EXAMINATION

14 May 2021

Subject A211 — Financial Mathematics

Time allowed:

Two hours and fifteen minutes – examination time

20 minutes (at the end of the examination) – scan and upload time

INSTRUCTIONS TO THE CANDIDATE

1. Once you have entered the ASSA Exam Platform, ensure that you have accessed the **Video Room** Invigilation link with both your camera and microphone on, **before you *attempt the examination.***
2. *Your PC must be placed, and camera angled, so that your writing area on your desk is visible to the invigilator.*
3. *Ensure that you have your candidate number handy to input **as part of the 2 hours 15 minutes examination**. Write your candidate number at the top of each page. (DO NOT WRITE YOUR NAME OR MEMBER NUMBER.)*
4. *Your cell phone that will be used to scan your final answer script must be switched **OFF** during the 2 hours and 15 minutes examination time. Place your cell phone at the top of your examination pad / writing pages in view of the invigilator.*
5. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may commence answering the paper whenever you are ready. You then have two hours to complete the paper.*
6. *Questions are only available in the ASSA Exam Platform and may not be printed or copied outside of the ASSA Exam Platform.*
7. *You are required to write your answers on a clean A4 examination pad. Write only on 1 side of the paper and number your pages.*

8. *Attempt all questions, beginning your answer to each question on a new page and numbering your answers clearly.*
9. *Write in black or dark blue pen.*
10. *You should show calculations where this is appropriate.*
11. *You MAY NOT use any computer program (e.g. email, MS Word or Excel), files or open any other browsers or browser tabs during the examination time.*
12. *You MAY NOT make use of a Formulae and Tables book during the examination. Any such information that may be required will be provided to you within the examination.*
13. *Mark allocations are shown in brackets.*
14. *You may use additional scrap paper to make notes where this is appropriate. This paper MUST NOT BE SCANNED as part of your answer script.*
15. *Assume that months are all of equal length, unless otherwise stated.*
16. *At the end of the 2 hours and 15 minutes examination time, you must stop writing and may start scanning and uploading your script.* ***Do not continue writing into upload time****.*
17. *Access to your PC will be opened-up after the examination time so you can access your scanned file. You may now also switch on your cell phone to scan.*
18. *Scan ALL your answer pages to .pdf so that your candidate number at the top of the page is clear.*
19. *Save your .pdf scanned file using your candidate number as file name. (DO NOT USE YOUR NAME OR MEMBER NUMBER AS FILE NAME.)*
20. *Transfer your .pdf script to your PC and click on the UPLOAD ANSWERS link below the examination paper link.*
21. *Upload your answer file into the ASSA Exam Platform and ensure you click on FINISH below the upload box and again on FINISH all and SUBMIT,* ***before*** *the 20 minute upload time is up. (If the status on the summary page indicates "Answer saved" your file was uploaded. You can click on Review attempt to see the file you have uploaded)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

The news of COVID-19 spreading across the world led a group of researchers to develop a model, called the $S-I-R$ model, to estimate the number of COVID-19 positive cases at any point in time. In the model

- S = Susceptible, those without the virus but can be infected
- I = Infected, those who have been infected by the virus and
- R = Removed, those who have either died or have recovered from the virus infection or are immune to the virus

Assume that:

- The population of any country is divided into the three categories of *S*, *I* and *R*.
- The number of new infections is a result of the interaction between members of the three categories.
- The rates of interaction between the categories are stipulated as α_1, α_2 and α_3, respectively.
- $S+I+R = N$ is constant (national population) at all *t* where *t > 0*.

List five factors to consider when assessing the suitability of the proposed modelling approach and for each listed factor explain whether this model meets the suitability requirements in this context.

[Total 7]

QUESTION 2

i. Prove that $\frac{1}{d^{(p)}} = \frac{1}{p} + \frac{1}{i^{(p)}}$, where *p* can be any natural number and

$d^{(p)}$ = nominal discount rate compounded p-thly per annum and

$i^{(p)}$ = nominal interest rate compounded p-thly per annum. [4]

ii. Explain how the identity in (i) can also describe the relationship between the present values of two perpetuities, each paying 1 per annum *p* times per year, where the one perpetuity is payable in advance and the other is payable in arrears. [3]

[Total 7]

QUESTION 3

Describe briefly the three most popular theories used to explain the term structure of interest rates.

[Total 4]

PLEASE TURN OVER

QUESTION 4

i. The force of interest $\delta(t)$, at time *t* is given by

$$\delta(t) = \begin{cases} 0.03 & \text{if } t \le 8 \\ 0.05 & \text{if } t > 8 \end{cases}$$

where *t* is measured in years.

Calculate the present value at time *t=0* of a continuous payment stream payable at a rate of $\rho(t) = 5e^{0.05t}$ per year from time *t=0* to time *t=15*. [10]

ii. The half-yearly effective rate of interest is 2.5%.

a. Calculate the equivalent annual nominal rate of discount, convertible every four months. [2]

b. Calculate the equivalent rate of interest denoted by $i^{\left(\frac{1}{4}\right)}$. [2]

[Total 14]

QUESTION 5

A listed company has liabilities of R100 million due in ten years' time and R200 million due in 15 years' time. It also owns assets consisting of two zero-coupon bonds, one paying R74.04 million in two years' time and the other paying R318.34 million in 25 years' time. The current interest rate is 7% per annum effective.

i. Show that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this company.

Work in millions and allow for rounding to two decimal places in your final answers. [7]

PLEASE TURN OVER

Compound Interest

n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
1	1.070 00	0.934 58	1.000 0	0.934 6	0.934 6	0.934 6	1
2	1.144 90	0.873 44	2.070 0	1.808 0	2.681 5	2.742 6	2
3	1.225 04	0.816 30	3.214 9	2.624 3	5.130 4	5.366 9	3
4	1.310 80	0.762 90	4.439 9	3.387 2	8.181 9	8.754 1	4
5	1.402 55	0.712 99	5.750 7	4.100 2	11.746 9	12.854 3	5
6	1.500 73	0.666 34	7.153 3	4.766 5	15.744 9	17.620 9	6
7	1.605 78	0.622 75	8.654 0	5.389 3	20.104 2	23.010 2	7
8	1.718 19	0.582 01	10.259 8	5.971 3	24.760 2	28.981 4	8
9	1.838 46	0.543 93	11.978 0	6.515 2	29.655 6	35.496 7	9
10	1.967 15	0.508 35	13.816 4	7.023 6	34.739 1	42.520 3	10
11	2.104 85	0.475 09	15.783 6	7.498 7	39.965 2	50.018 9	11
12	2.252 19	0.444 01	17.888 5	7.942 7	45.293 3	57.961 6	12
13	2.409 85	0.414 96	20.140 6	8.357 7	50.687 8	66.319 3	13
14	2.578 53	0.387 82	22.550 5	8.745 5	56.117 3	75.064 7	14
15	2.759 03	0.362 45	25.129 0	9.107 9	61.554 0	84.172 7	15
16	2.952 16	0.338 73	27.888 1	9.446 6	66.973 7	93.619 3	16
17	3.158 82	0.316 57	30.840 2	9.763 2	72.355 5	103.382 5	17
18	3.379 93	0.295 86	33.999 0	10.059 1	77.681 0	113.441 6	18
19	3.616 53	0.276 51	37.379 0	10.335 6	82.934 7	123.777 2	19
20	3.869 68	0.258 42	40.995 5	10.594 0	88.103 1	134.371 2	20
21	4.140 56	0.241 51	44.865 2	10.835 5	93.174 8	145.206 8	21
22	4.430 40	0.225 71	49.005 7	11.061 2	98.140 5	156.268 0	22
23	4.740 53	0.210 95	53.436 1	11.272 2	102.992 3	167.540 2	23
24	5.072 37	0.197 15	58.176 7	11.469 3	107.723 8	179.009 5	24
25	5.427 43	0.184 25	63.249 0	11.653 6	112.330 1	190.663 1	25
26	5.807 35	0.172 20	68.676 5	11.825 8	116.807 1	202.488 9	26
27	6.213 87	0.160 93	74.483 8	11.986 7	121.152 3	214.475 6	27
28	6.648 84	0.150 40	80.697 7	12.137 1	125.363 5	226.612 7	28
29	7.114 26	0.140 56	87.346 5	12.277 7	129.439 9	238.890 4	29
30	7.612 26	0.131 37	94.460 8	12.409 0	133.380 9	251.299 4	30
31	8.145 11	0.122 77	102.073 0	12.531 8	137.186 8	263.831 2	31
32	8.715 27	0.114 74	110.218 2	12.646 6	140.858 5	276.477 8	32
33	9.325 34	0.107 23	118.933 4	12.753 8	144.397 3	289.231 6	33
34	9.978 11	0.100 22	128.258 8	12.854 0	147.804 7	302.085 6	34
35	10.676 58	0.093 66	138.236 9	12.947 7	151.082 9	315.033 3	35
36	11.423 94	0.087 54	148.913 5	13.035 2	154.234 2	328.068 5	36
37	12.223 62	0.081 81	160.337 4	13.117 0	157.261 2	341.185 5	37
38	13.079 27	0.076 46	172.561 0	13.193 5	160.166 5	354.379 0	38
39	13.994 82	0.071 46	185.640 3	13.264 9	162.953 3	367.643 9	39
40	14.974 46	0.066 78	199.635 1	13.331 7	165.624 5	380.975 6	40
41	16.022 67	0.062 41	214.609 6	13.394 1	168.183 3	394.369 7	41
42	17.144 26	0.058 33	230.632 2	13.452 4	170.633 1	407.822 2	42
43	18.344 35	0.054 51	247.776 5	13.507 0	172.977 2	421.329 1	43
44	19.628 46	0.050 95	266.120 9	13.557 9	175.218 8	434.887 0	44
45	21.002 45	0.047 61	285.749 3	13.605 5	177.361 4	448.492 5	45
46	22.472 62	0.044 50	306.751 8	13.650 0	179.408 4	462.142 6	46
47	24.045 71	0.041 59	329.224 4	13.691 6	181.363 0	475.834 2	47
48	25.728 91	0.038 87	353.270 1	13.730 5	183.228 6	489.564 7	48
49	27.529 93	0.036 32	378.999 0	13.766 8	185.008 5	503.331 4	49
50	29.457 03	0.033 95	406.528 9	13.800 7	186.705 9	517.132 2	50
60	57.946 43	0.017 26	813.520 4	14.039 2	199.806 9	656.583 1	60
70	113.989 39	0.008 77	1 614.134 2	14.160 4	207.678 9	797.708 7	70
80	224.234 39	0.004 46	3 189.062 7	14.222 0	212.296 8	939.685 6	80
90	441.102 98	0.002 27	6 287.185 4	14.253 3	214.957 5	1 082.095 3	90
100	867.716 33	0.001 15	12 381.661 8	14.269 3	216.469 3	1 224.725 0	100

7%

i	0.070 000
$i^{(2)}$	0.068 816
$i^{(4)}$	0.068 234
$i^{(12)}$	0.067 850
δ	0.067 659
$(1+i)^{1/2}$	1.034 408
$(1+i)^{1/4}$	1.017 059
$(1+i)^{1/12}$	1.005 654
v	0.934 579
$v^{1/2}$	0.966 736
$v^{1/4}$	0.983 228
$v^{1/12}$	0.994 378
d	0.065 421
$d^{(2)}$	0.066 527
$d^{(4)}$	0.067 090
$d^{(12)}$	0.067 468
$i/i^{(2)}$	1.017 204
$i/i^{(4)}$	1.025 880
$i/i^{(12)}$	1.031 691
i/δ	1.034 605
$i/d^{(2)}$	1.052 204
$i/d^{(4)}$	1.043 380
$i/d^{(12)}$	1.037 525

PLEASE TURN OVER

The interest rate then increases immediately to 8% per annum effective.

ii. Determine the present value of the profit or loss that the company will make at the new increased interest rate. [2]

Compound Interest

8%

i	0.080 000
$i^{(2)}$	0.078 461
$i^{(4)}$	0.077 706
$i^{(12)}$	0.077 208
δ	0.076 961
$(1+i)^{1/2}$	1.039 230
$(1+i)^{1/4}$	1.019 427
$(1+i)^{1/12}$	1.006 434
v	0.925 926
$v^{1/2}$	0.962 250
$v^{1/4}$	0.980 944
$v^{1/12}$	0.993 607
d	0.074 074
$d^{(2)}$	0.075 499
$d^{(4)}$	0.076 225
$d^{(12)}$	0.076 715
$i/i^{(2)}$	1.019 615
$i/i^{(4)}$	1.029 519
$i/i^{(12)}$	1.036 157
i/δ	1.039 487
$i/d^{(2)}$	1.059 615
$i/d^{(4)}$	1.049 519
$i/d^{(12)}$	1.042 824

n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
1	1.080 00	0.925 93	1.000 0	0.925 9	0.925 9	0.925 9	1
2	1.166 40	0.857 34	2.080 0	1.783 3	2.640 6	2.709 2	2
3	1.259 71	0.793 83	3.246 4	2.577 1	5.022 1	5.286 3	3
4	1.360 49	0.735 03	4.506 1	3.312 1	7.962 2	8.598 4	4
5	1.469 33	0.680 58	5.866 6	3.992 7	11.365 1	12.591 1	5
6	1.586 87	0.630 17	7.335 9	4.622 9	15.146 2	17.214 0	6
7	1.713 82	0.583 49	8.922 8	5.206 4	19.230 6	22.420 4	7
8	1.850 93	0.540 27	10.636 6	5.746 6	23.552 7	28.167 0	8
9	1.999 00	0.500 25	12.487 6	6.246 9	28.055 0	34.413 9	9
10	2.158 92	0.463 19	14.486 6	6.710 1	32.686 9	41.124 0	10
11	2.331 64	0.428 88	16.645 5	7.139 0	37.404 6	48.262 9	11
12	2.518 17	0.397 11	18.977 1	7.536 1	42.170 0	55.799 0	12
13	2.719 62	0.367 70	21.495 3	7.903 8	46.950 1	63.702 8	13
14	2.937 19	0.340 46	24.214 9	8.244 2	51.716 5	71.947 0	14
15	3.172 17	0.315 24	27.152 1	8.559 5	56.445 1	80.506 5	15
16	3.425 94	0.291 89	30.324 3	8.851 4	61.115 4	89.357 9	16
17	3.700 02	0.270 27	33.750 2	9.121 6	65.710 0	98.479 5	17
18	3.996 02	0.250 25	37.450 2	9.371 9	70.214 4	107.851 4	18
19	4.315 70	0.231 71	41.446 3	9.603 6	74.617 0	117.455 0	19
20	4.660 96	0.214 55	45.762 0	9.818 1	78.907 9	127.273 2	20
21	5.033 83	0.198 66	50.422 9	10.016 8	83.079 7	137.290 0	21
22	5.436 54	0.183 94	55.456 8	10.200 7	87.126 4	147.490 7	22
23	5.871 46	0.170 32	60.893 3	10.371 1	91.043 7	157.861 8	23
24	6.341 18	0.157 70	66.764 8	10.528 8	94.828 4	168.390 5	24
25	6.848 48	0.146 02	73.105 9	10.674 8	98.478 9	179.065 3	25
26	7.396 35	0.135 20	79.954 4	10.810 0	101.994 1	189.875 3	26
27	7.988 06	0.125 19	87.350 8	10.935 2	105.374 2	200.810 4	27
28	8.627 11	0.115 91	95.338 8	11.051 1	108.619 8	211.861 5	28
29	9.317 27	0.107 33	103.965 9	11.158 4	111.732 3	223.019 9	29
30	10.062 66	0.099 38	113.283 2	11.257 8	114.713 6	234.277 7	30
31	10.867 67	0.092 02	123.345 9	11.349 8	117.566 1	245.627 5	31
32	11.737 08	0.085 20	134.213 5	11.435 0	120.292 5	257.062 5	32
33	12.676 05	0.078 89	145.950 6	11.513 9	122.895 8	268.576 4	33
34	13.690 13	0.073 05	158.626 7	11.586 9	125.379 3	280.163 3	34
35	14.785 34	0.067 63	172.316 8	11.654 6	127.746 6	291.817 9	35
36	15.968 17	0.062 62	187.102 1	11.717 2	130.001 0	303.535 1	36
37	17.245 63	0.057 99	203.070 3	11.775 2	132.146 5	315.310 3	37
38	18.625 28	0.053 69	220.315 9	11.828 9	134.186 8	327.139 1	38
39	20.115 30	0.049 71	238.941 2	11.878 6	136.125 6	339.017 7	39
40	21.724 52	0.046 03	259.056 5	11.924 6	137.966 8	350.942 3	40
41	23.462 48	0.042 62	280.781 0	11.967 2	139.714 3	362.909 6	41
42	25.339 48	0.039 46	304.243 5	12.006 7	141.371 8	374.916 3	42
43	27.366 64	0.036 54	329.583 0	12.043 2	142.943 0	386.959 5	43
44	29.555 97	0.033 83	356.949 6	12.077 1	144.431 7	399.036 6	44
45	31.920 45	0.031 33	386.505 6	12.108 4	145.841 5	411.145 0	45
46	34.474 09	0.029 01	418.426 1	12.137 4	147.175 8	423.282 4	46
47	37.232 01	0.026 86	452.900 2	12.164 3	148.438 2	435.446 7	47
48	40.210 57	0.024 87	490.132 2	12.189 1	149.631 9	447.635 8	48
49	43.427 42	0.023 03	530.342 7	12.212 2	150.760 2	459.848 0	49
50	46.901 61	0.021 32	573.770 2	12.233 5	151.826 3	472.081 4	50
60	101.257 06	0.009 88	1 253.213 3	12.376 6	159.676 6	595.293 1	60
70	218.606 41	0.004 57	2 720.080 1	12.442 8	163.975 4	719.464 8	70
80	471.954 83	0.002 12	5 886.935 4	12.473 5	166.273 6	844.081 1	80
90	1 018.915 09	0.000 98	12 723.938 6	12.487 7	167.480 3	968.903 3	90
100	2 199.761 26	0.000 45	27 484.515 7	12.494 3	168.105 0	1 093.821 0	100

60

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iii. Explain the reason for the profit or loss with reference to Redington's theory of immunisation. [4]

[Total 13]

QUESTION 6

An investor takes out a loan to be repaid monthly over 25 years.

The annual rate of interest is 8% compounded half-yearly and the monthly repayment is R300, payable in arrears.

i. Calculate the original size of the loan [3]

With each of the 84^{th} and 120^{th} repayments of R300, the borrower repays an extra R1,000. The borrower then renegotiates to repay the outstanding balance of the loan with equal monthly repayments over eight years immediately following the 120^{th} payment. A reduced annual effective interest rate of 7% will be applicable to the new time period.

ii. Calculate the loan outstanding at the time of the 120^{th} monthly repayment, after all payments then due have been received. [4]

Now assume that the loan outstanding in (ii) has been calculated as R29,400.

iii. Calculate the interest portion of the payment made exactly 15 years after the date of taking out the original loan. [6]

[Total 13]

PLEASE TURN OVER

QUESTION 7

The following investments were made on 15 January 2015.

Investment A
R10,000 was placed in a special savings account with a five-year term. The invested money accumulated at the following interest rates:

Year	Effective interest rate per annum
1	3.5%
2	4.5%
3	5.5%
4	6.5%
5	7.5%

Investment B
R10,000 was invested in a zero coupon bond with a five-year term. The redemption proceeds were increased by inflation as represented by the Retail Price Index (RPI) with a two month time lag. In addition, simple interest was paid on the amount originally invested at a rate of 2.75% per annum.

The RPI at various times was as follows:

Date	RPI	Date	RPI	Date	RPI
15 Jan 2014	236.4	15 March 2014	235.1	15 Nov 2014	237.6
15 Jan 2015	240.0	15 March 2015	240.2	15 Nov 2015	243.7
15 Jan 2016	250.0	15 March 2016	251.7	15 Nov 2016	249.1
15 Jan 2017	264.4	15 March 2017	264.2	15 Nov 2017	267.8
15 Jan 2018	266.6	15 March 2018	267.3	15 Nov 2018	270.2
15 Jan 2019	270.4	15 March 2019	271.0	15 Nov 2019	274.0
15 Jan 2020	275.6	15 March 2020	276.0	15 Nov 2020	280.2

Investment C
R10,000 was invested in a five-year annuity at an effective interest rate of 6.25% per annum. The annuity provided annual payments in arrears.

i. Calculate the annual annuity payment, provided to the investor, by the annuity in Investment C. [2]

ii. Calculate the annual effective real rate of return earned on Investment A and on Investment B over the period 15 January 2015 to 15 January 2020. [9]

iii. Show that the annual effective real rate of return earned on Investment C, over the period 15 January 2015 to 15 January 2020, is approximately 2.722%. [6]

[Total 17]

PLEASE TURN OVER

QUESTION 8

i. Describe the cashflows for an investor who purchases a share in a company. The company's shares are listed on the JSE.

Include reference to the certainty/uncertainty of the cashflow. [4]

ii. The value of a share is *P,* just after a dividend payment has been made. Let *D* be the amount of the dividend payment that has just been paid.

Assume that:

- Dividends are paid annually in arrears.
- Dividends grow at a rate of *g%* per annum effective.
- Dividends continue into perpetuity.
- No allowance is made for the possibility that the company may default and the dividend payments may cease.
- The interest rate used to value the share is *i%* per annum effective.

Derive a formula for *P* in terms of *D, g* and *i.* [4]

iii. An actuarial financial analyst is valuing two companies, Company A and Company B.

Company A is assumed to pay its first annual dividend after exactly six years. It is assumed that the dividend will be R1.08 per share at that time. Annual dividends are then assumed to grow at 10% per annum compound over the following six years and at 3% per annum compound in perpetuity thereafter.

Company B is expected to pay an annual dividend of R0.75 per share in exactly one year. Annual dividends are then expected to increase thereafter by R0.50 every year. Company B is expected to reach the end of its profitable business period after 30 years and at that time the share will be worthless.

The analyst values dividends from both shares at a rate of interest of 6% per annum effective.

a. Calculate the value of a share in Company A. [9]

b. Calculate the value of a share in Company B. [4]

Subsequently, there is a general rise in interest rates and the analyst decides it is appropriate to increase the valuation rate of interest to 7% per annum effective.

PLEASE TURN OVER

c. Without doing any further calculations, explain what the influence of the increase in the valuation rate of interest will be on the values calculated in (i) and (ii). Include reference to the relative changes in the values. [4]

Compound Interest

6%		n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
i	0.060 000	1	1.060 00	0.943 40	1.000 0	0.943 4	0.943 4	0.943 4	1
$i^{(2)}$	0.059 126	2	1.123 60	0.890 00	2.060 0	1.833 4	2.723 4	2.776 8	2
$i^{(4)}$	0.058 695	3	1.191 02	0.839 62	3.183 6	2.673 0	5.242 2	5.449 8	3
$i^{(12)}$	0.058 411	4	1.262 48	0.792 09	4.374 6	3.465 1	8.410 6	8.914 9	4
		5	1.338 23	0.747 26	5.637 1	4.212 4	12.146 9	13.127 3	5
δ	0.058 269	6	1.418 52	0.704 96	6.975 3	4.917 3	16.376 7	18.044 6	6
		7	1.503 63	0.665 06	8.393 8	5.582 4	21.032 1	23.627 0	7
$(1+i)^{1/2}$	1.029 563	8	1.593 85	0.627 41	9.897 5	6.209 8	26.051 4	29.836 8	8
$(1+i)^{1/4}$	1.014 674	9	1.689 48	0.591 90	11.491 3	6.801 7	31.378 5	36.638 5	9
$(1+i)^{1/12}$	1.004 868	10	1.790 85	0.558 39	13.180 8	7.360 1	36.962 4	43.998 5	10
		11	1.898 30	0.526 79	14.971 6	7.886 9	42.757 1	51.885 4	11
v	0.943 396	12	2.012 20	0.496 97	16.869 9	8.383 8	48.720 7	60.269 3	12
$v^{1/2}$	0.971 286	13	2.132 93	0.468 84	18.882 1	8.852 7	54.815 6	69.122 0	13
$v^{1/4}$	0.985 538	14	2.260 90	0.442 30	21.015 1	9.295 0	61.007 8	78.416 9	14
$v^{1/12}$	0.995 156	15	2.396 56	0.417 27	23.276 0	9.712 2	67.266 8	88.129 2	15
		16	2.540 35	0.393 65	25.672 5	10.105 9	73.565 1	98.235 1	16
d	0.056 604	17	2.692 77	0.371 36	28.212 9	10.477 3	79.878 3	108.712 3	17
$d^{(2)}$	0.057 428	18	2.854 34	0.350 34	30.905 7	10.827 6	86.184 5	119.539 9	18
$d^{(4)}$	0.057 847	19	3.025 60	0.330 51	33.760 0	11.158 1	92.464 3	130.698 1	19
$d^{(12)}$	0.058 128	20	3.207 14	0.311 80	36.785 6	11.469 9	98.700 4	142.168 0	20
		21	3.399 56	0.294 16	39.992 7	11.764 1	104.877 6	153.932 1	21
$i/i^{(2)}$	1.014 782	22	3.603 54	0.277 51	43.392 3	12.041 6	110.982 7	165.973 6	22
$i/i^{(4)}$	1.022 227	23	3.819 75	0.261 80	46.995 8	12.303 4	117.004 1	178.277 0	23
$i/i^{(12)}$	1.027 211	24	4.048 93	0.246 98	50.815 6	12.550 4	122.931 6	190.827 4	24
		25	4.291 87	0.233 00	54.864 5	12.783 4	128.756 5	203.610 7	25
i/δ	1.029 709	26	4.549 38	0.219 81	59.156 4	13.003 2	134.471 6	216.613 9	26
		27	4.822 35	0.207 37	63.705 8	13.210 5	140.070 5	229.824 4	27
$i/d^{(2)}$	1.044 782	28	5.111 69	0.195 63	68.528 1	13.406 2	145.548 2	243.230 6	28
$i/d^{(4)}$	1.037 227	29	5.418 39	0.184 56	73.639 8	13.590 7	150.900 3	256.821 3	29
$i/d^{(12)}$	1.032 211	30	5.743 49	0.174 11	79.058 2	13.764 8	156.123 6	270.586 1	30
		31	6.088 10	0.164 25	84.801 7	13.929 1	161.215 5	284.515 2	31
		32	6.453 39	0.154 96	90.889 8	14.084 0	166.174 2	298.599 3	32
		33	6.840 59	0.146 19	97.343 2	14.230 2	170.998 3	312.829 5	33
		34	7.251 03	0.137 91	104.183 8	14.368 1	175.687 3	327.197 6	34
		35	7.686 09	0.130 11	111.434 8	14.498 2	180.241 0	341.695 9	35
		36	8.147 25	0.122 74	119.120 9	14.621 0	184.659 6	356.316 9	36
		37	8.636 09	0.115 79	127.268 1	14.736 8	188.944 0	371.053 7	37
		38	9.154 25	0.109 24	135.904 2	14.846 0	193.095 1	385.899 7	38
		39	9.703 51	0.103 06	145.058 5	14.949 1	197.114 2	400.848 8	39
		40	10.285 72	0.097 22	154.762 0	15.046 3	201.003 1	415.895 1	40
		41	10.902 86	0.091 72	165.047 7	15.138 0	204.763 6	431.033 1	41
		42	11.557 03	0.086 53	175.950 5	15.224 5	208.397 8	446.257 6	42
		43	12.250 45	0.081 63	187.507 6	15.306 2	211.907 8	461.563 8	43
		44	12.985 48	0.077 01	199.758 0	15.383 2	215.296 2	476.947 0	44
		45	13.764 61	0.072 65	212.743 5	15.455 8	218.565 5	492.402 8	45
		46	14.590 49	0.068 54	226.508 1	15.524 4	221.718 2	507.927 2	46
		47	15.465 92	0.064 66	241.098 6	15.589 0	224.757 2	523.516 2	47
		48	16.393 87	0.061 00	256.564 5	15.650 0	227.685 1	539.166 2	48
		49	17.377 50	0.057 55	272.958 4	15.707 6	230.504 8	554.873 8	49
		50	18.420 15	0.054 29	290.335 9	15.761 9	233.219 2	570.635 7	50
		60	32.987 69	0.030 31	533.128 2	16.161 4	255.204 2	730.642 9	60
		70	59.075 93	0.016 93	967.932 2	16.384 5	269.711 7	893.590 9	70
		80	105.795 99	0.009 45	1 746.599 9	16.509 1	279.058 4	1 058.181 2	80
		90	189.464 51	0.005 28	3 141.075 2	16.578 7	284.973 3	1 223.688 3	90
		100	339.302 08	0.002 95	5 638.368 1	16.617 5	288.664 6	1 389.707 6	100

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[Total 25]

[GRAND TOTAL 100]

END OF EXAMINATION

