

# SPECIMEN EXAMINATION PAPER

April 2019

## Subject A211 — Financial Mathematics

*Time allowed: Two hours and 15 minutes reading time*

### **INSTRUCTIONS TO THE CANDIDATE**

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 2 hours to complete the paper.*
4. *The use of calculators is not permitted during the reading time.*
5. *Mark allocations are shown in brackets.*
6. *Attempt all questions, beginning your answer to each question on a new page.*
7. *You should show calculations where this is appropriate.*

### **AT THE END OF THE EXAMINATION**

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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### QUESTION 1

1. A sum of R1 500 is accumulated at a simple rate of interest of 5% per half-year for 2.5 years.

The resulting accumulated amount is then accumulated at a compound rate of discount of 16% per annum convertibly quarterly for a further  $T$  months. The total accumulated value of this investment is R2 495.18.

Calculate the value of  $T$ .

[Total 3]

### QUESTION 2

The force of interest per annum at time  $t$  years is given by

$$\delta_t = \begin{cases} 0.06 & 0 < t \leq 6 \\ 0.05 + 0.0002t^2 & 6 < t \leq 12 \end{cases}$$

Calculate the accumulated value at time  $t=12$  of a continuous payment stream of  $e^{-0.02t}$  per annum payable from time  $t=2$  to time  $t=6$ .

[Total 9]

### QUESTION 3

An ordinary share pays annual dividends on 1 May each year. An investor purchased a share on 1 March 2013 for a price of R21.50 and sold the share on 1 July 2014 for R32. The first dividend was R1.10 per share and thereafter dividends increased in line with an inflation index using a 2-month time lag.

The inflation index values are:

1/1/2013	1/3/2013	1/5/2013	1/3/2014	1/5/2014	1/7/2014
93	92	96	103	104	108

Show that annual effective real rate of return, earned on this transaction, is approximately 27.8%

[Total 7]

**PLEASE TURN OVER**

#### QUESTION 4

The annual effective forward rate applicable over the period  $t$  to  $t + r$  is defined as  $f_{t,r}$  where  $t$  and  $r$  are measured in years. The following forward rates are available:

$$f_{0,1} = 4\%, f_{1,1} = 4.25\%, f_{2,5} = 5\%, f_{2,9} = 7\%.$$

- i. Determine all the possible spot yields available from the data. [4]
- ii. Draw a detailed spot yield curve. Label your axis clearly. [2]
- iii. Explain the shape of the yield curve with reference to the expectation and the liquidity preference theory. [4]

[Total 10]

#### QUESTION 5

Describe briefly what is meant by a stochastic model and a deterministic model.

[Total 5]

#### QUESTION 6

- i. List the advantages and disadvantages of using models for actuarial work.

[6]

You have just built a simplistic model for a client.

- ii. Briefly explain the key factors that must be considered when communicating the results to the client.

[4]

[Total 10]

#### QUESTION 7

A loan is to be repaid over 20 years, with annual payments of  $R(3000-25t)$ . The first payment is made at  $t = 1$ . The rate of interest on the loan is 7.5% per annum effective.

- i. Determine the amount of the loan. [4]
- ii. Determine the capital and interest content of the 8<sup>th</sup> instalment by using the prospective method. [5]

[Total 9]

**PLEASE TURN OVER**

### QUESTION 8

An investor, who is liable to income and capital gains tax at 20% wants to earn a net redemption yield of 5.5% per annum, effective. A bond bearing coupons payable half-yearly in arrears at a rate 6.25% per annum, is available. The bond will be redeemed at 102% on a coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.

Determine the maximum price that the investor is willing to pay for the bond if he buys the bonds 4 years and 3 months into the term of the bond (i.e. 4 years and 3 months have expired since the bond was issued).

[Total 11]

### QUESTION 9

A fund has to provide an annuity of R60 000 per annum payable yearly in arrears for the next 9 years followed by a final payment of R750 000 in 10 years' time. The fund has earmarked cash assets exactly equal to the present value of the payments and the fund manager wants to invest these in two zero coupon bonds:

- Bond A which is repayable at the end of 5 years and
- Bond B which is repayable at the end of 20 years.

The fund manager wants both the assets and the liabilities to have the same volatility.

Determine how much the manager should invest in each of the bonds, given that an effective rate of interest of 7% per annum is used to value both assets and liabilities?

[Total 11]

### QUESTION 10

i. Let  $p$  and  $m$  be positive integers. Starting from  $(1+i)^{-1} = v = (1-d)$ , show that

$$d^{(p)} = p \left\{ 1 - \left( 1 + \frac{i^{(m)}}{m} \right)^{-\frac{m}{p}} \right\}.$$

[3]

ii. Hence, show that the value  $\frac{1}{d^{(m)}} - \frac{1}{i^{(m)}}$  does not depend on the rate of interest.

[6]

[Total 9]

**PLEASE TURN OVER**

## QUESTION 11

An insurance company is considering setting up a branch in a country in which it has previously not operated. The company is aware that access to capital may become difficult in twelve years' time. The following cashflows are generated in the development and operation of the branch.

### Cash Outflows

- Between the present time and the opening of the branch in three years' time the insurance company will spend R1.5m per annum on research, development and the marketing of products. This outlay is assumed to be a constant continuous payment stream.
- The rent on the branch building will be R0.3m per annum paid quarterly in advance for twelve years starting in three years' time.
- Staff costs are assumed to be R1m in the first year, R1.05m in the second year, rising by 5% per annum each year thereafter. Staff costs are assumed to be incurred at the beginning of each year starting in three years' time and assumed to be incurred for 12 years.

### Cash Inflows

- The company expects the sale of products to produce a net income at a rate of R1m per annum for the first three years after the branch opens rising to R1.9m per annum in the next three years and to R2.5m for the following six years. This net income is assumed to be received continuously throughout each year.
  - The company expects to be able to sell the branch operation 15 years from the present time for R8m.
- i. Determine whether this project has an internal rate of return greater than 9% per annum effective. [8]
- ii. Determine whether the discounted payback period, of this project, at a rate of 7% per annum effective is less than twelve years. [8]

[Total 16]

[Total 100]

**END OF EXAMINATION**