Actuarial Society of South Africa

EXAMINERS' REPORT

April 2019

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

It was noted that many candidates struggled to reach the end of the paper, this was taken into account in the marking process.

Question 1

(i)

$$\bar{a}_{\overline{n|}} = \int_0^n 1.v^t dt$$

(ii)

$$\bar{a}_{\overline{n|}} = \int_0^n e^{-\delta t} dt = \left.\frac{e^{-\delta t}}{-\delta}\right|_0^n = \frac{e^{-\delta n}}{-\delta} - \left(\frac{1}{-\delta}\right) = \frac{1-e^{-\delta n}}{\delta} = \frac{1-v^n}{\delta}$$

A large number of marginally candidates (candidates who are close to passing) could not reproduce this easy bookwork answer.

Question 2

(i)

- Descriptive analysis simply summarizes data into usable forms
- but draws no conclusions
- Inferential analysis uses sample data to draw
- conclusions about the population as a whole.

(ii)

- The researcher is using inferential analysis,
- since she is only surveying 20% of the workforce,
- but will be drawing conclusions for the entire company

(iii)

- The researcher can use stratified sampling.
- She can divide the workforce into strata based on level of employment and location
- and draw a 20% simple random sample from each.
- The aim of this method is to overcome the issue of simple random sampling which does not fully reflect characteristics of the population.

This question expected of candidates to apply the theory. A common mistake was quoting standard definitions which did not answer the question.

Candidates would do well in paying attention to the command verb in the question, i.e." explain and recommend" requires more than just stating standard definitions.

Question 3

- The objectives of the modelling exercise.
- The validity of the model for the purpose to which it is to be put.
- The validity of the data to be used.
- The validity of the assumptions.
- The possible errors associated with the model or parameters used not being a perfect representation of the real world situation being modelled.
- The impact of correlations between the random variables that 'drive' the model.
- The extent of correlations between the various results produced from the model.
- The current relevance of models written and used in the past.
- The credibility of the data input.
- The credibility of the results output.
- The dangers of spurious accuracy.
- The ease with which the model and its results can be communicated.
- Regulatory requirements.

This question was poorly answered by marginal candidates. Words like "validity" and "credibility" were used incorrectly and incorrect substitutes were used.

Question 4

$$r = \left[\frac{1.1}{(1.0045)^{12}}\right] - 1 = 4.2301571\%$$

Solve for X (representing real investment):

$$10,000(1+r)^2 + Y(1+r) = 13,000 \Rightarrow Y = 2,049.382273$$

Thus, he should invest an amount of $X = 2,049.382273(1.0045)^{12} = 2,162.829419$

Alternatively

$$10,000(1+0.1)^2 + X(1+0.1) = 13,000(1.0045)^{(2\times 12)}$$

$$X(1+0.1) = 2379.112361$$

$$\Rightarrow X = 2,162.829419$$

The concept of real and nominal interest rates were a problem for many candidates. Candidates that could work out the real value of X neglected to convert it to a nominal value.

Question 5

(i)

$$\left(1+\frac{i^{(4)}}{4}\right)^{4*3}=1.45$$

$$i^{(4)}=12.58\%$$

(ii)

$$A(0,5)=\frac{1.45*(1.7)}{1.3}=1.896156$$

$$1.896156=\left(1-\frac{d^{(12)}}{12}\right)^{-60}\Rightarrow\frac{d^{(12)}}{12}=1.0607\%$$

Question 6

$$A(2,9)=\int_2^9 e^{0.08+0.0002t^3}\exp\left(\int_t^9 0.09+0.0006s^2ds\right)dt=\int_2^9 e^{0.08+0.0002t^3}\exp\left(0.09s+\frac{0.0006s^3}{3}\right)\Bigg|_t^9 dt$$

$$=e^{1.0358}\int_2^9 e^{-0.09t}dt=e^{1.0358}\left[\frac{e^{-0.09t}}{-0.09}\right]_2^9=12.221458$$

$$A(9,15)=\exp\left(\int_9^{15}0.1836dt\right)=\exp\left[0.1836t\big|_9^{15}\right]=3.00897$$

$$A(15,17)=\exp\left(\int_{15}^{17}0.1086-0.005tdt\right)=\exp\left(0.1086t-\frac{0.005t^2}{2}\right)\Bigg|_{15}^{17}=1.05887$$

Total:
12.221458*3.00897*1.05887=38.93879759

Many well-prepared candidates scored full marks for this question whereas many marginal candidates made mistakes in setting up the boundary conditions for the integral.

Question 7

$$PV = \left[12(9,000)a_{\overline{1}|}^{(12)} + 12(10,000)a_{\overline{1}|}^{(12)}v + \ldots + 12(18,000)a_{\overline{1}|}^{(12)}v^9\right]_{@5\%}$$

$$+v_{5\%}^{10}\left[12(19,000)a_{\overline{1}|}^{(12)} + 12(20,000)a_{\overline{1}|}^{(12)}v^2 + \ldots + 12(28,000)a_{\overline{1}|}^{(12)}v^9\right]_{@8\%}$$

$$= 12a_{\overline{1}|5\%}^{(12)}\left[(9,000) + (10,000)v + \ldots + (18,000)v^9\right]_{@5\%}$$

$$+12a_{\overline{1}|8\%}^{(12)}v_{5\%}^{10}\left[(19,000) + (20,000)v + \ldots + (28,000)v^9\right]_{@8\%}$$

$$= 12a_{\overline{1}|5\%}^{(12)}\left[8,000\ddot{a}_{\overline{10}|5\%} + 1,000(I\ddot{a})_{\overline{10}|5\%}\right]$$

$$= 12a_{\overline{1}|8\%}^{(12)}v_{5\%}^{10}\left[18,000\ddot{a}_{\overline{10}|8\%} + 1,000(I\ddot{a})_{\overline{10}|8\%}\right]$$

$$= 2,412,816.949$$

Candidates lost marks by stating an incorrect formula as a first step. Easy marks could be gained by writing down the initial equation of value and then simplifying the equation to find the correct final equation.

Some candidate used a $(Ia)_{\overline{n}|}^{(12)}$ notation. This notation is not in the A211 syllabus and candidate will lose marks, in future, for using notation not in the A211 syllabus.

Question 8

(i)

$y_1 = 0.11$

$y_3 = [(1.11)\times(1.09)\times(1.07)]^{1/3} - 1 = 0.089878$

$y_5 = [(1+y_3)^3 \times (1.06)^2]^{1/5} - 1 = 0.077827$

(ii)

$(1+y_5)^5 = (1+f_0)\times(1+f_1)\times(1+f_{2,3})^3$

$(1+0.077827)^5 = (1.11)\times(1.09)\times(1+f_{2,3})^3 \Rightarrow f_{2,3} = 0.063323$

(iii)

Longer date bonds are more sensitive to interest rate movement than shorter dated bond
Risk averse investor will require higher return to invest in longer dated bond
This may explain the excess return offered on longer dated bond

For part (i) and (ii) many alternative solutions exist.

Part (iii) was one of the worst answered questions in the paper. Although liquidity preference theory occurs in other subjects as well, candidates should stay within the A211 syllabus and take account of the context of the question, which in this case was term structure of interest rates.

Question 9

(i)

$PV(L) = 100,000v^4 + 200,000v^{14} = 167,669.559199737$ .... $\boxed{1}$

$Numerator\ of\ \ DMT(L) = 100,000(4)v^4 + 200,000(14)v^{14} = 1,555,280.165558290$ .... $\boxed{2}$

$DMT\ \ = 9.27586482$

$PV(A) = 10,000(0.04)(5)a_{\overline{5}|} + 10,000(5)v^5 + Xv^n$

$= 45,787.6362 + Xv^n$ .... $\boxed{3}$

$Numerator\ of\ \ DMT(L) = 2,000(Ia)_{\overline{5}|} + 50,000(5)v^5 + nXv^n$

$= 211,108.368162160 + nXv^n$ .... $\boxed{4}$

Set $\boxed{1} = \boxed{3}$

$2,000a_{\overline{5}|} + 10,000(5)v^5 + Xv^n = 167,669.559199737 \Rightarrow Xv^n = 121,881.922985302$ .... $\boxed{5}$

Sub $\boxed{5}$ into $\boxed{4}$ and set equal to $\boxed{2}$

$211,108.368162160 + n(121,881.922985302) = 1,555,280.165558290$

$\Rightarrow n = 11.028475466$

Sub n into $\boxed{5}$

$Xv^n = 121,881.922985302 \Rightarrow X = 231,752.491538740$

(ii)

Redington's third condition does not hold, since the liabilities are more spread out around the discounted mean term than the assets.

Many well prepared candidates scored full marks for part (i). There are many alternative solutions to part (i), so if the candidate works consistently with the alternative solutions the same numerical answer will be obtained.

In part (ii) it was clear that the marginal candidates did not fully understand the concept of convexity, referring to spread of assets and liabilities rather that spread of assets and liabilities around the discounted mean term.

Question 10

(i)

Capital Gains Test: $i^{(2)} = 0.068816 > (1-0.25)\left(\frac{8}{100}\right) = 0.06$

Capital gain for investor, thus capital loss for borrower. Choose latest redemption date, 1 April 2028.

Thus, the purchase price for the original investor is

$$P = (1.07)^{-0.25} \times (100{,}000) \times \left[(1-0.25)(0.08)\ddot{a}^{(2)}_{\overline{12}|7\%} + v^{11.5}_{7\%}\right] - 0.3 \times (100{,}000 - P) \times v^{11.75}_{7\%}$$

$$= \frac{(100{,}000) * (1.07)^{-0.25}\left[(1-0.25)(0.08)\ddot{a}^{(2)}_{\overline{12}|7\%} + (1-0.3) * v^{11.5}_{7\%}\right]}{1 - 0.3 * v^{11.75}_{7\%}}$$

=R93,593.63

(ii)

$i^{(2)} = 0.049390153 < (1-0.25)\left(\frac{8}{100}\right) = 0.06 \Rightarrow$ capital loss

Thus, choose earliest redemption date

$$P = 100{,}000(1-0.25)(0.08)a^{(2)}_{\overline{4}|5\%} + 100{,}000v^{4}_{5\%} = 103{,}808.653$$

(iii)

The sale price is significantly higher than the redemption amount,
This increases the capital gain earned by the original investor.
This higher capital gain is realised sooner,
which will lead to a higher net redemption yield than 7% per annum effective.
Investor A decision to sell the bond was financially sound.

The most common mistake in part (i) was calculating the term to redemption incorrectly, as this bond was purchased on a non-coupon date.

Part (ii) was one of the worst answered questions in the paper with many candidates just subtracting two years from the term to redemption calculated in part (i).

However, in part (ii) the test for a capital gain must be repeated as a new investor, with new circumstances, is purchasing the bond. The new investor wants to determine the price for a minimum yield, which represents a worst case scenario for the new investor.

Question 11

(working in millions)

(i)

$$PV = 20 + (12)(2.5)a^{(12)}_{\overline{7/12}|} + v^{7/12}\left(60 + 12(6)a^{(12)}_{\overline{9/12}|}\right) = 144.430958 \quad @\ i = 8\%$$

(ii)

$$(12)(0.8)\ddot{a}^{(12)}_{\overline{7/12}|} + v^{7/12}1(12)\ddot{a}^{(12)}_{\overline{9/12}|} = 13.88183665 \quad @\ i = 8\%$$

(iii)

$$PV = -35 - 144.430958 + 13.88183665 + v^{16/12}(4)(12)\ddot{a}^{(12)}_{\overline{n}|} \geq 0 \quad @ i = 8\%$$

$$v^{16/12}(8)(12)\ddot{a}^{(12)}_{\overline{n}|} \geq 165.5491251 \Rightarrow \ddot{a}^{(12)}_{\overline{n}|} = 3.82164779$$

$$\Rightarrow n = 4.508447 \text{years}, 55 \text{ number of payments}$$

Thus DPP will be at the 55th income receipt (with first payment being at 16th month), thus at t = 70 months or 5 years and 10 months

(iv)
Accumulated profit in millions at 4% effective investment rate

$$(12)(4)\ddot{s}^{(12)}_{\overline{49/12}|4\%} + 70 = 282.9149$$

(v)

Choose Option A.
It has an earlier DPP and so loan will be paid off earlier with less interest paid on loan.
More time to earn interest, thus larger accumulated value.

This was the question were the reading time, if appropriately used, would have been most beneficial due to the large amount of information that needed to be digested.

Very few candidates did this question as time seemed to be a factor.

The question was broken up into sections to aid in the answering. Few candidates realized that the answers to part (i) and (ii) could directly be used in part (iii).

In part (i) and (ii) the South phase and North phase needed to be taken into account in the cashflows calculations.

Part (v) could be answered even if no calculations were done in this question.

