Actuarial Society of South Africa

Examiner's Report

September 2020

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

QUESTION 1

d = 0.090913; i = 0.105,

FV = $2{,}500\times(1-d)^{-\frac{1}{2}}\times(1+i)^{\frac{1}{2}} = R2{,}756.25$

$i^{(2)} = 0.1$

FV = $2{,}500(1+\frac{i^{(2)}}{2})^2$ = R2,756.25

Both options lead to the same future value, so the student can choose any of the two options.

This question was answered well by most candidates.

QUESTION 2

i.

$$PV = R20{,}000(1.04)^{-5}[(1.06)^{-6} + (1.06)^{-7} + (1.06)^{-8}]$$

$$= R32{,}834.84$$

ii.

$$FV_{real} = R32{,}834.84(1.04)^{5}(1.06)^{6} = R56{,}667.85$$

$$FV_{Money} = R56{,}667.85(1.05)^{11} = R96{,}921.26$$

The topic of real and nominal interest rates remains a challenging one for most candidates.

Candidates struggle to identify real and nominal interest rates, as well as where to use them correctly.

QUESTION 3

$$\text{AV} = 100\times e^{\int_2^{10}\delta(s)ds} + 600\times e^{\int_8^{10}\delta(s)ds}$$

$$= 100\exp\left(\int_2^3 0.01ds\right)\times\exp\left(\int_3^7 (0.05-0.01s)\ ds\right)\times\exp\left(\int_7^{10}(0.01s-0.02)\ ds\right)$$

$$+600\times\exp\left(\int_8^{10}(0.01s-0.02)\ ds\right)$$

$$= 100\times\exp\left(0.01s\ \Big|_2^3\right)\times\exp\left((0.05s-0.005s^2)\ \Big|_3^7\right)\times\exp\left((0.005s^2-0.02s)\ \Big|_7^{10}\right)$$

$$+600\times\exp\left((0.005s^2-0.02s)\ \Big|_8^{10}\right)$$

$$= 100\times\exp\left[0.01+(0.105-0.105)+(0.3-0.105)\right]+600\times\exp(0.3-0.16)$$

$$= 100\times\exp\left(0.205\right)\ +600\times\exp\left(0.14\right)\ = 122.753+690.164 = 812.917$$

This question was answered well by most candidates.

QUESTION 4

i

$$\ddot{a}^{(p)}_{\overline{n}|} = \sum_{j=0}^{np-1} \frac{1}{p}(1+i)^{-\frac{j}{p}} = \sum_{j=0}^{np-1} \frac{1}{p} v^{\frac{j}{p}} = \frac{1}{p} + \frac{1}{p} v^{1/p} + \frac{1}{p} v^{2/p} + \ldots \frac{1}{p} v^{n-\frac{1}{p}}$$

ii.

$$\ddot{a}^{(p)}_{\overline{n+m}|} = \frac{1-v^{m+n}}{d^{(p)}}$$

$$= \frac{1-v^m+v^m-v^{m+n}}{d^{(p)}} = \frac{1-v^m}{d^{(p)}} + v^m\left(\frac{1-v^n}{d^{(p)}}\right)$$

$$= \ddot{a}^{(p)}_{\overline{m}|} + v^m \ddot{a}^{(p)}_{\overline{n}|}$$

> *This question was answered poorly with many candidates not able to define the annuity.*
>
> *The common mistake in part (i) was taking the payment as 1 instead of* $\frac{1}{p}$.
>
> *There are alternative ways to prove part (ii).*

QUESTION 5

i.

Let y_n be the n-year annual spot rate.

$y_1 = f_{0,1} = 5\%$ and $y_2 = f_{0,2} = 5.5\%$

$$(1+y_3)^3 = (1+y_1)(1+f_{1,2})^2$$

$$(1+y_3) = \left((1+f_{1,2})^2(1+y_1)\right)^{\frac{1}{3}} = \left(1.057^2 \times 1.05\right)^{\frac{1}{3}} = 1.05466$$

The n-year par yield is the value of C in the equation

$$1 = C\left((1+y_1)^{-1} + (1+y_2)^{-2} + (1+y_3)^{-3}\right) + 1 \times (1+y_3)^{-3}$$

$$\Rightarrow C = \frac{0.147566}{2.70327} = 0.054588 = 5.4588\%$$

ii.

The theory posits that risk averse investors will require compensation for the greater risk of loss on longer term bonds and thus long-term bonds should always offer higher yields.

The first two spot rates show an increasing term structure followed by a slight decrease in the three-year spot rate.

This cannot be fully explained by the liquidity preference theory.

Part (i) of this question was well answered by most candidates.

Part (ii) was answered less well as it not only required an explanation of the liquidity preference theory but also an application of this theory to the problem in part (i).

Another common problem in part (ii) was referencing supply and demand. No credit was awarded for this, as supply and demand does not form part of liquidity preference theory.

QUESTION 6

Given that $\delta = 6.5\%$, then $i = e^{\delta} - 1 = 0.067159$.

We have

$$PV(L) = 9,000\,a_{\overline{4}|} + 11,000v^5 + 11,000v^6$$

$$= 46,076.86875$$

Let A = the amount invested in asset A and B = the amount invested in asset B

Then

$$PV_A = A + B$$

$$PV(A) = A + B = 46,076.86875$$

Now, numerator of DMT (L) is

$$DMT_{numerator}(L) = 9,000\left(Ia\right)_{\overline{4}|} + 11,000 \times (5) \times v^5 + 11,00 \times (6) \times v^6 = 158,638.476$$

$$DMT_{numerator}(A) = 10B = 158,638.476 \Rightarrow B = 15,863.8476$$

$$PV(A) = A + B = 46,076.86875 \Rightarrow A + 15,863.8476 = 46,076.86875$$

$$\Rightarrow A = 30,213.0212$$

This question was generally well-answered by most candidates. With a good exam technique 10 to 12 marks could easily be gained even with some technical errors.

The most common errors were to include the cash in the calculation of the discounted mean term of the assets and to calculate the nominal amount of asset B instead of the amount invested at time 0.

QUESTION 7

i.

$$i^{(4)} = 11.25\% \quad \Rightarrow \quad i^{(12)} = 11.1461489\%$$

$$Xa_{\overline{25}|}^{(12)} = 275,000$$

$$X = 32,693.0028 \text{ .}$$

The monthly repayment is thus $\frac{X}{12} = 2,724.42$

The outstanding balance after the 84^{th} payment (7 years) is

$$L_{84} = Xa_{\overline{18}|}^{(12)} \ @ i^{(12)} = 0.111461489$$

$$L_{84} = 253\ 499.199073$$

$$i = 13\% \Rightarrow i^{(12)} = 0.122842132$$

Let Y be the new annual repayment after the rate change. Then

$$Ya_{\overline{18}|}^{(12)} = 253\,499.199073 \quad @ i^{(12)} = 0.122842132$$

$$Y = \frac{253{,}499.199073}{7.23845859} = 35{,}021.1579$$

The new monthly repayment is thus $\frac{Y}{12} = 2{,}918.43$

The monthly increase is $\frac{Y}{12} - \frac{X}{12} = 194.012900198973 \approx 194$

ii.

Instalments increased to cover the additional interest due to the effective interest rate charged on the loan increasing from 11.73% to 13% for the last 18 years.

Common errors in part (i) were not converting the interest rates to the payment period of the loan, as well as using incorrect time periods in calculating the loan payments and outstanding loan.

QUESTION 8

i.

R = redemption, P = price , D =annual coupons payable p times per year

$i^{(p)}$ =net redemption yield

$$R > P$$

$$R > (1-t_1)Da_{\overline{n}|}^{(p)} + Rv^n$$

$$R > (1-t_1)D\left[\frac{1-v^n}{i^{(p)}}\right] + Rv^n$$

$$(R - Rv^n) = R(1-v^n) \;>\; (1-t_1)D\left[\frac{1-v^n}{i^{(p)}}\right]$$

$$R \;>\; (1-t_1)D\left(\frac{1}{i^{(p)}}\right)$$

$$i^{(p)} > \;(1-t_1)\left(\frac{D}{R}\right) \Rightarrow capital \text{ gain}$$

ii.

$i = 0.06 \Rightarrow i^{(2)} = 0.059126$

$0.6 \times \frac{0.08}{1.33} = 0.0360902 < i^{(2)}$ Thus, there is gain on capital

Assume worst case scenario for investor thus redeem as late as possible thus n=25

P = $0.6 \times 0.08 \times 1,200,000 a_{\overline{25|}}^{(2)} + 1.33 \times 1,200,000 \times v^{25}$

$= 57,600 a_{\overline{25|}}^{(2)} + 1,596,000 v^{25}$

$= 747,205.2534 + 371,865.8143 = 1,119,071.07$

iii.

The investor chose the worst-case scenario (where the net yield would be at its lowest value) and calculated the price for that scenario.

If the borrower now redeems at a different date it is no longer the worst-case scenario for the investor

And the net yield for the investor > 6%.

Part (i) was poorly answered by most candidates.

Part (ii) was answered well by well-prepared candidates. Candidates should note that half of the marks in this part of the question is allocated to determination of the redemption date, so all necessary information (as shown above) should be stated.

Even though no capital gains tax is payable, candidates should still determine whether a capital gain/loss is made as this will determine the redemption date corresponding to the worst-case scenario.

Part (iii) was poorly answered by most candidates as this relied on an understanding of how the redemption date is chosen. Candidates will notice that no new information is needed to answer part (iii).

QUESTION 9

i.

PV (Expenses) = $210,000\ddot{a}_{\overline{3}|} = 595,012.4599502$

Let t = DPP . Therefore

$$-595,012.4599502 + \int_0^t 30,000 v_{0.06}^s (1.055)^s \, ds = 0$$

New interest rate: $\frac{1.055}{1.06} < 1 \quad \Rightarrow \frac{1.055}{1.06} = \frac{1}{1+j} \quad \Rightarrow j = 0.004739336$

$$-595,012.4599502 + 30,000 \int_0^t v_j^s ds = 0$$

Now $\int_0^t v_j^s ds = \bar{a}_{\overline{t}|j}$ thus

$$-595,012.4599502 + 30,000\bar{a}_{\overline{t}|j} = 0$$

$$-595,012.4599502 + 30,000\left[\frac{1-v_j^t}{0.004728141}\right] = 0$$

$$1 - v_j^t = 0.093776764$$

$$\Rightarrow t = 20.82628289$$

ii.

Rate of payment: $30,000(1.055)^{20.82628289} = 91,492.09177$

Time to end of project: $25 - 20.82628289 = 4.173717111$

Profit: $91,492.09177\int_0^{4.173717111} (1.045)^{4.173717111-t} \times (1.055)^t \, dt$

$$= 91,492.09177 \times (1.045)^{4.173717111} \int_0^{4.173717111} \left[\frac{1.055}{1.045}\right]^t dt$$

New interest rate : $\frac{1.055}{1.045} = (1 + j^{'}) \quad \Rightarrow j^{'} = 0.009569378 \quad \Rightarrow \delta = 0.009523882$

$$= 91,492.09177 \times (1.045)^{4.173717111} \times \bar{s}_{\overline{4.173717111}|} \;@\; j^{'} = R468,115.20$$

This was the most poorly answered question in the paper.

In part (i) candidates ignored the information that the growth of the net revenue would be continuous and made the increases discrete.

Although this seemed to simplify the question, the solution corresponding to the assumption of annual increases became more complex, including the solution to part (ii).

QUESTION 10

The key steps in a modelling process can be described as follows:

Develop a well-defined set of objectives which need to be met by the modelling process.

Plan the modelling process and how the model will be validated.

Collect and analyse the necessary data for the model.

Define the parameters for the model and consider appropriate parameter values.

Define the model initially by capturing the essence of the real-world system.

Involve experts on the real-world system you are trying to imitate to get feedback on the validity of the conceptual model.

Decide on whether a simulation package or a general-purpose language is appropriate for the implementation of the model. Choose a statistically reliable random number generator that will perform adequately in the context of the complexity of the model

Write the computer program for the model.

Debug the program to make sure it performs the intended operations in the model definition.

Test the reasonableness of the output from the model.

Review and carefully consider the appropriateness of the model in the light of small changes in input parameters.

Analyse the output from the model.

Ensure that any relevant professional guidance has been complied with.

Communicate and document the results and the model.

This question was answered much better than similar questions in previous sittings.

