Actuarial Society of South Africa

EXAMINERS' REPORT

May 2020

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

In the paper there were several questions on areas of the syllabus that have not been tested recently. Candidates will do well to remember that the complete syllabus is examinable.

QUESTION 1

(i)

Any six of the following**:**

- Requires considerable investment of time and expertise.
- For a stochastic model, each run gives only estimates of outputs, so several runs required.
- Models are best used for studying input variations rather than for optimizing outputs.
- Models can look impressive and give a false sense of confidence.
- Models rely heavily on data quality. If data quality is poor, outputs will be flawed.
- Models cannot incorporate all future events, e.g. change in legislation may invalidate results.
- Outputs of the model may be difficult to interpret, e.g. may only be valid in relative terms.
- Users must understand the model and uses it can safely be put to – danger of using it like a black box from which it is assumed that all results are valid

(ii)

Experts on the real-world system are asked to compare several sets of real-world and model data, without being told which are which.
If they can distinguish between these datasets, their methods can be used to improve the existing model.

Many marginal candidates missed out on easy marks which could have made a difference in their result.

QUESTION 2

(i)

The issuer of the loan has a

- large –‘ve cashflow (outflow)
- initially at t = 0
- timing and amount certain

In return, the issuer

- receives regular +’ve cashflow (of interest payments)
- relatively small amounts compared to loan amount (initial negative cashflow)
- the timing of which are certain and already known
- amounts may be fixed (known in advance) or variable (unknown in advance) depending on whether interest rate is fixed or variable.

Finally,

- large +’ve cashflow (inflow)
- amount is certain = the initial amount of the loan
- timing known or unknown depending on whether early repayment is allowed.

(ii)

28-Jun-16	1-Jul-16	1-Jul-17	1-Jul-18	1-Jul-19	1-Aug-19
-55,000	0	6,050	6,655	7,320.50	82,500

Both (i) and (ii) were question types which candidates may not have seen before. The instructions for parts (i) and (ii) were clearly set out. Many candidates paid little attention to these details.

Common errors in part (i) included producing answers from the borrower’s perspective, taking account of default risk and leaving out information specifically asked for.

A common error in part (ii) was including the dividend on 1 July 2016 for the ex-dividend share.

QUESTION 3

The principle of consistency implies that the proceeds of an investment in a consistent market will not depend on the reinvestment actions of the investor.

Thus, $A(t_0, t_n) = A(t_0, t_1)A(t_1, t_2) \ldots A(t_2, t_n)$,

where $A(t_k, t_n)$ is the accumulated value at time t_n of R1 invested at time t_k.

Candidates had to describe both the financial and mathematical aspects of this principle.

Many candidates had some form of mathematical explanation but very few candidates discussed both aspects.

QUESTION 4

The equation of value after 8 years is

$$20{,}000\left(1+\frac{j}{4}\right)^{16} \times \left(1+\frac{3j}{12}\right)^{48} = 50{,}000$$

$$\left(1+\frac{j}{4}\right)^{64} = 2.5 \quad \Rightarrow \left(1+\frac{j}{4}\right)^{16} = (2.5)^{\frac{1}{4}} = 1.25743$$

$$20{,}000\left(1+\frac{j}{4}\right)^{16} = 20{,}000 \times 1.25743 \quad = 25{,}148.70$$

Well done by a majority of candidates.

A common error was solving j through linear interpolation. Linear interpolation leads to an approximate solution and should only be used when there is no algebraic solution to the equation of value.

QUESTION 5

(i)

2015:

Money Coupon: $10\left(\frac{110}{102}\right) = 10.7843$

Real Coupon: $10\left(\frac{110}{102}\right)\left(\frac{104}{112}\right) = 10.01401$

(ii)

There is a gradual decrease in the inflation rate from year to year (even though the index is increasing).

Thus, due to the lag in calculation, the money coupon is adjusted for a higher inflation rate than is actually experienced over the period.

This results in real coupons that are slightly higher than the 10% that the indexation is attempting to lock in.

Due to the higher real coupons, and because the purchase and redemption payments are at par, the realised real yield on this bond will be slightly greater than 10%.

Part (i) was well done by most candidates.

Part (ii) was one of the worst answered question in the paper. Many candidates failed to realise that the answer to part (ii) is provided by the numerical values of part (i).

QUESTION 6

(i)

$$(1 + y_3)^3 = e^{3Y_3} \Rightarrow Y_3 = \ln(1 + y_3) = 0.0478$$

(ii)

$$F_t = -\frac{d}{dt} \ln P_t$$

$$= -\frac{d}{dt} \ln(1 + y_t)^{-t}$$

$$= \frac{d}{dt} t \ln\left(1.04 + \frac{t^2}{1{,}000}\right)$$

Therefore

$$F_3 = \ln\left(1.04 + \frac{t^2}{1{,}000}\right) + \left.\frac{\frac{2}{1{,}000} t^2}{1.04 + \frac{t^2}{1{,}000}}\right|_{t=3}$$

$$= 0.06499$$

(iii)

$$1 = c_3((1 + y_1)^{-1} + (1 + y_2)^{-2} + (1 + y_3)^{-3}) + 1(1 + y_3)^{-3}$$

with
$y_1 = 0.041; y_2 = 0.044$ and $y_3 = 0.049$

thus
$1 = c_3(0.9606 + 0.9175 + 0.8663) + 0.8663$

$c_3 = 0.048713$

Part (ii) was the worst answered question on the paper with many candidates not producing an attempt. Candidates that attempted this part of the question often confused "instantaneous" with "continuous".

Part (iii) was well done by most candidates

QUESTION 7

(i)

$PV_L = 10{,}000 + 20{,}000v^{0.75} + 25{,}000v^1 + 40{,}000v^2 + 40{,}000v^4$

$= 113{,}688.2115$

(ii)

DMT_L(numerator)
$= 0 + (0.75)20{,}000v^{0.75} + (1)(25{,}000)v^1 + 40{,}000(2)v^2 + 40{,}000(4)v^4$

$= 217{,}679.3807$

$DMT_L = \frac{217{,}679.3807}{113688.2115} = 1.914705$ years

(iii)

X and Y is nominal value invested in A and B, respectively

$PV_{Aa} = X(5a_{\overline{4}|} + 100v^4) = 87.0411X$

$PV_{Ab} = Y(100v) = 91.7431Y$

$PV_L = 113{,}688.2115 = X(87.0411) + Y(91.7431) = PV_A$

$X = \dfrac{113{,}688.21 - Y(91.74)}{87.04}$ … … (1)

Numerator of DMT_{Aa}
$= X(5(Ia)_{\overline{4}|} + 400v^4) = 322.1253X$

Numerator of DMT_{Ab}
$= Y(100v) = 91.7431Y$

$DMT_L(\text{num}) = 217{,}679.3807 = 322.1253X + 91.7431Y$

$= DMT_A(\text{num})$ … … (2)

Sub (1) into (2)

$217{,}679.3807 = \left(\frac{113{,}688.21 - Y(91.74)}{87.04}\right)(322.1253) + Y(91.7431)$

Solve for Y:
$Y = 819.52$

Solve for X:
$X = 442.36$

Thus, amount invested in A: $442.36(87.0411) = R38{,}503.11$
And amount invested in B is: $819.52(91.7431) = R75{,}185.0966$

(iv)

$$c(0.09)_L(\text{numerator}) = 0 + (0.75)(1.75)20{,}000v^{2.75} + (1)(2)(25{,}000)v^3 + 40{,}000(2)(3)v^4 + 40{,}000(4)(5)v^6$$
$$= 706{,}356.34$$

$$c(0.09)_L = \frac{706{,}356.34}{113{,}688.2115} = 6.2131$$

$$c(0.09)_A(\text{numerator}) = X(5(1(2)v^3 + 2(3)v^4 + 3(4)v^5) + 4(5)(105)v^6) + Y(100(1)(2)v^3)$$
$$= X(67.9704 + 1252.1613) + Y(154.4366)$$
$$= 710{,}531.1$$

$$c(0.09)_A = \frac{710{,}531.1}{113{,}688.2115} = 6.2498$$

$$c(0.09)_A > c(0.09)_L$$

∴ The portfolio is immunised against small changes in the interest rate.

For part (ii) examiners will expect the time unit to be added to the discounted mean term, in the future.

Part (iii) was well done by well-prepared candidates with a good exam strategy. The common error was providing the nominal amounts of the assets, instead of the amounts that should be invested in each asset.

Part (iv) was poorly done as many candidates made comments about the spread of assets and liabilities around discounted mean term. Candidates should always calculate the convexity (or at least the numerator of the convexity), unless specifically asked not to.

QUESTION 8

Capital gains test :

$i = 0.11 \Rightarrow i^{(4)} = 0.105733$

$\frac{D}{R} \times (1 - t_1) = \frac{10}{110} \times (1 - 0.35) = 0.059091$

$0.059091 < i^{(4)} = 0.105733$

There is a capital gain for the investor.

Investor should choose worst case scenario for himself ⇒ price at the later redemption date to lock in the minimum yield required.

Thus, price at 10 years.

$$P = 10a_{\overline{10}|}^{(4)} - 3.5a_{\overline{10}|}v^{\frac{1}{12}} + 110v^{10} - 0.2(110 - P)v^{10+\frac{1}{12}}$$

$$P = \frac{10a_{\overline{10}|}^{(4)} - 3.5a_{\overline{10}|}v^{\frac{1}{12}} + 110v^{10} - 22v^{10+\frac{1}{12}}}{1 - 0.2v^{10+\frac{1}{12}}} = R77.2913357$$

This question was answered well by most candidates.

Common errors were mistakes made in the equation of value with regard to the income and capital gains tax.

QUESTION 9

(i)

$\text{flat rate} = 0.05 = \frac{60 * \text{PMT} - 1{,}000{,}000}{1{,}000{,}000 \times 5}$

$\text{PMT} = R20{,}833.35$

(ii)

$1{,}000{,}000 = 20{,}800 a_{\overline{60}|@\frac{i^{(12)}}{12}}$

Solve for i by using interpolation.

We know that the Annual Percentage Rate of charge (APR) is approximately twice the flat rate. Start with $i = 0.1$.

$$i = 0.1 \Rightarrow \frac{i^{(12)}}{12} = (1.1)^{\frac{1}{12}} - 1 = 0.007974$$
$$\Rightarrow 20{,}800 \cdot a_{\overline{60}|@\frac{i^{(12)}}{12}} = 20{,}800 \cdot (47.538687) = 988{,}804.69 = P_1$$

This is nearly equal to 1,000,000, so we need to select a slightly lower interest rate. Try $i = 0.09$.

$$\Rightarrow 20{,}800 \cdot a_{\overline{60}|@\frac{i^{(12)}}{12}} = 20{,}800 \cdot (48.571680) = 1{,}010{,}290.94 = P_2$$

Then by interpolation:

$$\begin{aligned} i &= i_1 + \left(\frac{P - P_1}{P_1 - P_2}\right) \cdot (i_1 - i_2) \\ &= 0.10 + \left(\frac{1{,}000{,}000 - 988{,}804.69}{988{,}804.69 - 1{,}010{,}290.94}\right) \cdot (0.1 - 0.09) \\ &= 0.0948 \end{aligned}$$

The APR is 9.5% (rounded to 1 DP)

(iii)

Using the prospective method, find the number of payments that would cover 500,000.

$$i = 9.4738 \quad \Rightarrow \frac{i^{(12)}}{12} = 0.75714416\%$$

$$\text{OR } i = 9.5\% \Rightarrow \frac{i^{(12)}}{12} = 0.759153429\%$$

$$20{,}800 a_{\overline{n}|@\frac{i^{(12)}}{12}} \leq 500{,}000$$

$n = 26.6342459$ (if use $i = 9.5\% \Rightarrow n = 26.642115037$)

$n = 26$

Subtract this from 60 to get the number of payments that would result in a capital outstanding of less than 500,000.

The number of payments to pay more than half the initial capital is 60-26 = 34

(iv)

The number of payments to repay the first half of the loan is more than the number of payments to repay the second half.

This is because the interest portion of the repayment is very large in the beginning (and loan outstanding still large)

and thus capital is repaid more slowly than in later years when the interest portion of the repayment is lower (because loan outstanding is lower).

Part (i) was very poorly done with most candidates showing very little understanding of flat rates of interest.

Part (ii) was well done by most candidates. A common error in part (ii) was not rounding the APR correctly.

Part (iii) and (iv) were done poorly which was surprising as it is based on basic loan theory. A common error in part (iii) was taking the 26 payments as the final solution.

QUESTION 10

(i)

$i = 6.09\%$

PV of outflows:

$2\text{m} + 1.5\text{m}\, v^{\frac{7}{12}}$
$= R3{,}449{,}153.86$

Running cost:

$i^{(4)} = 0.059556626$

$150{,}000 a^{(4)}_{\overline{2}|i^{(4)}}[1 + (1.04)^2 v^2_{6.09\%} + (1.04)^4 v^4_{6.09\%} + \cdots + (1.04)^{10} v^{10}_{6.09\%}]$

$\left[\frac{1.04}{1.0609}\right]^2 = \frac{1}{1+j} \Rightarrow j = 0.040596163$

$= 150{,}000 \times a^{(4)}_{\overline{2}|i^{(4)}}[1 + v^1_{j\%} + v^2_{j\%} + \cdots + v^5_{j\%}] = 150{,}000 \times a^{(4)}_{\overline{2}|i^{(4)}} \times \ddot{a}_{\overline{6}|j\%}$

$= 1{,}529{,}095.56$

PV Inflows:

$v^2_{6.09\%} \times [200{,}000\bar{a}_{\overline{10}|\delta=5.91176\%} + 100{,}000(I\bar{a})_{\overline{10}|\delta=5.91176\%}] + 3{,}000{,}000 \times v^{12}_{6.09\%}$

$= R6{,}181{,}457.38$

NPV:

$R6{,}181{,}457.38 - R3{,}449{,}154.86 - \text{R}1{,}529{,}095.56$

$= R1{,}203{,}207.96$

(ii)

In 1st t bullet the IRR would fall and in 2nd bullet it would increase.

However, the relative decrease in IRR would be a lot more than the relative increase in IRR.

This is because the R1 million outflow occurs early on in the life of the project as thus is a bigger cashflow in PV terms.

In part (i) easy marks were on offer for candidates with a good exam strategy. Such a strategy would include:

- *Separating your solution into different sections (as in marking schedule)*
- *Calculate and write down intermediate annuity values*
- *Writing down all formulas i.e. NPV = PV income – PV outgo.*

Common errors in part (i) was using $i^{(2)} = 6\%$ *as* $i = 6\%$ *and incorrectly applying the growth to the running cost.*

In the calculation of the running cost, alternatives to the new annuity ($\ddot{a}_{\overline{6}|j\%}$ *) method were*

- *Geometric series or*
- *Calculating value directly by adding the 6 discounted values.*

