Actuarial Society of South Africa

WRITTEN EXAMINATION

23 April 2019

Subject A213 — Contingencies

Time allowed: Two hours and 15 minutes reading time
Maximum: 100 marks

INSTRUCTIONS TO THE CANDIDATE

1. Enter all the candidate and examination details as requested on the front of your answer booklet.

2. You must not start writing your answers in the booklet until instructed to do so by the supervisor.

3. You have 15 minutes at the start of the examination to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 2 hours to complete the paper.

4. The use of calculators is not permitted during the reading time.

5. Mark allocations are shown in brackets.

6. Attempt all questions, beginning your answer to each question on a new page.

7. You should show calculations where this is appropriate.

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION:

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.
QUESTION 1

The following is an extract from a life table used in an African country:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( d_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>2 569</td>
</tr>
<tr>
<td>65</td>
<td>2 896</td>
</tr>
<tr>
<td>66</td>
<td>3 436</td>
</tr>
<tr>
<td>67</td>
<td>3 977</td>
</tr>
</tbody>
</table>

The number of lives alive at age 65 is 98 529.

Calculate \( 0.8 \ p_{65.5} \) assuming:

i. Uniform distribution of deaths; and
ii. Constant force of mortality

[Total 8]

QUESTION 2

Banks in South Africa generally require customers to obtain life insurance before granting a home loan. A male life aged exactly 40, interested in acquiring a home loan, obtains a 20-year term assurance policy from Coastal Life with the following benefit:

An initial sum assured of R500 000 is paid at the end of year of death. On the twelve policy anniversary, the sum assured decreases by R100 000 and then, every five years thereafter, the sum assured decreases by R100 000.

i. Calculate the expected present value of the assurance benefit.  [7]

**Basis:**
- Interest  4% per annum
- Mortality AM92 Ultimate

The male life acquired a home loan of R500 000. After seventeen years the male life finds another insurer offering a cheaper policy. He has managed to reduce the outstanding amount of his loan by R425 000 at this stage and obtains a three-year level term assurance policy with a sum assured equal to the outstanding loan amount.

ii. Calculate the expected present value of the new policy.  [3]

**Basis:**
- Interest  6% per annum
- Mortality AM92 Select

[Total 10]
QUESTION 3

i. Define the dependent probability of decrement, \( (aq)_{x}^{\alpha} \) \[2\]

ii. Define the independent probability of decrement, \( q_{x}^{\alpha} \) \[2\]

The following multi-state model is used for the purposes of a population projection:

Assume constant transition intensities.

iii. Assuming that \( \rho_{x} = 0 \) and \( \nu_{x} = 0 \), show that \( (aq)_{x}^{\prime} = \frac{\sigma}{\sigma + \mu} \left( 1 - e^{-\left(\mu + \sigma\right)} \right) \),

stating any relevant additional assumptions that you make. \[6\]

[Total 10]

QUESTION 4

From first principles and first defining appropriate random variables, show that:

\[ a_{1}A_{x} = A_{x} - A_{x}^{1} = \nu_{x}^{a} p_{x} A_{x+n} . \]

[Total 8]
QUESTION 5

A life company sells 20-year with-profit endowment assurance policies to lives aged 35 exactly. The basic sum assured is R1 000 000 and compound bonuses of 1.9231% are added to the sum assured at the end of the year. The death benefit is payable at the end of the year of death after the bonus amount for the current year has been added. Level premiums are payable monthly in advance.

_Basis:_

Mortality: AM92 Ultimate
Interest: 6% per annum
Expenses
  - Initial: 45% of premiums payable in the first year
  - Renewal: 3% of each premium from the second year onward

i. Determine the monthly premium. [10]

ii. Calculate the gross premium prospective reserve at the start of the 16th policy year, assuming that the bonuses have been declared according to the initial assumptions. Use a monthly premium of R3500 which was determined on a different basis to the above. [9]

iii. State why an insurance company will set up reserves for a regular premium endowment policy. [3]

[Total 22]

QUESTION 6

First-for-Females insurance company issues 15-year term assurance policies, in return for a single premium. On 1 January 2018, it sells 20 000 such policies to females aged 50 exactly.

The sum assured under the policy is R 1 500 000 and the benefit is payable at the moment of death.

As at 1 January 2018, the company uses the following basis to calculate the premiums and reserves:

_Basis:_

Interest rate: 6% per annum
Mortality: AM92 Ultimate
Expenses and commission: Nil

During the year, 20 policyholders die.

The Statutory actuary revises the reserving basis at the end of the year to the following:

Interest rate: 4% per annum
Mortality: AM92 Ultimate with a 3-year age deduction
Expenses and commission: Nil

Calculate the total mortality profit or loss arising during the year using the latest basis. [Total 9]

PLEASE TURN OVER
QUESTION 7

A married couple (Peter and Sandy) have recently decided to buy a joint whole-life policy to cover the estate duty that might be levied on their estate. Peter is currently exactly aged 52 and Sandy is exactly age 50.

The joint whole-life policy pays a sum assured of R500 000 on the earlier of the death of Peter or Sandy. The benefit is payable immediately on death.

Premiums are payable monthly in advance while both lives are alive, but subject to a maximum of 5 years.

i. Calculate the monthly premium payable for this policy.

Basis:
Interest: 4% per annum
Mortality: PA92C20
Initial Expense: R30 000

ii. List the main events that would normally lead to the premiums being stopped under a policy involving two lives.

iii. Explain, with reasons, the likely impact on the premium if premiums are changed to be paid annually in arrears.

[Total 21]

QUESTION 8

Two lives, Cyril and Helen, are exactly aged 60. They have decided to purchase an annuity paying an income of R20 000 monthly in advance. The monthly payments are only being made while at least one of them is alive and there is a maximum of 300 payments.

Calculate the expected present value of the annuity using PMA92C20 mortality for Cyril and PFA92C20 for Helen and interest of 4% per annum.

[Total 12]

[GRAND TOTAL 100]