EXAMINATION

11 April 2019

Subject A211 — Financial Mathematics

Time allowed: Two hours and 15 minutes reading time

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 2 hours to complete the paper.*

4. *The use of calculators is not permitted during the reading time.*

5. *Mark allocations are shown in brackets.*

6. *Attempt all questions, beginning your answer to each question on a new page.*

7. *You should show calculations where this is appropriate.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

QUESTION 1

i.   Define $\bar{a}_{\overline{n}|}$ algebraically.   [1]

ii. Hence, prove that $\bar{a}_{\overline{n}|} = \dfrac{1-v^n}{\delta}$.   [2]

                                                                                                         [Total 3]

QUESTION 2

A researcher is interested in the dietary habits of employees at a big multinational company with thousands of employees. The researcher wishes to know if there is a difference in the dietary habits of employees at different levels of seniority and in different locations to make recommendations to the company on their wellness program. For this purpose, a survey is compiled and sent to 20% of the company's workforce.

i.   Explain the difference between descriptive analysis and inferential analysis.
                                                                                                         [3]
ii. Explain which of the forms of data analysis mentioned in (i), the researcher is using.
                                                                                                         [2]
iii. Recommend which data collection (sampling) method the researcher should use to ensure that the data appropriately reflects all the employees at the company.
                                                                                                         [3]

                                                                                                         [Total 8]

QUESTION 3

List 10 factors to consider when deciding on the suitability of a model for a particular exercise.

                                                                                                         [Total 6]

QUESTION 4

An individual invests R10,000 now and RX in one year's time. The investment earns interest at an effective interest rate of 10% per annum for two years. The individual's goal is to have R13,000 in real terms after two years. Inflation is assumed to be at an effective rate of 0.45% per month for the two-year period.

Determine the value of X.

                                                                                                         [Total 5]

PLEASE TURN OVER

QUESTION 5

Consider the following accumulation factors where $A(t_1, t_2)$ is the accumulation at time t_2 of an investment of 1 made at time t_1.

$$A(0,3) = 1.45$$
$$A(3,7) = 1.7$$
$$A(5,7) = 1.3$$

i. Calculate the constant nominal interest rate per annum, compounded quarterly, applicable between time 0 and 3.

[1]

ii. Calculate the constant monthly effective discount rate earned, over a five-year period, on an investment of R1 made at time 0.

[3]

[Total 4]

QUESTION 6

The force of interest is given by:

$$\delta(t) = \begin{cases} 0.09 + 0.0006t^2 & 0 \le t < 9 \\ 0.1836 & 9 \le t < 15 \\ 0.1086 - 0.005t & 15 \le t < 20 \\ 0.1086 & 20 \le t \end{cases}$$

where t is measured in years.

Calculate the accumulated value at time 17 of a continuous payment stream with rate of payment $\rho(t) = e^{0.08 + 0.0002t^3}$, payable for seven years and starting at time 2.

[Total 10]

QUESTION 7

A loan is repayable over 20 years by monthly instalments in arrears. Interest is charged at an effective rate of 5% per annum for the first 10 years, increasing to 8% per annum effective for the remaining term.

The monthly instalments start at R9,000 per month in the first year and increase to R10,000 per month in the second year, R11,000 per month in the third year, and so on.

Calculate the original amount of the loan.

[Total 10]

PLEASE TURN OVER

QUESTION 8

The following yield curve is assumed for expected one-year risk-free forward rates:
- 11% at the moment,
- 9% in one year's time,
- 7% in two years' time,
- 6% in three years' time and
- it is then expected to remain 6% indefinitely after that.

i. Bond yields over all terms to maturity are assumed only to reflect expectations of future interest rates.

   Calculate the gross redemption yields from one-year, three-year and five-year zero coupon bonds.
   [3]

ii. Calculate $f_{2,3}$.
   [3]

iii. Describe the liquidity preference theory.
   [2]

   [Total 8]

QUESTION 9

An insurance company has liabilities of R100,000 due in four years' time and R200,000 due in 14 years' time.

The company owns assets consisting of five coupon bearing bonds of R10,000 nominal each and one zero coupon bond paying RX in n years' time. The coupon bearing bonds pay 4% coupons annually in arrear for five years and are redeemable at par.

i. The current effective interest rate is 6% per annum and an attempt has been made to immunise the portfolio against small movements in interest rates.

   Determine X and n.
   [8]

ii. Without doing further calculations, state whether Redington's third condition for immunisation holds, giving a reason for your answer.
   [2]

   [Total 10]

PLEASE TURN OVER

QUESTION 10

A fixed-interest bond bears coupons of 8% per annum payable half-yearly and is redeemable at par on any 1 April between 1 April 2022 and 1 April 2028, inclusive, at the option of the borrower. Coupons are paid on 1 April and 1 October of each year.

On 1 July 2016 Investor A purchased R100,000 nominal of the bond. Investor A is liable to pay tax at a rate of 25% on the coupon payments and 30% on capital gains.

i.   Calculate the price at which the bond was bought by Investor A, if Investor A wanted to earn a minimum net yield of 7% per annum effective on the bond.

[7]

On 1 April 2018, directly after the coupon payment then due, Investor A sold the holding to Investor B who is also liable to pay tax at a rate of 25% on the coupon payments but is not liable to pay capital gains tax. Investor B bought the holding at a price that would earn him a minimum net yield of 5% per annum effective.

ii.   Calculate the price at which the bond was sold by Investor A on 1 April 2018.

[4]

iii. Without doing further calculations, explain whether Investor A made a sound financial decision by selling the bond on 1 April 2018 for the price calculated in (ii).

[3]

[Total 14]

PLEASE TURN OVER

QUESTION 11

BLD, a property development and management company, will purchase and renovate a block of occupied offices. The purchase price of the property is R35 million and it will be sold for R70 million after 10 years.

Currently, the rental income from the non-renovated wings, received at the beginning of each month, is:
- South wing: R0.4 million
- North wing: R0.8 million

TYZ bank provides loan financing for the project as required. TYZ bank has offered BLD a special borrowing effective rate of 8% per annum and BLD will earn an effective interest rate of 4% per annum on any income after all debt has been repaid. All income from the project is immediately used to repay the outstanding loan amount.

BLD will undertake renovation in two phases:
- South phase, which involves the south wing of the building, and
- North phase, which involves the prestigious north wing of the building.

The second phase of renovation starts immediately after the first phase has been completed.

South phase
The South phase will take seven months to complete. The developer will incur renovation costs of R20 million at the beginning of the renovations, followed by R2.5 million at the end of each month during this renovation phase.

The north wing will be occupied and deliver income while the south wing is unoccupied and undergoing renovation. After the South phase is complete, the rent from the renovated offices of the South wing will be R1 million per month, received at the beginning of each month.

North phase
The North phase will take nine months to complete. The developer will incur costs of R60 million at the beginning of the renovations, followed by R6 million at the end of each month during this renovation phase.

The south wing will be occupied and deliver income while the north wing is unoccupied and undergoing renovation. After the North phase is complete, the rent from the renovated offices of the North wing will be R3 million per month, received at the beginning of each month.

The company has two options:

Option A – Start renovation with the South phase.
Option B – Start renovation with the North phase.

PLEASE TURN OVER

i. Determine the present value of the total renovation costs of Option A.

[4]

ii. Determine the present value of the rental income received during the renovation period of Option A.

[4]

iii. Determine the discounted payback period of Option A. The discounted payback period must be calculated to the nearest month.

[7]

iv. Assume the accumulated profit for Option A is exactly equal to zero at the discounted payback period calculated in (iii).

Calculate the accumulated profit for Option A.

[5]

The discounted payback period for Option B is longer than the discounted payback period for Option A.

v. Explain (with reasons) which option BLD should choose.

[2]

[Total 22]

[GRAND TOTAL 100]

END OF EXAMINATION