# EXAMINATION

22 September 2020

# Subject A211 — Financial Mathematics

*Time allowed:*

*Two hours and fifteen minutes – exam time*

*20 minutes (at the end of the exam) – scan and upload time*

***INSTRUCTIONS TO THE CANDIDATE***

1. *Once you have entered the Exam Platform, ensure that you have accessed the **Video Room** Invigilation link with both your camera and microphone on, before you attempt the exam.*
2. *Your PC must be placed, and camera angled, so that your writing area on your desk is visible to the invigilator.*
3. *Ensure that you have your candidate number handy to input as part of the exam. Write your candidate number at the top of each page. (DO NOT WRITE YOUR NAME.)*
4. *Your cell phone that will be used to scan your final answer scrip must be switched **OFF** during the 2 hours and 15 minutes exam time. Place your cell phone at the top of your exam pad / writing pages in view of the invigilator.*
5. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may commence answering the paper whenever you are ready. You then have two hours to complete the paper.*
6. *Questions are only available in the Exam platform and may not be printed or copied outside of the Exam platform.*
7. *You are required to write your answers on a clean A4 exam pad. Write only on 1 side of the paper. Write your candidate number at the top of each page and number your pages.*
8. *Attempt all questions, beginning your answer to each question on a new page and numbering your answers clearly.*
9. *Write in black or dark blue pen.*
10. *You should show calculations where this is appropriate.*
11. *You MAY NOT use any computer program (e.g. email, MS Word or Excel), files or open any other browsers or browser tabs during the examination time.*
12. *Mark allocations are shown in brackets.*

 

13. *You may use additional scrap paper to make notes where this is appropriate. This paper MUST NOT BE SCANNED as part of your answer script.*
14. *Assume that months are all of equal length, unless otherwise stated.*
15. *At the end of the 2 hours and 15 minutes exam time, you must stop writing and may start scanning and uploading your script.* ***Do not continue writing into upload time****.*
16. *Access to your PC will be opened-up after the exam time so you can access your scanned file. You may now also switch on your cell phone to scan.*
17. *Scan ALL your answer pages to .pdf so that your candidate number at the top of the page is clear.*
18. *Save your .pdf scanned file using your candidate number as file name. (DO NOT USE YOUR NAME AS FILE NAME)*
19. *Transfer your .pdf script to your PC and click on the UPLOAD ANSWERS link below the exam paper link.*
20. *Upload your answer file into the Exam Platform and ensure you click on FINISH below the upload box and again on FINISH all and SUBMIT,* ***before*** *the 20 minute upload time is up. (If the status on the summary page indicates "Answer saved" your file was uploaded. You can click on Review attempt to see the file you have uploaded.)*

**Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.**

***END OF INSTRUCTIONS***

 

**QUESTION 1**

A local bank has introduced a new student savings account that gives a student two options:

- Deposit R2,500 for one year and earn an effective rate of discount of 9.0913% per annum for the first six months and an effective interest rate of 10.5% per annum for the last six months or

- Deposit R2,500 for one year and earn 10% per annum compounded half-yearly for the entire year.

Which option, if any, is best for the student from a financial perspective? Show your calculations and justify your answer fully. [4]

**QUESTION 2**

Sandra has a daughter aged eight, exactly, on 1 January 2020. She would like to provide her daughter with R20,000 (in real terms) on each 1 January of her three years at university (when her daughter will turn 19, 20, and 21).

Sandra can earn an effective real interest rate on her investments of 4% per annum for the next five years, and 6% per annum, thereafter.

The inflation rate is expected to remain constant at 5% per annum for the foreseeable future.

i. Calculate the amount of money Sandra should invest on 1 January 2020. [3]

Sandra invests the amount calculated in part (i), but then decides to give her daughter a single lump sum at the start of her first year of university (instead of the three instalments).

ii. Calculate the lump sum the daughter can expect to receive. [3]

[Total 6]

**QUESTION 3**

The force of interest $\delta(t)$ at any time $t$, where $t$ is measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.01 & if \quad 0 \le t \le 3 \\ 0.05 - 0.01t & if \quad 3 < t \le 7 \\ 0.01t - 0.02 & if \quad t > 7 \end{cases}$$

If R100 is invested at $t = 2$ and a further R600 is invested at $t = 8$, calculate the accumulated amount at time $t = 10$.

[Total 9]

**PLEASE TURN OVER**

 

**QUESTION 4**

Let $i$ be the annual effective interest rate and $n$ and $p$ be any natural numbers.

i. Define algebraically, in a summation format, $\ddot{a}^{(p)}_{\overline{n}|\,i}$. [1]

ii. Prove that $\ddot{a}^{(p)}_{\overline{m+n}|\,i} = \ddot{a}^{(p)}_{\overline{m}|\,i} + v^m \ddot{a}^{(p)}_{\overline{n}|\,i}$ [3]

[Total 4]

**QUESTION 5**

The $n$-year forward rate for transactions beginning at time $t$ and maturing at time $t+n$ is denoted by $f_{t,n}$. You are given:

$f_{0,1} = 5.0\%$ per annum

$f_{0,2} = 5.5\%$ per annum

$f_{1,2} = 5.7\%$ per annum

i. Calculate the 3-year par yield. [4]

ii. Outline briefly the liquidity preference theory and then, explain whether (or not) the theory can be used to fully explain the term structure of spot rates above.

[3]

[Total 7]

**QUESTION 6**

A small investor has the following liabilities:

- R9,000 payable at the end of years one, two, three and four, and
- R11,000 payable at the end of years five and six.

The investor has exactly enough money to cover the liability based on a constant force of interest of 6.5% per annum. This force of interest is also used in the market to determine the price of all assets. He wishes to invest the money in a combination of the following two assets:

Asset A: Cash account

Asset B: 10-year zero coupon bond redeemable at par

Calculate the amounts that should be invested in Asset A and B to immunise the portfolio against small changes in the interest rate. Focus only on the first two conditions of the immunisation theory.

[Total 15]

**PLEASE TURN OVER**

 

**QUESTION 7**

i. On 1 January 1997, a bank issued a loan of R275,000, repayable by level monthly instalments payable in arrears. The loan was issued at a rate of interest of 11.25% per annum compounded quarterly and the last monthly instalment will be paid on 1 January 2022.

On 1 January 2004, immediately after the monthly instalment then due, the effective annual rate of interest was changed to 13% per annum.

Calculate the change in the monthly instalment due to the change in the interest rate assuming that the original term of the loan was unchanged. [7]

ii. Comment on your answer in (i). [1]

[Total 8]

**PLEASE TURN OVER**

 

# Compound Interest

| 6% | | $n$ | $(1+i)^n$ | $v^n$ | $s_{\overline{n}\rceil}$ | $a_{\overline{n}\rceil}$ | $(Ia)_{\overline{n}\rceil}$ | $(Da)_{\overline{n}\rceil}$ | $n$ |
|---|---|---|---|---|---|---|---|---|---|
| $i$ | 0.060 000 | 1 | 1.060 00 | 0.943 40 | 1.000 0 | 0.943 4 | 0.943 4 | 0.943 4 | 1 |
| $i^{(2)}$ | 0.059 126 | 2 | 1.123 60 | 0.890 00 | 2.060 0 | 1.833 4 | 2.723 4 | 2.776 8 | 2 |
| $i^{(4)}$ | 0.058 695 | 3 | 1.191 02 | 0.839 62 | 3.183 6 | 2.673 0 | 5.242 2 | 5.449 8 | 3 |
| $i^{(12)}$ | 0.058 411 | 4 | 1.262 48 | 0.792 09 | 4.374 6 | 3.465 1 | 8.410 6 | 8.914 9 | 4 |
| | | 5 | 1.338 23 | 0.747 26 | 5.637 1 | 4.212 4 | 12.146 9 | 13.127 3 | 5 |
| $\delta$ | 0.058 269 | 6 | 1.418 52 | 0.704 96 | 6.975 3 | 4.917 3 | 16.376 7 | 18.044 6 | 6 |
| | | 7 | 1.503 63 | 0.665 06 | 8.393 8 | 5.582 4 | 21.032 1 | 23.627 0 | 7 |
| | | 8 | 1.593 85 | 0.627 41 | 9.897 5 | 6.209 8 | 26.051 4 | 29.836 8 | 8 |
| $(1+i)^{1/2}$ | 1.029 563 | 9 | 1.689 48 | 0.591 90 | 11.491 3 | 6.801 7 | 31.378 5 | 36.638 5 | 9 |
| $(1+i)^{1/4}$ | 1.014 674 | 10 | 1.790 85 | 0.558 39 | 13.180 8 | 7.360 1 | 36.962 4 | 43.998 5 | 10 |
| $(1+i)^{1/12}$ | 1.004 868 | 11 | 1.898 30 | 0.526 79 | 14.971 6 | 7.886 9 | 42.757 1 | 51.885 4 | 11 |
| | | 12 | 2.012 20 | 0.496 97 | 16.869 9 | 8.383 8 | 48.720 7 | 60.269 3 | 12 |
| | | 13 | 2.132 93 | 0.468 84 | 18.882 1 | 8.852 7 | 54.815 6 | 69.122 0 | 13 |
| $v$ | 0.943 396 | 14 | 2.260 90 | 0.442 30 | 21.015 1 | 9.295 0 | 61.007 8 | 78.416 9 | 14 |
| $v^{1/2}$ | 0.971 286 | 15 | 2.396 56 | 0.417 27 | 23.276 0 | 9.712 2 | 67.266 8 | 88.129 2 | 15 |
| $v^{1/4}$ | 0.985 538 | 16 | 2.540 35 | 0.393 65 | 25.672 5 | 10.105 9 | 73.565 1 | 98.235 1 | 16 |
| $v^{1/12}$ | 0.995 156 | 17 | 2.692 77 | 0.371 36 | 28.212 9 | 10.477 3 | 79.878 3 | 108.712 3 | 17 |
| | | 18 | 2.854 34 | 0.350 34 | 30.905 7 | 10.827 6 | 86.184 5 | 119.539 9 | 18 |
| | | 19 | 3.025 60 | 0.330 51 | 33.760 0 | 11.158 1 | 92.464 3 | 130.698 1 | 19 |
| $d$ | 0.056 604 | 20 | 3.207 14 | 0.311 80 | 36.785 6 | 11.469 9 | 98.700 4 | 142.168 0 | 20 |
| $d^{(2)}$ | 0.057 428 | 21 | 3.399 56 | 0.294 16 | 39.992 7 | 11.764 1 | 104.877 6 | 153.932 1 | 21 |
| $d^{(4)}$ | 0.057 847 | 22 | 3.603 54 | 0.277 51 | 43.392 3 | 12.041 6 | 110.982 7 | 165.973 6 | 22 |
| $d^{(12)}$ | 0.058 128 | 23 | 3.819 75 | 0.261 80 | 46.995 8 | 12.303 4 | 117.004 1 | 178.277 0 | 23 |
| | | 24 | 4.048 93 | 0.246 98 | 50.815 6 | 12.550 4 | 122.931 6 | 190.827 4 | 24 |
| | | 25 | 4.291 87 | 0.233 00 | 54.864 5 | 12.783 4 | 128.756 5 | 203.610 7 | 25 |
| $i/i^{(2)}$ | 1.014 782 | 26 | 4.549 38 | 0.219 81 | 59.156 4 | 13.003 2 | 134.471 6 | 216.613 9 | 26 |
| $i/i^{(4)}$ | 1.022 227 | 27 | 4.822 35 | 0.207 37 | 63.705 8 | 13.210 5 | 140.070 5 | 229.824 4 | 27 |
| $i/i^{(12)}$ | 1.027 211 | 28 | 5.111 69 | 0.195 63 | 68.528 1 | 13.406 2 | 145.548 2 | 243.230 6 | 28 |
| | | 29 | 5.418 39 | 0.184 56 | 73.639 8 | 13.590 7 | 150.900 3 | 256.821 3 | 29 |
| $i/\delta$ | 1.029 709 | 30 | 5.743 49 | 0.174 11 | 79.058 2 | 13.764 8 | 156.123 6 | 270.586 1 | 30 |
| | | 31 | 6.088 10 | 0.164 25 | 84.801 7 | 13.929 1 | 161.215 5 | 284.515 2 | 31 |
| $i/d^{(2)}$ | 1.044 782 | 32 | 6.453 39 | 0.154 96 | 90.889 8 | 14.084 0 | 166.174 2 | 298.599 3 | 32 |
| $i/d^{(4)}$ | 1.037 227 | 33 | 6.840 59 | 0.146 19 | 97.343 2 | 14.230 2 | 170.998 3 | 312.829 5 | 33 |
| $i/d^{(12)}$ | 1.032 211 | 34 | 7.251 03 | 0.137 91 | 104.183 8 | 14.368 1 | 175.687 3 | 327.197 6 | 34 |
| | | 35 | 7.686 09 | 0.130 11 | 111.434 8 | 14.498 2 | 180.241 0 | 341.695 9 | 35 |
| | | 36 | 8.147 25 | 0.122 74 | 119.120 9 | 14.621 0 | 184.659 6 | 356.316 9 | 36 |
| | | 37 | 8.636 09 | 0.115 79 | 127.268 1 | 14.736 8 | 188.944 0 | 371.053 7 | 37 |
| | | 38 | 9.154 25 | 0.109 24 | 135.904 2 | 14.846 0 | 193.095 1 | 385.899 7 | 38 |
| | | 39 | 9.703 51 | 0.103 06 | 145.058 5 | 14.949 1 | 197.114 2 | 400.848 8 | 39 |
| | | 40 | 10.285 72 | 0.097 22 | 154.762 0 | 15.046 3 | 201.003 1 | 415.895 1 | 40 |
| | | 41 | 10.902 86 | 0.091 72 | 165.047 7 | 15.138 0 | 204.763 6 | 431.033 1 | 41 |
| | | 42 | 11.557 03 | 0.086 53 | 175.950 5 | 15.224 5 | 208.397 8 | 446.257 6 | 42 |
| | | 43 | 12.250 45 | 0.081 63 | 187.507 6 | 15.306 2 | 211.907 8 | 461.563 8 | 43 |
| | | 44 | 12.985 48 | 0.077 01 | 199.758 0 | 15.383 2 | 215.296 2 | 476.947 0 | 44 |
| | | 45 | 13.764 61 | 0.072 65 | 212.743 5 | 15.455 8 | 218.565 5 | 492.402 8 | 45 |
| | | 46 | 14.590 49 | 0.068 54 | 226.508 1 | 15.524 4 | 221.718 2 | 507.927 2 | 46 |
| | | 47 | 15.465 92 | 0.064 66 | 241.098 6 | 15.589 0 | 224.757 2 | 523.516 2 | 47 |
| | | 48 | 16.393 87 | 0.061 00 | 256.564 5 | 15.650 0 | 227.685 1 | 539.166 2 | 48 |
| | | 49 | 17.377 50 | 0.057 55 | 272.958 4 | 15.707 6 | 230.504 8 | 554.873 8 | 49 |
| | | 50 | 18.420 15 | 0.054 29 | 290.335 9 | 15.761 9 | 233.219 2 | 570.635 7 | 50 |
| | | 60 | 32.987 69 | 0.030 31 | 533.128 2 | 16.161 4 | 255.204 2 | 730.642 9 | 60 |
| | | 70 | 59.075 93 | 0.016 93 | 967.932 2 | 16.384 5 | 269.711 7 | 893.590 9 | 70 |
| | | 80 | 105.795 99 | 0.009 45 | 1 746.599 9 | 16.509 1 | 279.058 4 | 1 058.181 2 | 80 |
| | | 90 | 189.464 51 | 0.005 28 | 3 141.075 2 | 16.578 7 | 284.973 3 | 1 223.688 3 | 90 |
| | | 100 | 339.302 08 | 0.002 95 | 5 638.368 1 | 16.617 5 | 288.664 6 | 1 389.707 6 | 100 |

**PLEASE TURN OVER**

 

**QUESTION 8**

i. Derive a capital gains test for an investment in a fixed interest bond. The test must be in terms of the net redemption yield on the fixed interest bond.

The following notation may be used:
$R$ = redemption %, $P$ = price paid, $D$ = annual coupons payable $p$ times per year

$i^{(p)}$ = net redemption yield, $n$ = term of fixed interest bond, $t_1$ = income tax and

$t_2$ = capital gains tax [4]

A fixed interest bond of nominal amount of R1,200,000 is issued with 8% coupons per annum, payable half-yearly in arrears. Redemption will be at 133% on any coupon date between 15 and 25 years after the date of issue. The date of redemption is at the option of the borrower.

ii. An investor who is liable for paying income tax at 40% wishes to purchase the entire nominal value of this fixed interest bond at the date of issue. The investor does not pay any capital gains tax.

Calculate the price the investor should pay to ensure a net effective yield of at least 6% per annum? [8]

iii. Suppose that the investor pays the price calculated in (ii) but the borrower redeems the fixed interest bond at a different date from the one chosen in (ii).

Explain, without doing any additional calculations, what the change will be on the investor's net effective yield. [3]

[Total 15]

**PLEASE TURN OVER**

 

**QUESTION 9**

An investor is considering the following project.

At the beginning of each of the first three years of the project, R210,000 will be invested. The investor does not have sufficient capital but can borrow money from the bank at an effective interest rate of 6% per annum.

Net revenue is received from the start of the project and will be received continuously. The initial rate of payment is R30,000 per annum, but is expected to grow continuously at an effective rate of 5.5% per annum. The length of the project is 25 years.

The investor can earn interest at an effective rate of 4.5% per annum on any money invested in his bank account. The investor has the option to repay the loan as early as possible with revenue received from the project.

i. Calculate the discounted payback period for the project. [10]

ii. Calculate the profit/loss the investor will make at the end of the project. [10]

[Total 20]

**QUESTION 10**

Describe ten key steps that should be followed in the modelling process.

[Total 12]

[GRAND TOTAL: 100]

## END OF EXAMINATION

 