0.999212 * 0.990099010\ ) + (0.998250758 * 0.980296049) * 15.95367664$$

=17.60128803

(ii) EPV=600*180 000*17.601

=R1 900 908 000

QUESTION 5

Generally, a disappointing result where students failed to apply the basic theory of joint life calculations.

Average mark: 35-40%

(i) Under constant force of mortality, the instantaneous rate of mortality is assumed to take the same value between consecutive integer ages. This enables us to calculate survival probabilities over non-integer time periods and from non-integer ages

$$(ii)\ \Pr ob = \int_0^\infty {}_t p_{40}\mu_{40+t}\ {}_t p_{30}\ {}_{10}q_{30+t}\,dt$$

$$= \int_0^\infty {}_t p_{40}\mu_{40+t}\ {}_t p_{30}(1 - {}_{10}p_{30+t})\,dt$$

$$= \int_0^\infty {}_t p_{40}\mu_{40+t}({}_t p_{30} - {}_t p_{30}\ {}_{10}p_{30+t})\,dt$$

$$= \int_0^\infty {}_t p_{40}\mu_{40+t}({}_t p_{30} - {}_{t+10}p_{30})\,dt$$

where ${}_t p_x = e^{-\mu t}$ *under C.F.M assumption*

$$= \int_0^\infty e^{-0.02t}.0.02.(e^{-0.02t} - e^{-0.02(t+10)})\,dt$$

$$= \int_0^\infty 0.02.(e^{-0.04t} - e^{-0.02(2t+10)})\,dt$$

$$= 0.02\left(\left[\frac{e^{-0.04t}}{-0.04}\right]_0^\infty - \left[\frac{e^{-0.02(2t+10)}}{-0.04}\right]_0^\infty\right)$$

$$= \frac{0.02}{0.04}\left((e^0 - e^{-\infty}) - (e^{-0.02(0+10)} - e^{-\infty})\right)$$

$$= 0.5(1 - e^{-0.2})$$

$$= 0.090635$$

QUESTION 6

Well prepared students scored well on this question which tested reasonably standard application. As the premium answer was again given, student could have used the opportunity to fix their calculations where their result did not agree with the given answer.

Average mark: 40-50%

Let P denote the monthly premiums

EPV of premiums=$12P\ddot{a}^{(12)}_{45:\overline{20|}}$

where $\ddot{a}^{(12)}_{45:\overline{20|}} = \ddot{a}_{45:\overline{20|}} - \frac{11}{24}\left(1 - v^{20}_{4\%}\frac{l_{65}}{l_{45}}\right)$

$$=13.780-\frac{11}{24}\left(1-(1.04)^{-20}\frac{8821.2612}{9801.3123}\right)$$

$$=13.50993$$

$\therefore$ EPV of premiums=12*P*13.50993=162.11916P

EPV expenses=$1500+250\bar{A}_{45} + (0.015)*12P*\ddot{a}^{(12)}_{45:\overline{20|}} + 0.005P$

$$=1500+250*\sqrt{1.04}*0.27605 + (0.015*12)*P*13.50993 + 0.005P$$

$$=1\ 570.379217+2.4367874P$$

EPV Benefits =$1\ 200\ 000(v^{0.5}q_{45} + v^{1.5}\ {}_{1|}q_{45}(1.04) + v^{2.5}\ {}_{2|}q_{45}*(1.04)^2 + ...)$

$$=1\ 200\ 000v^{0.5}(q_{45} + v*(1.04)*\ {}_{1|}q_{45} + v^2*(1.04)^2*\ {}_{2|}q_{45} + ...)$$

$$=\frac{1200000}{\sqrt{1.04}}\left(q_{45} + {}_{1|}q_{45} + {}_{2|}q_{45} + ...\right)$$ expression in brackets sums to 1

$$=1\ 176\ 696.81$$

now

EPV Premiums=EPV Benefits +EPV Expenses

$162.11916P = 1\ 176\ 696.81 + 1\ 570.379217+2.4367874P$

$159.682373P = 1\ 178\ 267.19$

P=7 378.82

(ii) Since the same premium is to be charged,the epv of expenses and premiums will be the same as before. We can then equate the previous EPV of benefits to to the new one

let b denote the simple bonus rate payable

$$\text{EPV Benefits(new)}=1\ 200\ 000*\frac{b}{100}*I\bar{A}_{45}+1\ 200\ 000*(1-\frac{b}{100})*\bar{A}_{45}$$

$$=1200000*\frac{b}{100}*\sqrt{1.04}*8.33628+1200000*(1-\frac{b}{100})*\sqrt{1.04}*0.27605$$

$$=10\ 201\ 645.054*\frac{b}{100}-337\ 820.24081*\frac{b}{100}+337\ 820.24081$$

$$=9\ 863\ 824.813*\frac{b}{100}+337\ 820.24081$$

equating the values for the benefits

$$1\ 176\ 696.81=9\ 863\ 824.813*\frac{b}{100}+337\ 820.24081$$

$$\frac{b}{100}=\frac{1\ 176\ 696.81-337\ 820.24081}{9\ 863\ 824.813}$$

$$\therefore b=0.0850457694*100=8.50\%$$

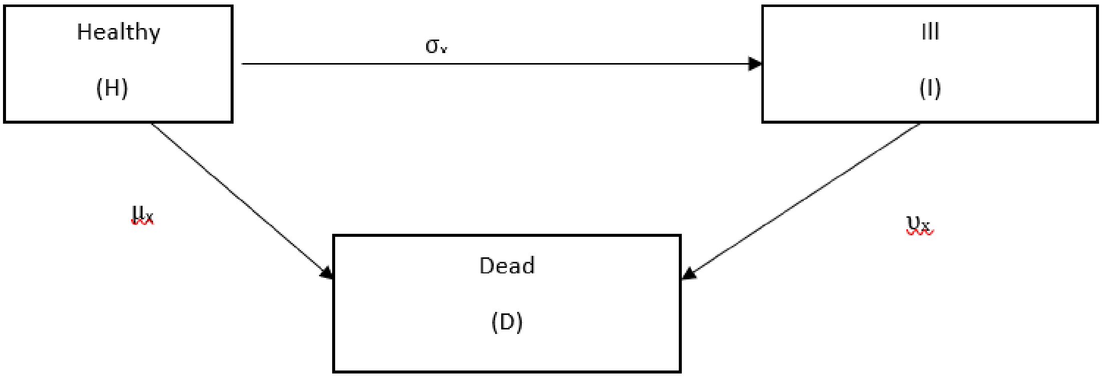
(iii) The sum assured and bonuses increases more slowly than under the other methods for the same ultimate benefit, enabling the office to retain surplus for longer. This method rewards longer standing policyholders and discourages surrenders relative to other methods.

QUESTION 7

This question was generally well done.

Average mark: 65-70%

i)



(ii)

$$\text{EPV of Benefis} = 100000\int_0^{10} e^{-\delta t}\,{}_tp_{55}^{HH}\left(\mu_{55+t}+\sigma_{55+t}\right)dt$$

$$=100000\int_0^{10} e^{-0.1t}\,{}_tp_{55}^{HH}\left(0.003+0.005\right)dt$$

$$=100000\times 0.008.\int_0^{10} e^{-0.1t}\,{}_tp_{55}^{HH}\,dt$$

where ${}_tp_{55}^{HH} = e^{-(\mu_{55+t}+\sigma_{55+t})t} = e^{-0.008t}$

$$\text{EPV of Benefis} = 800\int_0^{10} e^{-0.1t}e^{-0.008t}dt$$

$$=800\int_0^{10} e^{-0.108t}dt$$

$$=\frac{800}{0.108}\left[-e^{-0.108t}\right]_0^{10}$$

$$=\frac{800}{0.108}\left(1-e^{-0.108(10)}\right)$$

$$=4891.88$$

QUESTION 8

The question covered a more complicated joint life case. That said, well prepared students would have done many similar types of joint live and reversionary annuity questions as part of their preparation and therefore scored well.

Average mark: 35-40%

The first condition is the difference betweena last survivor annuity and a joint life annuity

let this portion of the benefit be denoted by A

$$A = 200000\left(a_{\overline{50:50}} - a_{50:50}\right)$$

$$A = 200000(a_{50}^{(m)} + a_{50}^{(f)} - a_{50:50} - a_{50:50})$$

$$A = 200000(a_{50}^{(m)} + a_{50}^{(f)} - 2a_{50:50})$$

$$A = 200000\left(a_{50|50}^{m\ \ f} + a_{50|50}^{f\ \ m}\right)$$

$$A = 200\,000\left(a_{50|50}^{m\ \ f} + a_{50|50}^{f\ \ m}\right)$$

$$= 200\,000\left(a_{50}^{f} - a_{\underset{m\ f}{50:50}} + a_{50}^{m} - a_{\underset{m\ f}{50:50}}\right)$$

$$= 200\,000\left(a_{50}^{f} + a_{50}^{m} - 2\,a_{50:50}^{m\ \ f}\right)$$

= same as step 3.

Second part of benefit is paid at the end of the year of death of the survivor

let B denote this benefit

$$B = 200000\ddot{a}_{\overline{5|}}A_{\overline{50:50}}$$

$$a_{50}^{(m)} = \ddot{a}_{50}^{(m)} - 1 = 18.843 - 1 = 17.843$$

$$a_{50}^{(f)} = \ddot{a}_{50}^{(f)} - 1 = 19.539 - 1 = 18.539$$

$$a_{50:50} = \ddot{a}_{50:50} - 1 = 17.688 - 1 = 16.688$$

$$\ddot{a}_{\overline{50:50}} = \ddot{a}_{50}^{(m)} + \ddot{a}_{50}^{(f)} - \ddot{a}_{50:50} = 18.843 + 19.539 - 17.688 = 20.694$$

where

$$d = \frac{0.04}{1.04} = 0.038461538$$

$$\ddot{a}_{\overline{5|}} = \frac{(1 - v_{4\%}^{5})}{d} = 4.62989$$

$$A_{\overline{50:50}} = 1 - d\ddot{a}_{\overline{50:50}} = 1 - (0.038461538 * 20.694) = 0.204076923$$

$$now\, A = 200000(17.843 + 18.539 - 2 * 16.688)$$

$$= 200000 * 3.006 = 601\,200$$

$$B = 200000 * 4.62989 * 0.204076923 = 188\,970.74$$

$$EPV = A + B$$

$$= 601200 + 188970.74 = 790\,170.74$$

QUESTION 9

Many students lost easy marks for not doing a "retrospective" reserve as the question stated. The rest of the question saw well prepared students being able to successfully complete the reasonably straight forward calculations.

Average mark: 45-50%

$$\text{Reserve}=(1+i)^5 * \frac{l_{[55]}}{l_{60}} * \left(P\ddot{a}_{[55]:\overline{5}|} - 500000\bar{A}^{\ \ 1}_{[55]:\overline{5}|} \right)$$

$\ddot{a}_{[55]:\overline{5}|} = 4.59$; $v^5 = 0.821927107$; $(1+i)^5 = 1.216652902$

$l_{60} = 9287.2164$; $l_{[55]} = 9\,545.993$

$$\bar{A}^{\ \ 1}_{[55]:\overline{5}|} = \bar{A}_{[55]} - v^5 * \frac{l_{60}}{l_{[55]}} * \bar{A}_{60}$$

$$= \sqrt{1.04} * (0.38879 - 0.821927107 * \frac{9287.2164}{9\,545.993} * 0.4564)$$

$$= 0.024304$$

$$\therefore \text{Reserve}=1.216652902 * \frac{9\,545.993}{9287.2164} * \left(2574.61 * 4.59 - 500\,000 * 0.024304\right)$$

$$= -418.07$$

(ii) The product provides more cover in the first 5 years than what is paid for by the premiums in those years

Holding a negative reserve would imply treating the reserve as an asset. If policy lapsed during the first 5 years of the policy, then the insurer would suffer a loss that is not possible to recover from the policyholder.

(iii) Mortality profit = EDS-ADS

Death Strain at Risk $=500000(1.04)^{\frac{1}{2}} - (-418.07) = 510\,320.02$

$$EDS = (500 - 15) * q_{59} * 510\,320.02$$

$$= 485 * 0.00714 * 510\,320.02 = 1\,767\,187.2$$

$$ADS = 13 * 510\,320.02 = 6\,634\,160.26$$

$$\textit{Mortality profit} = 1\,767\,187.2 - 6\,634\,160.26 = -4\ 866\,973 \text{(a mortality loss)}$$

QUESTION 10

This question was generally poorly done with evidence that students started to run out of time. Careful time management in this exam is a critical part of the assessment and student should plan their exam day accordingly.

Average mark: 20-25%

(i) There is a mismatch in the timing at which the insurer receives a premium and the policyholder starts to receive benefits. If the premium that was not required to pay benenfits early in the contract were spent by the company, then later in the contract, if the policyholder is still alive, the company may not be able to find funds to pay the benenfits.

(ii) A=EPV of annuity benefits

$$=10\ 000 * v_{6\%}^{5} * \frac{l_{60}^{AM92}}{l_{55}^{AM92}} * \ddot{a}_{60}^{PMA92@4\%}$$

where $i=\frac{1.06}{1.01923}-1=0.04;\ l_{60}^{AM92}=9\ 287.2164\ ;\ l_{55}^{AM92}=9\ 557.8179;\ \ddot{a}_{60}^{PMA92@4\%}=15.632$

$\therefore\ A=10\ 000*1.06^{-5}*\frac{9\ 287.2164}{9\ 557.8179}*15.632=113\ 504.23$

(iii) The value of the past five years premiums repayable on death in the next five years have the value

$$\frac{\left(11\ 818*\ddot{s}_{\overline{5|}}^{1.923\%}\right)}{l_{55}^{AM92}} * \left[\frac{1.01923}{1.06}*d_{55}+\left(\frac{1.01923}{1.06}\right)^{2}*d_{56}+...+\left(\frac{1.01923}{1.06}\right)^{5}*d_{59}\right]$$

$$=11\ 818*\ddot{s}_{\overline{5|}}^{1.923\%} * \left[v_{4\%}*\frac{d_{55}}{l_{55}^{AM92}}+v_{4\%}^{2}*\frac{d_{56}}{l_{55}^{AM92}}+...+v_{4\%}^{5}*\frac{d_{59}}{l_{55}^{AM92}}\right]$$

$=11\ 818*\ddot{s}_{\overline{5|}}^{1.923\%}*A^{1}_{55:\overline{5|}}$ where the assurance function is valued at 4%

Now

$$A^{1}_{55:\overline{5|}} = A_{55}-v_{4\%}^{5}*\frac{l_{60}^{AM92}}{l_{55}^{AM92}}*A_{60}$$

$$=0.3895-(1.04)^{-5}*\frac{9\ 287.2164}{9\ 557.8179}*0.45640=0.024993101$$

and

$\ddot{s}_{\overline{5|}}^{1.923\%}=\frac{(1.01923)^{5}-1}{d}$ where $d=\frac{0.01923}{1.01923}=0.018867184$

$\therefore\ \ddot{s}_{\overline{5|}}^{1.923\%}=5.295953362$

finally

EPV of Death Benefit=11 818*5.295953362 * 0.024993101 = 1 564.26

(*iv*) Reserve =113 504.23 + 1 564.26=115 068.49