Actuarial Society of South Africa

EXAMINATION

18 May 2020

Subject A211 — Financial Mathematics

Time allowed:

Two hours and fifteen minutes – exam time

20 minutes (at the end of the exam) – scan and upload time

INSTRUCTIONS TO THE CANDIDATE

1. *Ensure that you have logged into your ProctorU account before you attempt the exam. Your PC must be placed so that your writing area is visible to proctor.*
2. *Ensure that you have your candidate number handy to input as part of the exam. Write your candidate number at the top of each page. (DO NOT WRITE YOUR NAME.)*
3. *Your cell phone that will be used to scan your final answer scrip must be switched **OFF** during the 2hours and 15 minutes exam time. Place your cell phone at the top of your exam pad / writing pages in view of the proctor.*
4. *Questions are only available in this Moodle learning platform and may not be printed.*
5. *You are required to write your answers on a clean A4 exam pad. Write only on 1 side of the paper.*
6. *Attempt all questions, beginning your answer to each question on a new page and numbering your answers clearly.*
7. *Write in black or dark blue pen.*
8. *You should show calculations where this is appropriate.*
9. *You MAY NOT use any computer program (e.g. email, MS Word or Excel), files or open any other browsers during the examination time.*

10. *You have 15 minutes at the start of the exam to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have two hours to complete the paper.*
11. *Mark allocations are shown in brackets.*
12. *You may use additional scrap paper to make notes where this is appropriate. This paper MUST NOT BE SCANNED as part of your answer script.*
13. *Assume that months are all of equal length, unless otherwise stated.*
14. *At the end of the 2 hours and 15 minutes exam time, the exam part will close, and you will have **20 minutes to scan and upload your answer script** into the Moodle learning platform.*
15. *Access to your PC will be opened-up after the exam time so you can access your scanned file. You may now also switch on your cell phone to scan.*
16. *Scan ALL your answer pages to .pdf so that your candidate number at the top of the page is clear.*
17. *Save your .pdf scanned file using your candidate number as file name. (DO NOT USE YOUR NAME AS FILE NAME)*
18. *Transfer your .pdf script to your PC and click on the UPLOAD ANSWERS link below the exam paper link.*
19. *Upload your answer file into the Moodle Learning Platform and ensure you click on FINISH below the upload box and again on FINISH all and SUBMIT, **before** the 20 minute upload time is up. (If the status on the summary page indicates "Answer saved" your file was uploaded. You can click on Review attempt to see the file you've uploaded.)*

Note: The Actuarial Society of South Africa will not be held responsible for loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

i. List six drawbacks of models that must be understood when interpreting the output from a model. [4]

ii. When analysing the output from a model against results of a real-world system, a 'Turing' test may be used.

Briefly describe a Turing Test and how it can be used to improve a model. [3]

[Total 7]

QUESTION 2

i. Describe the cashflows involved for the issuer of an *interest-only* loan, referring to certainty of payment, timing of payment, sign of the payments (+'ve or –'ve) and relative magnitude of cashflows.

Ignore the possibility of default in your answer. [4]

ii. Company X is an aerospace company that pays annual dividends on ordinary shares. Dividends are payable each year on 1 July if approved by the Board of Company X.

On 28 June 2016, an investor purchased 10,000 shares, ex-dividend, in Company X at a price of R5.50 each. A dividend of 55c per share was declared on 1 July 2016. Due to the development of a new cost-effective alloy, Company X generated greater profits and subsequent dividend declarations increased by 10% every year. On 1 August 2019, the investor sold his holding of Company X shares for a 50% capital gain on the initial price paid.

Show the investor's cashflows and dates thereof, in a table. [3]

[Total 7]

QUESTION 3

Explain the principle of consistency in the context of time value of money.

Define any notation used. [Total 4]

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QUESTION 4

An individual deposits R20,000 into a bank account. He earns interest at $j\%$ per annum convertible quarterly for the first four years and at $3\times j\%$ per annum convertible monthly, thereafter. He has accumulated R50,000 in his account after eight years.

Calculate how much the individual has in the bank account after four years.

[Total 4]

QUESTION 5

The table below sets out the annual inflation index of a certain country, which is published on 1 January from 2010 to 2021.

2010	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**
102	104	106	108	110	112	114	116	118	120	122	124

The government issued a 10-year index-linked bond on 1 January 2011, with a 1-year time lag. The bond pays nominal coupons of 10% per annum in arrears and is redeemable at par.

i. Calculate the coupon payable, on a R100 nominal, on 1 January 2015 in money and real terms. [2]

Assume that an investor purchased the bond at issue for a price equal to par and that the investor will hold the bond until maturity.

ii. Comment on the effect that the time-lag in the coupon calculation will have on the real yield earned by this investor. [4]

[Total 6]

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QUESTION 6

Assume that spot interest rates are given by the following function, where *t* is measured in years:

$$y_t = 0.04 + \frac{t^2}{1{,}000}$$

i. Calculate the continuous-time 3-year spot rate. [1]

ii. Calculate the instantaneous forward rate, F_3. [4]

iii. Calculate the 3-year par yield. [4]

[Total 9]

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Compound Interest

n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
1	1.090 00	0.917 43	1.000 0	0.917 4	0.917 4	0.917 4	1
2	1.188 10	0.841 68	2.090 0	1.759 1	2.600 8	2.676 5	2
3	1.295 03	0.772 18	3.278 1	2.531 3	4.917 3	5.207 8	3
4	1.411 58	0.708 43	4.573 1	3.239 7	7.751 0	8.447 6	4
5	1.538 62	0.649 93	5.984 7	3.889 7	11.000 7	12.337 2	5
6	1.677 10	0.596 27	7.523 3	4.485 9	14.578 3	16.823 1	6
7	1.828 04	0.547 03	9.200 4	5.033 0	18.407 5	21.856 1	7
8	1.992 56	0.501 87	11.028 5	5.534 8	22.422 5	27.390 9	8
9	2.171 89	0.460 43	13.021 0	5.995 2	26.566 3	33.386 1	9
10	2.367 36	0.422 41	15.192 9	6.417 7	30.790 4	39.803 8	10
11	2.580 43	0.387 53	17.560 3	6.805 2	35.053 3	46.609 0	11
12	2.812 66	0.355 53	20.140 7	7.160 7	39.319 7	53.769 7	12
13	3.065 80	0.326 18	22.953 4	7.486 9	43.560 0	61.256 6	13
14	3.341 73	0.299 25	26.019 2	7.786 2	47.749 5	69.042 8	14
15	3.642 48	0.274 54	29.360 9	8.060 7	51.867 6	77.103 5	15
16	3.970 31	0.251 87	33.003 4	8.312 6	55.897 5	85.416 0	16
17	4.327 63	0.231 07	36.973 7	8.543 6	59.825 7	93.959 7	17
18	4.717 12	0.211 99	41.301 3	8.755 6	63.641 6	102.715 3	18
19	5.141 66	0.194 49	46.018 5	8.950 1	67.336 9	111.665 4	19
20	5.604 41	0.178 43	51.160 1	9.128 5	70.905 5	120.793 9	20
21	6.108 81	0.163 70	56.764 5	9.292 2	74.343 2	130.086 2	21
22	6.658 60	0.150 18	62.873 3	9.442 4	77.647 2	139.528 6	22
23	7.257 87	0.137 78	69.531 9	9.580 2	80.816 2	149.108 8	23
24	7.911 08	0.126 40	76.789 8	9.706 6	83.849 9	158.815 4	24
25	8.623 08	0.115 97	84.700 9	9.822 6	86.749 1	168.638 0	25
26	9.399 16	0.106 39	93.324 0	9.929 0	89.515 3	178.567 0	26
27	10.245 08	0.097 61	102.723 1	10.026 6	92.150 7	188.593 6	27
28	11.167 14	0.089 55	112.968 2	10.116 1	94.658 0	198.709 7	28
29	12.172 18	0.082 15	124.135 4	10.198 3	97.040 5	208.908 0	29
30	13.267 68	0.075 37	136.307 5	10.273 7	99.301 7	219.181 6	30
31	14.461 77	0.069 15	149.575 2	10.342 8	101.445 2	229.524 4	31
32	15.763 33	0.063 44	164.037 0	10.406 2	103.475 3	239.930 7	32
33	17.182 03	0.058 20	179.800 3	10.464 4	105.395 9	250.395 1	33
34	18.728 41	0.053 39	196.982 3	10.517 8	107.211 3	260.912 9	34
35	20.413 97	0.048 99	215.710 8	10.566 8	108.925 8	271.479 8	35
36	22.251 23	0.044 94	236.124 7	10.611 8	110.543 7	282.091 5	36
37	24.253 84	0.041 23	258.375 9	10.653 0	112.069 2	292.744 5	37
38	26.436 68	0.037 83	282.629 8	10.690 8	113.506 6	303.435 3	38
39	28.815 98	0.034 70	309.066 5	10.725 5	114.860 0	314.160 9	39
40	31.409 42	0.031 84	337.882 4	10.757 4	116.133 5	324.918 2	40
41	34.236 27	0.029 21	369.291 9	10.786 6	117.331 1	335.704 8	41
42	37.317 53	0.026 80	403.528 1	10.813 4	118.456 6	346.518 2	42
43	40.676 11	0.024 58	440.845 7	10.838 0	119.513 7	357.356 1	43
44	44.336 96	0.022 55	481.521 8	10.860 5	120.506 1	368.216 6	44
45	48.327 29	0.020 69	525.858 7	10.881 2	121.437 3	379.097 8	45
46	52.676 74	0.018 98	574.186 0	10.900 2	122.310 5	389.998 0	46
47	57.417 65	0.017 42	626.862 8	10.917 6	123.129 1	400.915 6	47
48	62.585 24	0.015 98	684.280 4	10.933 6	123.896 0	411.849 2	48
49	68.217 91	0.014 66	746.865 6	10.948 2	124.614 3	422.797 4	49
50	74.357 52	0.013 45	815.083 6	10.961 7	125.286 7	433.759 1	50
60	176.031 29	0.005 68	1 944.792 1	11.048 0	130.016 2	543.911 2	60
70	416.730 09	0.002 40	4 619.223 2	11.084 4	132.378 6	654.617 2	70
80	986.551 67	0.001 01	10 950.574 1	11.099 8	133.530 5	765.557 2	80
90	2 335.526 58	0.000 43	25 939.184 2	11.106 4	134.082 1	876.596 1	90
100	5 529.040 79	0.000 18	61 422.675 5	11.109 1	134.342 6	987.676 6	100

9%

i	0.090 000
$i^{(2)}$	0.088 061
$i^{(4)}$	0.087 113
$i^{(12)}$	0.086 488
δ	0.086 178
$(1+i)^{1/2}$	1.044 031
$(1+i)^{1/4}$	1.021 778
$(1+i)^{1/12}$	1.007 207
v	0.917 431
$v^{1/2}$	0.957 826
$v^{1/4}$	0.978 686
$v^{1/12}$	0.992 844
d	0.082 569
$d^{(2)}$	0.084 347
$d^{(4)}$	0.085 256
$d^{(12)}$	0.085 869
$i/i^{(2)}$	1.022 015
$i/i^{(4)}$	1.033 144
$i/i^{(12)}$	1.040 608
i/δ	1.044 354
$i/d^{(2)}$	1.067 015
$i/d^{(4)}$	1.055 644
$i/d^{(12)}$	1.048 108

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QUESTION 7

A company has the following liability payments in the future:

- R10,000 immediately
- R20,000 after nine months
- R25,000 after one year
- R40,000 after two years
- R40,000 after four years

i. Calculate the present value of the liability payments using an effective interest rate of 9% per annum. [1]

ii. Calculate the discounted mean term of the liability payments using an effective interest rate of 9% per annum. [3]

The company wishes to immunise its asset-liability portfolio at an effective interest rate of 9% per annum by investing in two assets:

Asset A: A fixed-interest security that pays annual coupons of 5% in arrears and is redeemable at par in four years' time.

Asset B: A 1-year zero-coupon bond.

iii. Calculate the amounts that should be invested in asset A and B to ensure immunisation of the asset-liability portfolio. (You must assume that assets can be bought and sold in fractions of a unit.) [10]

iv. Confirm whether the asset-liability portfolio is immunised when the amounts, as calculated in (iii), are invested in assets A and B. [4]

[Total 18]

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QUESTION 8

A company issued a bond that is redeemable at 110% of par. The bond pays coupons of 10% per annum, quarterly in arrears. The company may redeem the bond at their own discretion on any coupon date between five and ten years from the date of issue. An investor subject to 35% income tax and 20% capital gains tax purchased the bond at issue on 1 April 2010.

Calculate the price that the investor would have agreed to pay for the bond if he required a minimum net yield of 11% per annum effective. The investor pays all taxes due on income and capital gains, over the previous 12 months, on 1 May each year.

[Total 15]

QUESTION 9

Consider a R1 million, 5-year loan that is issued at a flat rate of interest of 5% per annum. Repayments are made monthly in arrears.

i. Calculate the monthly repayment on the loan. [1]

Assume the monthly repayment on the loan is R20,800.

ii. Calculate, through linear interpolation, the annual percentage rate of return applicable on the loan. [3]

iii. Calculate the number of payments required, after which the capital outstanding will be less than half of the original loan. [4]

Consider the number of payments required to repay half of the capital, as calculated in (iii).

iv. Explain why this value is not equal to half the total number of repayments required. [3]

[Total 11]

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QUESTION 10

A small hydroponics company sets up a new venture on 1 January 2020. The initial investment on 1 January 2020 is R2 million with a further R1.5 million required on 1 August 2020. The running costs for the venture start immediately in 2020 at R150,000 per annum payable quarterly in arrears. The running costs will increase at an effective rate of 4% per annum compounded. The increases will occur every second year with the first increase taking place at the start of the third year.

It is expected that on 1 January 2022, income will begin at the rate of R300,000 per annum and that the rate will increase by R100,000 per annum on 1 January of each subsequent year. It is assumed that the income will be received continuously throughout the project.

The company expects to sell the business on 1 January 2032 for R3 million.

i. Calculate the net present value of the venture on 1 January 2020 at a rate of interest of 6% per annum, convertible half-yearly. [16]

The original effective internal rate of return on the project was 9.55% per annum. However, it is now assumed that the following two additional cashflows will be incurred:

- the project will require a further investment of R1 million on 1 January 2021 and,
- due to increased popularity of hydroponic farming techniques, the income in the final year of the project will increase by R1 million.

ii. Discuss, without further calculations, the combined effect of the additional cashflows on the effective internal rate of return, both in terms of the magnitude and direction of change.

[3]

[Total 19]

[GRAND TOTAL: 100]

END OF EXAMINATION

