EXAMINATION

14 September 2021

Subject A211 — Financial Mathematics

Time allowed:

Two hours and fifteen minutes – examination time

20 minutes (at the end of the examination) – scan and upload time

INSTRUCTIONS TO THE CANDIDATE

1. *Once you have entered the ASSA Exam Platform, ensure that you have accessed the* ***Video Room*** *Invigilation link with both your camera and microphone on,* ***before you attempt the examination.***
2. *Your PC must be placed, and camera angled, so that your writing area on your desk is visible to the invigilator. Readjust your camera if you bump or move your PC by accident.*
3. *Ensure that you have your candidate number handy to input* ***as part of the 2 hours 15 minutes examination****. Write your candidate number at the top of each page. (DO NOT WRITE YOUR NAME OR MEMBER NUMBER.)*
4. *Your cell phone that will be used to scan your final answer script must be switched* ***OFF*** *during the 2 hours and 15 minutes examination time. Place your cell phone at the top of your examination pad / writing pages in view of the invigilator.*
5. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may commence answering the paper whenever you are ready. You then have two hours to complete the paper.*
6. *Questions are only available in the ASSA Exam Platform and may not be printed or copied outside of the ASSA Exam Platform.*
7. *You are required to write your answers on a clean A4 examination pad. Write only on 1 side of the paper and number your pages.*

8. *Attempt all questions, beginning your answer to each question on a new page and numbering your answers clearly.*
9. *Write in black or dark blue pen.*
10. *You should show calculations where this is appropriate.*
11. *You may not use any other computer program (e.g. Email, MS Word or Excel) or files, nor open any other browser during the examination.*
12. *You MAY NOT make use of a Formulae and Tables book during the examination. Any such information that may be required will be provided to you within the examination.*
13. *Mark allocations are shown in brackets.*
14. *You may use additional scrap paper to make notes where this is appropriate. This paper MUST NOT BE SCANNED as part of your answer script.*
15. *Assume that months are all of equal length, unless otherwise stated.*
16. *At the end of the 2 hours and 15 minutes examination time, you must stop writing and may start scanning and uploading your script.* ***Do not continue writing into upload time****.*
17. *After the examination time, you can access your phone to scan your file, transfer it to your PC and upload it to the ASSA Exam Platform. You may NOT continue to write or review your script during this time.*
18. *Scan ALL your answer pages to .pdf so that your candidate number at the top of the page is clear.*
19. *Save your .pdf scanned file using your candidate number as file name. (DO NOT USE YOUR NAME OR MEMBER NUMBER AS FILE NAME.)*
20. *Transfer your .pdf script to your PC and click on the UPLOAD ANSWERS link below the examination paper link.*
21. *Upload your answer file into the ASSA Exam Platform and ensure you click on FINISH below the upload box and again on FINISH all and SUBMIT,* ***before*** *the 20 minute upload time is up. (A summary will be shown after submission of the number of files successfully submitted.)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

The distribution of death claim amounts (in Rands), submitted to an insurance company, is being modelled.

i. Describe why a model that accurately predicts the mean claim amount, may not be the best model for predicting the future claims experience of the insurance company. [4]

ii. When using a stochastic model, the desired results may be obtained via Monte Carlo simulation or, under certain conditions, by analytical methods.

   a. Describe the advantages of the analytical methods over the Monte Carlo simulation. [2]

   b. How can the analytical methods supplement the Monte Carlo simulation? [1]

iii. Explain, with reason(s), whether the following statement is true or false:
In a model of the South African economy, which predicts GDP as one of the outputs, the investigator runs the model over all possible values of future inflation. This is an example of scenario testing. [3]

[Total 10]

QUESTION 2

The annual Retail Price Index (RPI) for the year ending 31 December is given below:

Year	**2016**	**2017**	**2018**	**2019**	**2020**
RPI	110	108	104	98	90

i. Comment on the annual inflation rates (without doing any calculations) as summarised by the RPI index for the time period 2016 to 2020. [3]

An investor won a R15,000 cash prize on 31 December 2016. He decided to immediately invest his money in a savings account offering an annual effective rate of interest of 3% for four years.

ii. Calculate the annual real rate of return earned on this investment over the four-year period. [3]

iii. Comment on your answer in (ii) with reference to the purchasing power of money. [3]

[Total 9]

PLEASE TURN OVER

QUESTION 3

The force of interest $\delta(t)$, at time *t* is given by

$$\delta(t) = \begin{cases} 0.075 + 0.01t & \text{if} \quad 0 \le t < 4 \\ 0.09 & \text{if} \quad 4 \le t < 8 \\ 0.01 + 0.001t^2 & \text{if} \quad t \ge 8 \end{cases}$$

Calculate the present value of the following cashflows:

- R5,000 at time t =11 and
- A continuous income stream of $\rho(t) = 1{,}000 \times \exp(0.005t^2)$ between time t =3 and t =4.

[Total 14]

QUESTION 4

i. Write down equations stating the relationship between:

- the discrete-time forward rate agreed at time *0* for an investment made at time *t*, $t > 0$, for a period of *r* years and
- the appropriate discrete-time spot rates and
- the appropriate zero-coupon bond prices.

Clearly define all the symbols used. [4]

You have been provided with the following information:

- $y_1 = 0.04$
- $F_{1,3} = 0.06$
- $f_{2,3} = 0.065$

The current price of a 2-year zero-coupon bond with redemption value R100 is R90.

The redemption yield on a five-year bond bearing annual coupons of 5% and redeemable at 104% of par is 6% per annum effective.

ii. Calculate the five-year spot yield curve. [7]

[Total 11]

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QUESTION 5

Company J has to meet the following liabilities:

- R10,000 payable in three years' time
- R10,000 payable in five years' time
- R80,000 payable in nine years' time

The company holds two bonds in their asset portfolio:

- A fixed interest bond paying coupons of R3,000 per annum in arrears and is redeemable at R*X* after seven years.
- A zero-coupon bond paying R*Y* in 11 years' time.

Assets and liabilities are valued at a rate of 8% per annum effective. The company has invested in the two bonds so that it is protected against small changes in the interest rate.

Calculate the values of *X* and *Y*.

[Total 12]

PLEASE TURN OVER

Compound Interest

8%		n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
i	0.080 000	1	1.080 00	0.925 93	1.000 0	0.925 9	0.925 9	0.925 9	1
$i^{(2)}$	0.078 461	2	1.166 40	0.857 34	2.080 0	1.783 3	2.640 6	2.709 2	2
$i^{(4)}$	0.077 706	3	1.259 71	0.793 83	3.246 4	2.577 1	5.022 1	5.286 3	3
$i^{(12)}$	0.077 208	4	1.360 49	0.735 03	4.506 1	3.312 1	7.962 2	8.598 4	4
		5	1.469 33	0.680 58	5.866 6	3.992 7	11.365 1	12.591 1	5
δ	0.076 961	6	1.586 87	0.630 17	7.335 9	4.622 9	15.146 2	17.214 0	6
		7	1.713 82	0.583 49	8.922 8	5.206 4	19.230 6	22.420 4	7
		8	1.850 93	0.540 27	10.636 6	5.746 6	23.552 7	28.167 0	8
$(1+i)^{1/2}$	1.039 230	9	1.999 00	0.500 25	12.487 6	6.246 9	28.055 0	34.413 9	9
$(1+i)^{1/4}$	1.019 427	10	2.158 92	0.463 19	14.486 6	6.710 1	32.686 9	41.124 0	10
$(1+i)^{1/12}$	1.006 434	11	2.331 64	0.428 88	16.645 5	7.139 0	37.404 6	48.262 9	11
		12	2.518 17	0.397 11	18.977 1	7.536 1	42.170 0	55.799 0	12
		13	2.719 62	0.367 70	21.495 3	7.903 8	46.950 1	63.702 8	13
v	0.925 926	14	2.937 19	0.340 46	24.214 9	8.244 2	51.716 5	71.947 0	14
$v^{1/2}$	0.962 250	15	3.172 17	0.315 24	27.152 1	8.559 5	56.445 1	80.506 5	15
$v^{1/4}$	0.980 944	16	3.425 94	0.291 89	30.324 3	8.851 4	61.115 4	89.357 9	16
$v^{1/12}$	0.993 607	17	3.700 02	0.270 27	33.750 2	9.121 6	65.710 0	98.479 5	17
		18	3.996 02	0.250 25	37.450 2	9.371 9	70.214 4	107.851 4	18
		19	4.315 70	0.231 71	41.446 3	9.603 6	74.617 0	117.455 0	19
d	0.074 074	20	4.660 96	0.214 55	45.762 0	9.818 1	78.907 9	127.273 2	20
$d^{(2)}$	0.075 499	21	5.033 83	0.198 66	50.422 9	10.016 8	83.079 7	137.290 0	21
$d^{(4)}$	0.076 225	22	5.436 54	0.183 94	55.456 8	10.200 7	87.126 4	147.490 7	22
$d^{(12)}$	0.076 715	23	5.871 46	0.170 32	60.893 3	10.371 1	91.043 7	157.861 8	23
		24	6.341 18	0.157 70	66.764 8	10.528 8	94.828 4	168.390 5	24
		25	6.848 48	0.146 02	73.105 9	10.674 8	98.478 9	179.065 3	25
$i/i^{(2)}$	1.019 615	26	7.396 35	0.135 20	79.954 4	10.810 0	101.994 1	189.875 3	26
$i/i^{(4)}$	1.029 519	27	7.988 06	0.125 19	87.350 8	10.935 2	105.374 2	200.810 4	27
$i/i^{(12)}$	1.036 157	28	8.627 11	0.115 91	95.338 8	11.051 1	108.619 8	211.861 5	28
		29	9.317 27	0.107 33	103.965 9	11.158 4	111.732 3	223.019 9	29
		30	10.062 66	0.099 38	113.283 2	11.257 8	114.713 6	234.277 7	30
i/δ	1.039 487	31	10.867 67	0.092 02	123.345 9	11.349 8	117.566 1	245.627 5	31
		32	11.737 08	0.085 20	134.213 5	11.435 0	120.292 5	257.062 5	32
$i/d^{(2)}$	1.059 615	33	12.676 05	0.078 89	145.950 6	11.513 9	122.895 8	268.576 4	33
$i/d^{(4)}$	1.049 519	34	13.690 13	0.073 05	158.626 7	11.586 9	125.379 3	280.163 3	34
$i/d^{(12)}$	1.042 824	35	14.785 34	0.067 63	172.316 8	11.654 6	127.746 6	291.817 9	35
		36	15.968 17	0.062 62	187.102 1	11.717 2	130.001 0	303.535 1	36
		37	17.245 63	0.057 99	203.070 3	11.775 2	132.146 5	315.310 3	37
		38	18.625 28	0.053 69	220.315 9	11.828 9	134.186 8	327.139 1	38
		39	20.115 30	0.049 71	238.941 2	11.878 6	136.125 6	339.017 7	39
		40	21.724 52	0.046 03	259.056 5	11.924 6	137.966 8	350.942 3	40
		41	23.462 48	0.042 62	280.781 0	11.967 2	139.714 3	362.909 6	41
		42	25.339 48	0.039 46	304.243 5	12.006 7	141.371 8	374.916 3	42
		43	27.366 64	0.036 54	329.583 0	12.043 2	142.943 0	386.959 5	43
		44	29.555 97	0.033 83	356.949 6	12.077 1	144.431 7	399.036 6	44
		45	31.920 45	0.031 33	386.505 6	12.108 4	145.841 5	411.145 0	45
		46	34.474 09	0.029 01	418.426 1	12.137 4	147.175 8	423.282 4	46
		47	37.232 01	0.026 86	452.900 2	12.164 3	148.438 2	435.446 7	47
		48	40.210 57	0.024 87	490.132 2	12.189 1	149.631 9	447.635 8	48
		49	43.427 42	0.023 03	530.342 7	12.212 2	150.760 2	459.848 0	49
		50	46.901 61	0.021 32	573.770 2	12.233 5	151.826 3	472.081 4	50
		60	101.257 06	0.009 88	1 253.213 3	12.376 6	159.676 6	595.293 1	60
		70	218.606 41	0.004 57	2 720.080 1	12.442 8	163.975 4	719.464 8	70
		80	471.954 83	0.002 12	5 886.935 4	12.473 5	166.273 6	844.081 1	80
		90	1 018.915 09	0.000 98	12 723.938 6	12.487 7	167.480 3	968.903 3	90
		100	2 199.761 26	0.000 45	27 484.515 7	12.494 3	168.105 0	1 093.821 0	100

PLEASE TURN OVER

QUESTION 6

Consider a loan of size L that is repayable by level annual instalments of size P over a period of n years at an annual effective interest rate of i. The level annual payment P is payable in arrears and consists of a capital portion, P_C, and an interest portion, P_I, such that $P = P_C + P_I$.

i. Draw rough plots (on the same graph) of the interest portions, P_I, and capital portions, P_C, respectively over the life of the loan. Provide appropriate labels for the plot. [5]

The borrower decides to change his repayment schedule for the loan of size L. The level annual instalments of size P are replaced by level quarterly instalments of size $P_{1/4}$, payable in arrears, over a period of n years at an annual effective interest rate of i.

ii. Explain, with reason(s), how the quarterly instalment, $P_{1/4}$, compares to the original annual instalment amount of P. [3]

The borrower decides to repay the loan of size L by annual instalments of Q, payable in arrears, over a period of $2n$ years at an annual effective interest rate i.

iii. Explain, with reason(s), what happens to the total interest payable on the loan (in monetary terms), as compared to the total interest payable under the original loan agreement (in monetary terms). [2]

[Total 10]

QUESTION 7

On 1 October 2018, a bond of R100,000 nominal, paying half-yearly coupons of 6% per annum in arrears, is put up for auction. The bond has a term of eight years and is redeemable at par.

Income tax is only payable on 28 February of each year on the previous 12 months' income. In addition, capital gains tax, on the capital gain at the date when the bond was sold or redeemed, is only payable on 28 February following the date of sale or redemption.

An investor, subject to income tax of 32% and capital gains tax of 20%, requires a minimum net yield to redemption of 7% per annum effective on this investment.

i. Calculate the maximum price that the investor would be willing to bid for R100,000 nominal of the bond, assuming a capital gain is made on redemption. [7]

The tax system of the country changes and tax will in the future be paid immediately on receipt of any cashflow.

PLEASE TURN OVER

ii. Explain, with reason(s), whether the investor would be willing to pay more or less, compared to your answer in (i), for the bond under the new tax system. No additional calculations are required. [3]

[Total 10]

Compound Interest

n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
1	1.070 00	0.934 58	1.000 0	0.934 6	0.934 6	0.934 6	1
2	1.144 90	0.873 44	2.070 0	1.808 0	2.681 5	2.742 6	2
3	1.225 04	0.816 30	3.214 9	2.624 3	5.130 4	5.366 9	3
4	1.310 80	0.762 90	4.439 9	3.387 2	8.181 9	8.754 1	4
5	1.402 55	0.712 99	5.750 7	4.100 2	11.746 9	12.854 3	5
6	1.500 73	0.666 34	7.153 3	4.766 5	15.744 9	17.620 9	6
7	1.605 78	0.622 75	8.654 0	5.389 3	20.104 2	23.010 2	7
8	1.718 19	0.582 01	10.259 8	5.971 3	24.760 2	28.981 4	8
9	1.838 46	0.543 93	11.978 0	6.515 2	29.655 6	35.496 7	9
10	1.967 15	0.508 35	13.816 4	7.023 6	34.739 1	42.520 3	10
11	2.104 85	0.475 09	15.783 6	7.498 7	39.965 2	50.018 9	11
12	2.252 19	0.444 01	17.888 5	7.942 7	45.293 3	57.961 6	12
13	2.409 85	0.414 96	20.140 6	8.357 7	50.687 8	66.319 3	13
14	2.578 53	0.387 82	22.550 5	8.745 5	56.117 3	75.064 7	14
15	2.759 03	0.362 45	25.129 0	9.107 9	61.554 0	84.172 7	15
16	2.952 16	0.338 73	27.888 1	9.446 6	66.973 7	93.619 3	16
17	3.158 82	0.316 57	30.840 2	9.763 2	72.355 5	103.382 5	17
18	3.379 93	0.295 86	33.999 0	10.059 1	77.681 0	113.441 6	18
19	3.616 53	0.276 51	37.379 0	10.335 6	82.934 7	123.777 2	19
20	3.869 68	0.258 42	40.995 5	10.594 0	88.103 1	134.371 2	20
21	4.140 56	0.241 51	44.865 2	10.835 5	93.174 8	145.206 8	21
22	4.430 40	0.225 71	49.005 7	11.061 2	98.140 5	156.268 0	22
23	4.740 53	0.210 95	53.436 1	11.272 2	102.992 3	167.540 2	23
24	5.072 37	0.197 15	58.176 7	11.469 3	107.723 8	179.009 5	24
25	5.427 43	0.184 25	63.249 0	11.653 6	112.330 1	190.663 1	25
26	5.807 35	0.172 20	68.676 5	11.825 8	116.807 1	202.488 9	26
27	6.213 87	0.160 93	74.483 8	11.986 7	121.152 3	214.475 6	27
28	6.648 84	0.150 40	80.697 7	12.137 1	125.363 5	226.612 7	28
29	7.114 26	0.140 56	87.346 5	12.277 7	129.439 9	238.890 4	29
30	7.612 26	0.131 37	94.460 8	12.409 0	133.380 9	251.299 4	30
31	8.145 11	0.122 77	102.073 0	12.531 8	137.186 8	263.831 2	31
32	8.715 27	0.114 74	110.218 2	12.646 6	140.858 5	276.477 8	32
33	9.325 34	0.107 23	118.933 4	12.753 8	144.397 3	289.231 6	33
34	9.978 11	0.100 22	128.258 8	12.854 0	147.804 7	302.085 6	34
35	10.676 58	0.093 66	138.236 9	12.947 7	151.082 9	315.033 3	35
36	11.423 94	0.087 54	148.913 5	13.035 2	154.234 2	328.068 5	36
37	12.223 62	0.081 81	160.337 4	13.117 0	157.261 2	341.185 5	37
38	13.079 27	0.076 46	172.561 0	13.193 5	160.166 5	354.379 0	38
39	13.994 82	0.071 46	185.640 3	13.264 9	162.953 3	367.643 9	39
40	14.974 46	0.066 78	199.635 1	13.331 7	165.624 5	380.975 6	40
41	16.022 67	0.062 41	214.609 6	13.394 1	168.183 3	394.369 7	41
42	17.144 26	0.058 33	230.632 2	13.452 4	170.633 1	407.822 2	42
43	18.344 35	0.054 51	247.776 5	13.507 0	172.977 2	421.329 1	43
44	19.628 46	0.050 95	266.120 9	13.557 9	175.218 8	434.887 0	44
45	21.002 45	0.047 61	285.749 3	13.605 5	177.361 4	448.492 5	45
46	22.472 62	0.044 50	306.751 8	13.650 0	179.408 4	462.142 6	46
47	24.045 71	0.041 59	329.224 4	13.691 6	181.363 0	475.834 2	47
48	25.728 91	0.038 87	353.270 1	13.730 5	183.228 6	489.564 7	48
49	27.529 93	0.036 32	378.999 0	13.766 8	185.008 5	503.331 4	49
50	29.457 03	0.033 95	406.528 9	13.800 7	186.705 9	517.132 2	50
60	57.946 43	0.017 26	813.520 4	14.039 2	199.806 9	656.583 1	60
70	113.989 39	0.008 77	1 614.134 2	14.160 4	207.678 9	797.708 7	70
80	224.234 39	0.004 46	3 189.062 7	14.222 0	212.296 8	939.685 6	80
90	441.102 98	0.002 27	6 287.185 4	14.253 3	214.957 5	1 082.095 3	90
100	867.716 33	0.001 15	12 381.661 8	14.269 3	216.469 3	1 224.725 0	100

7%

i	0.070 000
$i^{(2)}$	0.068 816
$i^{(4)}$	0.068 234
$i^{(12)}$	0.067 850
δ	0.067 659
$(1+i)^{1/2}$	1.034 408
$(1+i)^{1/4}$	1.017 059
$(1+i)^{1/12}$	1.005 654
v	0.934 579
$v^{1/2}$	0.966 736
$v^{1/4}$	0.983 228
$v^{1/12}$	0.994 378
d	0.065 421
$d^{(2)}$	0.066 527
$d^{(4)}$	0.067 090
$d^{(12)}$	0.067 468
$i/i^{(2)}$	1.017 204
$i/i^{(4)}$	1.025 880
$i/i^{(12)}$	1.031 691
i/δ	1.034 605
$i/d^{(2)}$	1.052 204
$i/d^{(4)}$	1.043 380
$i/d^{(12)}$	1.037 525

PLEASE TURN OVER

QUESTION 8

Derive from first principle the following relationship:

$$D = \frac{1}{n}\left[1-(1+i)^{-n}\right]$$

Where

- D is annual rate of commercial discount,
- i is the effective rate of interest per annum and
- $n<1$.

[Total 3]

QUESTION 9

A company is considering investing in one of two projects, Project A or Project B. Both projects will run for a period of eight years.

Project A

Outflows

- R100,000 at time 0
- A series of regular maintenance costs of R50,000 per annum payable continuously for three years, starting at time 0.

Inflows

- Income starting at R30,000 payable annually from the end of the second year onwards until the project ends. The annual income increases by 5% per annum compound, with the first increase being applied to the second payment.
- A lump sum of R100,000 at the end of year eight.

Project B

Outflows

- R80,000 invested annually in advance for five years, starting immediately.

Inflows

- Annual income receivable in arrears for eight years. The income level starts at R60,000 per annum in the first year and increases by R10,000 per annum every year thereafter.

i. Show that the internal rate of return for Project A is approximately 6.615% per annum effective. [9]

PLEASE TURN OVER

ii. Determine whether the internal rate of return for Project B is higher or lower than the internal rate of return for Project A. [9]

The company will need to borrow money from a bank to invest in either project.

iii. Explain, with reason(s), the range of the interest rates on the loan which would allow the company to make a profit on both projects. [3]

[Total 21]

[GRAND TOTAL 100]

END OF EXAMINATION

