

THE CAPITAL-ASSET PRICING MODEL: THE CASE OF SOUTH AFRICA

By **TL Reddy and RJ Thomson**

ABSTRACT

This paper tests the empirical validity of the capital-asset pricing model (CAPM) for the South African share market. For the investigation, quarterly total returns from ten sectoral indices listed on the JSE Securities Exchange from 30 June 1995 to 30 June 2009, were used. As expressed in the securities market line, the CAPM suggests that higher risk, as measured by beta, is associated with higher expected returns. In addition, the theoretical underpinnings of the CAPM are that it explains expected excess return, and that the relationship between expected return and beta is linear. In this investigation the above-mentioned predictions of the CAPM were tested. Direct tests of the securities market line were made, using both prior betas and in-period betas. A nonparametric test was also made. Regression analysis was used to test hypotheses based on both individual sectoral indices and portfolios constructed from those indices according to their betas. These tests were made for individual years as well as for all periods combined. It was found that while, on the assumption that the residuals of the return-generating function are normally distributed, the CAPM could be rejected for certain periods, the use of the CAPM for long-term actuarial modelling in the South African market can be reasonably justified.

KEYWORDS

Capital-asset pricing model; beta; JSE Securities Exchange; excess return

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Announcements of the 'death' of beta seem premature.

Fischer Black

1. INTRODUCTION

1.1 The capital-asset pricing model (CAPM) has played an important role in modern finance and, in particular, in modern capital theory. The attraction of the CAPM is that it offers powerful and intuitively pleasing predictions about how to measure risk and the relationship between expected return and risk (Fama & French, 1992). According to the CAPM, investors aim to maximise the expected return of their portfolios for a given variance. The standard version of the CAPM, as developed by Sharpe (1964) and Lintner (1965), relates the expected rate of return of an individual security to a measure of its systematic risk. Systematic risk, as measured by beta, captures that aspect of investment risk which cannot be eliminated by diversification. One property of the CAPM is that investors are compensated with a higher expected return only by accepting systematic risk; in particular, it suggests that higher-beta securities are expected to give higher expected returns than lower-beta securities because they are more risky (Elton & Gruber, 1995).

1.2 The standard version of the CAPM expresses returns relative to risk-free rates. Black (1972) extended it to allow for the expression of returns relative to a zero-beta portfolio. As pointed out by Fama & French (1992):

When there is risk-free borrowing and lending, the expected return on assets that are uncorrelated with the market return [i.e. on the zero-beta portfolio], must equal the risk-free rate...

Under those circumstances the zero-beta version reduces to the standard CAPM.

1.3 The last half-century has witnessed the proliferation of empirical studies testing the validity of the CAPM. Reinganum (1981) remarked that the adequacy of the CAPM of Sharpe (op. cit.), Lintner (op. cit.) and Black (op. cit.) as empirical representations of capital-market equilibrium was even then being seriously challenged. While empirical work has been predominant in the literature on the CAPM over the past 30 years, the theory itself has been criticised and scholarly debate has questioned whether it is valid, whether it is useful and whether it is testable in the first place. As argued by Bailey, Alexander & Sharpe (1998) (cf. Michailidis et al., 2006), the empirical testing of CAPM has 'two broad purposes':

to test whether or not the theories should be rejected; and to provide information that can aid financial decisions.

These purposes relate explicitly to the questions whether it is valid and whether it is useful. The argument of Roll (1977) regarding its testability is discussed in ¶2.9.6 below. Although the CAPM is an important tool in finance, the empirical record of the model

is poor—poor enough to invalidate the way it is generally used. In particular, as Fama & French (2004) stated: “If betas do not suffice to explain expected returns, the market portfolio is not efficient, and the CAPM is dead in its tracks.”

1.4 The main question this study aimed to answer is: Is the CAPM valid in the South African market? In particular, does the CAPM explain expected excess return? (Here ‘excess return’ is the excess of the return over the risk-free rate; it is defined more formally in ¶2.2.1 below.) Is the relationship between return and beta linear? These questions are addressed with an actuarial audience in mind. In the first place, unlike most studies, this study uses yearly intervals. It is envisaged that the CAPM will be used, not for day-to-day trading, but for the pricing and risk-management of long-term financial instruments. This means that short-term effects, which may dominate models based on daily intervals, are relatively unimportant. It may be argued that nearly all the excess return experienced in a year comes in a few days in the year, and that this makes the short term important. However, if decisions (such as the revision of investment benchmarks in line with retirement-fund liabilities) are being made annually, it is only the aggregate contribution of short-term effects to the annual return that is of importance. In other words, it is the annual return itself that needs to be modelled.

1.5 Secondly, it is envisaged that the focus of actuarial activities such as the benchmarking of investment performance relative to liabilities will be on major sectors of the equity market, rather than on individual equities. For that reason, the analysis is undertaken with reference to the major sectoral indices on the JSE Securities Exchange rather than to individual equities. In particular, are those sectors with higher systematic risk (as measured by beta) associated with higher expected return?

1.6 The tests of the CAPM were first performed on the individual sectoral indices. However, as explained below, the use of individual indices to test the validity of the CAPM leads to certain problems in the estimation of betas. In order to improve the precision of the beta estimates, the individual sectoral indices were grouped into portfolios according to their beta, and the tests were repeated.

1.7 The CAPM is an *ex-ante* model: it is expressed in terms of investors’ subjective *ex-ante* expectations of beta and of expected returns. However, in order to test the theoretical underpinnings of the CAPM *ex-post* data are generally used. Therefore, the conclusions obtained will be in terms of an *ex-post* CAPM. A rejection of the *ex-post* CAPM is not necessarily a rejection of the *ex-ante* CAPM. This is discussed further below.

1.8 The rest of the paper is organised as follows. In Section 2, a discussion of the ‘rational-expectations hypothesis’ (REH) is provided and literature on the results of tests of the CAPM in other markets is reviewed. In Section 3, a description of the data used for this study is provided. The method used is described and explained in Section 4.

Section 5 presents preliminary observations of the data. In Section 6 the results of the empirical tests are presented and discussed. The results are summarised and concluding comments are made in Section 7.

2. LITERATURE REVIEW

2.1 THE RATIONAL-EXPECTATIONS HYPOTHESIS

2.1.1 To explain fairly simply how expectations work, Muth (1961) advanced the hypothesis that they are essentially the same as the predictions of the relevant economic theory. In particular, the hypothesis asserts that information is scarce, that the economy generally does not waste information, and that expectations depend specifically on the structure of the relevant system describing the economy. This hypothesis is referred to as the REH. As explained in Lovell (1986), the REH assumes that:

- the prediction error of the value of an economic variable (such as the price of an asset) predicted by an agent (such as an investor) is distributed independently of the information set available to the agent at the time of the prediction;
- it is also distributed independently of the predicted value; and
- its expected value is zero (i.e. expectations are unbiased).

2.1.2 The implication of the REH—particularly of the assumption that expectations are unbiased—is that estimates based on *ex-post* realisations of asset prices are unbiased estimates of *ex-ante* expectations.

2.1.3 Prescott (1977) has argued that the REH is not amenable to direct empirical tests. He stated:

... like utility, expectations are not observed, and surveys cannot be used to test the REH. One can only test if some theory, whether it incorporates rational expectations or not, for that matter, irrational expectations, is or is not consistent with observations.

This is a contestable statement. Like utility, agents' expectations can be elicited. However, just as the elicitation of utility functions is fraught with difficulties (Thomson, 2003), so is the elicitation of *ex-ante* expectations. It is for this reason that most tests of the CAPM rely on *ex-post* observations, implicitly assuming the REH. Thus, rejection of the null hypothesis that the CAPM and the REH apply does not necessarily necessitate rejection of the CAPM; it may be that the REH is false. Nevertheless, the tests can be constructed so as to reduce, as far as possible, the influence of the REH. This is discussed further below.

2.2 DIRECT TESTS OF THE SECURITIES MARKET LINE

2.2.1 It is intuitively appealing to test the CAPM using the securities market line:

$$E\{R_i\} = R_F + \beta_i [E\{R_M - R_F\}]; \quad (1)$$

where:

R_i , R_F and R_M are the returns on security i , on the risk-free asset F and on the market portfolio M respectively;

$$\beta = \frac{\sigma_{iM}}{\sigma_{MM}} ;$$

$$\sigma_{iM} = \text{cov}\{R_i, R_M\}; \text{ and}$$

$$\sigma_{MM} = \text{var}\{R_M\} .$$

For this purpose the return-generating process may be expressed as:

$$r_{it} = \beta_{it} r_{Mt} + \varepsilon_{it} ; \quad (2)$$

where:

$$r_{it} = R_{it} - R_{ft} \text{ is the 'excess return' on index } i; \quad (3)$$

$$\beta_{it} = \frac{\sigma_{iMt}}{\sigma_{MMt}} ; \quad (4)$$

$$\varepsilon_{it} \sim N(0, \sigma_{\varepsilon it}^2);$$

$$\sigma_{ijt} = \text{cov}(R_{it}, R_{jt});$$

$$\sigma_{\varepsilon it}^2 = \text{var}(\varepsilon_{it}); \text{ and}$$

$$\text{cov}(\varepsilon_{it}, \varepsilon_{iu}) = 0 \text{ for } t \neq u .$$

2.2.2 A problem with this approach, though, is that it implies that *ex-post* estimates not only of β_{it} (which are implicitly assumed to be more stable than *ex-post* estimates of the expected return on the market portfolio) but also of $E\{R_{Mt}\}$, are unbiased estimates of *ex-ante* expectations. Although the CAPM is an *ex-ante* model, *ex-ante* returns are unobservable and “large-scale systematic data on expectations do not exist” (Elton & Gruber, op. cit.: 341). According to Rosenberg & Guy (1976), we never observe the true beta; thus the underlying value of beta that generated the observed outcomes must be estimated. Not only can we not observe the true (*ex-ante*) betas, we also cannot observe the true expected returns. Brenner & Smidt (1977) stated that the ‘almost universal practice’ had been to regress the realised return of a security against the return on a market portfolio. As this is only an estimation of these parameters, their true values will remain unknown. Therefore, researchers rely on realised returns and almost all tests of the CAPM have used *ex-post* values for the variables. As pointed out by Galagedera (2007), “The empirical question arises: Do the past security returns conform to the CAPM?” Levy (1974) concluded that there was correlation between historical beta coefficients and subsequent security performance. Thus, historical betas are a proxy for future betas and it may be assumed for the purposes of the CAPM that they are stable and can be used in the CAPM to calculate expected returns. In this study, *ex-post* estimates not only of β_{it} were found to be more stable than *ex-post* estimates of the expected return on the market portfolio, relative in each case to their respective levels—cf. ¶¶5.1.4–5. Historical beta coefficients were therefore used.

2.2.3 Secondly, while the above approach permits the testing of the standard version of the CAPM in terms of equation (1)—in particular with reference to the return on the risk-free asset—it does not permit the testing of the zero-beta version. Another

problem is that it introduces the assumption that the error term is normally distributed, which is not a requirement of the CAPM.

2.3 TESTS OF RETURNS RELATIVE TO BETA

2.3.1 To reduce the confounding effect of the REH and to allow extension to the zero-beta version of the CAPM, a less explicitly expressed statement may be tested, viz. that:

$$E\{R_i\} = \gamma_0 + \gamma_1\beta_i \tag{5}$$

2.3.2 The null hypothesis for a given period t may be expressed in terms of the equation:

$$R_{it} = \gamma_{R0} + \gamma_{R1}\beta_{it} + \varepsilon_{it} \tag{6}$$

or:

$$r_{it} = \gamma_{r0} + \gamma_{r1}\beta_{it} + \varepsilon_{it} \tag{7}$$

2.3.3 The tests may involve regression analyses of one of equations (6) and (7) itself or tests of that equation against alternative hypotheses. Various alternative hypotheses may be used. A major advantage of such tests as against the direct tests considered in section 2.2 is that they do not use *ex-post* expected values of R_{Mt} . Also, the zero-beta asset is not necessarily the risk-free asset. In the standard form of the CAPM, equation (6) implies that:

$$\gamma_{R0} = R_{Ft} \text{ ; and}$$

$$\gamma_{R1} = E\{r_{Mt}\}$$

are constant. Similarly, equation (7) implies that:

$$\gamma_{r0} = 0 \text{ ; and}$$

$$\gamma_{r1} = E\{r_{Mt}\}.$$

2.3.4 In terms of the zero-beta version of the CAPM, it is not necessary to refer to the risk-free asset; instead the variance of the return on zero-beta portfolios may be determined and the zero-beta portfolio with the lowest variance may be used. It must be borne in mind, though, that, if the expected return on the zero-beta portfolio is less than the return on the riskless asset, then the latter should be used (at least for investors who cannot hold short positions in such a portfolio).

2.3.5 In the zero-beta form of the CAPM, equation (6) implies that γ_{R0} is the return on the zero-beta portfolio, and equation (7) implies that γ_{r0} is the expected excess return on that portfolio.

2.3.6 Using 100 equities on the New York Stock Exchange from 1931 to 1965, Black, Jensen & Scholes (1972) tested equation (6). They found that the relationship between expected return and beta was very close to linear and that portfolios with high betas had high average returns and portfolios with low betas had low average returns (Elton & Gruber, op. cit.). However, as pointed out in Black (1993), Black, Jensen & Scholes (op. cit.) also found that returns on low-beta stocks were better than those implied

by the CAPM while those on high-beta stocks were worse, giving a flatter securities market line than that implied by the CAPM. Similar findings were reported from tests performed by Friend & Blume (1973) and Fama & French (1992). This evidence is further confirmed in tests performed by Friend & Blume (1970) and Stambaugh (1982). The intercept in time-series regressions of excess asset returns on the excess market return was positive (Fama & French, 1992).

2.3.7 Reinganum (op. cit.) investigated empirically whether equities with different estimated betas have different average rates of return. Daily share returns of all companies that were traded on the New York Stock Exchange or the American Stock Exchange from July 1962 to December 1979 were used. A two-step strategy was employed for the investigation. First, in period A, individual-security betas were estimated and each security placed into one of ten portfolios based on the rank of its estimated beta. Then, in period B, the returns of the ten portfolios were calculated by applying an equal weighting to the returns of the component securities within each portfolio. A multivariate statistical procedure was invoked to test whether the ten portfolios had significantly different average returns. It was found that the average returns of high-beta securities were not reliably different from the average returns of low-beta securities.

2.3.8 Fama & Macbeth (1973) found a positive relationship between return and risk. The data used for this study were monthly percentage returns (including dividends and capital gains) for all equities traded on the New York Stock Exchange from January 1926 to June 1968. An extended model of returns was used to arrive at their conclusion. Using two to five years of prior monthly returns, they estimated the beta for every equity on the New York Stock Exchange (1928–2003), the American Stock Exchange (1963–2003) and the NASDAQ (1972–2003). Ten portfolios were derived from these estimated betas and their returns for the next twelve months were computed. The process was repeated for each year from 1928 to 2003. The results confirmed earlier evidence that the relationship between beta and average return for the ten portfolios is much flatter than that predicted by the CAPM, as the returns on low-beta portfolios were too high and the returns on high-beta portfolios were too low.

2.3.9 On the other hand, Sharpe & Cooper (1972) tested the CAPM to see if higher return has been associated with higher risk (as measured by beta), over longer periods of time. For each year from 1931 to 1967, beta was measured using 60 months of previous data and all equities on the New York Stock Exchange were divided into deciles based on their betas. Their work shows that there was a positive relationship between return and beta and that the relationship was both strong and linear (Elton & Gruber, op. cit.).

2.3.10 Fama & Macbeth (op. cit.) tested the relationship between average return and risk for equities on the New York Stock Exchange. The theoretical basis of the tests was the two-parameter (expected value and dispersion of return) portfolio model and models of market equilibrium derived from the two-parameter portfolio model. They tested the following three implications from equation (2):

- that the relationship between the expected return on a security and its risk in any efficient portfolio is linear;
- that β_{it} is a complete measure of the risk of security i in any efficient portfolio; and

- that, in a market of risk-averse investors, higher risk should be associated with higher expected return.

Their model of returns included a measure of the risk of security i that is not deterministically related to β_i . To test linearity they also included a term in β_i^2 . The summary results for the regression did not reject the condition that the relationship between expected return and β_i is linear. A similar method for testing the linear relationship between the expected return and beta of security i was used by Michailidis et al. (op. cit.).

2.4 OTHER EXPLANATORY VARIABLES

2.4.1 Many authors have investigated other possible variables than the CAPM's beta. Banz (1981) challenged the CAPM by demonstrating that firm size explains the variation in average returns on a particular collection of assets better than beta. That author concluded that, in fact, equities of small firms (with low values of equity) yielded higher average returns than those of large firms. Fundamental variables such as the earnings yield (Basu, 1977), the ratio of book value to market value (Rosenberg, Reid & Lanstein, 1985), macroeconomic variables and the price-earnings ratio (Basu, op.cit.) considerably explain cross-sectional variation in expected returns. Van Rensburg & Robertson (2003) stated that attempts to empirically verify the predictions of the CAPM had produced numerous inconsistencies with the theory. Most notable is the evidence that other variables such as book-to-market ratios, market capitalisation, price-earnings ratios and leverage are able to predict security returns beyond that explained by beta. The evidence presented in their study showed that small size earns a higher return on the JSE Securities Exchange but has a lower beta. It was further documented that portfolios containing shares with low price-earnings ratios earn higher average returns and also have lower betas. When two-way portfolios were created by sorting on both the size and the price-earnings attributes, the findings suggested that to a large extent the small size and low price-earnings effects operate independently of each other. The findings of this study show that variables other than beta may help to explain expected returns in the South African context. This is further discussed in ¶2.9.5 below.

2.4.2 In an efficient market one would expect that the price of a share would not have any effect on the variability of that price. In one study, Clendenin (1951) found that, other factors being constant, low-price shares do not fluctuate more widely than high-price shares. However, he did not employ a statistical analysis and used only two independent variables, namely the share price and the credit rating of the company. Heins & Allison (1966) also found that, if other factors were held proportionately constant, share-price variability bore no relationship to the average price of the share. This result was obtained by using data from 1 January 1959 to 31 December 1959, and performing a regression analysis on share price and the credit rating of the company as well as price-earnings ratios and turnover. Blume & Husic (1973) found that price was, in some sense, a better predictor of future returns than the historically estimated beta. They expressed the annual returns in 1969, on the New York Stock Exchange and the American Stock Exchange, as a function of 1968 year-end price and the historically estimated beta. In order to assess how adequately price measured beta, the correlation between price and

historically estimated beta was calculated for American listed securities for each of the years from 1964 to 1968, and they found that these correlations were unexpectedly close to zero. These early studies illustrate the more general observation that, while the CAPM may hold true in some markets at some times, it does not hold true in all markets at all times. As reported in Black (1993), Fama & French (2004) and Michailidis et al. (op. cit.) for example, later studies have continued to support this observation.

2.4.3 As Reinganum (op. cit.) stated: "... the [then] current consensus [seemed] to be that a security's beta [was] still an important determinant of equilibrium pricing even though it [might] not be the sole determinant." Whilst subsequent work cited above has explored other explanatory variables, this observation remains valid.

2.5 THE ZERO-BETA VERSION OF THE CAPM

2.5.1 Friend & Blume (1973) explained why the relationship between expected return and risk, as implied by the CAPM, is unable to explain differential returns in the securities market. Monthly returns for each equity listed on the New York Stock Exchange during five-year periods from January 1950 to December 1968 were used. Risk–return trade-offs implied by these securities were estimated for three different periods: from January 1955 to December 1959, from January 1960 to December 1964 and from January 1965 to December 1968. The evidence in this paper seemed to require rejection of the CAPM.

2.5.2 In the same paper, the standard CAPM assumption of unlimited lending and borrowing at a unique risk-free interest rate is discussed, as well as the assumption (common to the standard and zero-beta versions) of unlimited short positions in risky assets. The authors argue that, even in the absence of short positions in risky assets, disequilibria could be corrected by the issuance of capital, whereas no similar adjustment is possible in risk-free assets. It should also be observed that there is no such thing as risk-free borrowing by a corporate market participant. As explained in Elton & Gruber (op. cit.: 318–21), the zero-beta version of the CAPM is therefore more plausible.

2.6 HIGHER-ORDER MOMENTS

The standard version of the CAPM assumes that investors are not concerned with higher-order moments, particularly skewness and kurtosis. Many researchers have investigated the validity of the CAPM in the presence of higher-order co-moments and their effects on asset prices. In particular, researchers such as Kraus & Litzemberger (1976), Friend & Westerfield (1980) and Sears & Wei (1988), among others, investigated the effect of skewness on asset pricing models. It has also been documented that skewness and kurtosis cannot be diversified away (Arditti, 1967).

2.7 PORTFOLIO TESTS

2.7.1 Earlier studies such as Lintner (op. cit.) and Douglas (1969) on the CAPM were based, primarily, on returns on individual equities. The *ex-post* betas for each individual equity were calculated using the realised returns. This procedure was inappropriate since the *ex-post* betas may differ substantially from the *ex-ante* betas, thus,

the realised returns on individual securities will be poor estimates of the *ex-ante* expected returns. Miller & Scholes (1972) also highlighted the statistical problems encountered when using individual securities in testing the validity of the CAPM. Fama & French (2004) remarked that estimates of beta for individual assets are imprecise, which creates a measurement-error problem when these estimates are used to explain average returns.

2.7.2 To improve the precision of beta estimates and to cope with the measurement-error problem, portfolios rather than individual securities are used by researchers such as Friend & Blume (1973) and Black, Jensen & Scholes (op. cit.). This problem can be mitigated by sorting securities by beta to form portfolios, the first containing equities with the lowest betas and the last containing those with the highest. Lau, Quay & Ramsey (1974) found that such grouping “greatly [reduced] the standard errors on both the intercept and the slope of the ... regression”:

Without this reduction, a meaningful ... comparison between the expected return and the systematic risk could not [have been] made.

2.8 TESTS OF THE SOUTH AFRICAN AND EGYPTIAN SHARE MARKETS

2.8.1 Barr & Bradfield (1988) stated that the CAPM appears to be a reasonable model in the South African context. However, Bowie & Bradfield (1998) warned that the choice of the wrong market proxy would reduce the predictive ability of the CAPM. Affleck-Graves & Bradfield (1993) concluded that the power of tests on smaller markets may be even less than that of tests on the New York Stock Exchange. Thus it is possible to argue that the CAPM may be valid on the JSE Securities Exchange but that we are unable to test that validity.

2.8.2 One method of overcoming the power problem is to use a long period (say 30 years) of data in the tests of the CAPM. This is impracticable as it would be difficult to compile total returns for that period of time for a significant sample on the JSE Securities Exchange (Davidson, unpublished). Davidson (op. cit.) investigated the theoretical underpinnings of the CAPM within the context of the JSE Securities Exchange. That author found that evidence in favour of the CAPM had been sparse.

2.8.3 Omran (unpublished) investigated the relationship between the returns dynamics in the Egyptian stock market and risk, as measured by beta. The Egyptian stock market is comparable to the JSE Securities Exchange to the extent that it is illiquid and growing. However, unlike the JSE Securities Exchange, the Egyptian Stock Exchange does not have historical data on equity returns for long periods of time. Hence, weekly prices of active securities (which were determined using weekly volume of trade and number of transactions from March 2001 to October 2001) from December 2001 to December 2002 were used to examine the observed risk–return trade-off during this period. Nevertheless, given the constraints, the author concluded that risk seemed to play a significant role in the returns dynamics in the Egyptian Stock Exchange.

2.9 ACTUARIAL APPLICATIONS

2.9.1 For the purposes of the pricing of liabilities in an incomplete market, arbitrage pricing is inadequate; an equilibrium model is required. As set out in Thomson

(2005), Thomson (unpublished) and Thomson & Gott (2009), the CAPM may be used for such purposes.

2.9.2 For actuarial applications it may be preferable to measure returns in real terms (i.e. net of consumer price inflation). While excess returns are identical whether they are measured in nominal or real terms, the covariances of returns on assets with returns on the market portfolio, and the variance of the latter are different, and this produces different betas. This may be shown as follows. For real returns, the covariance of the return on security i with that on the market portfolio is:

$$\text{cov}\{r_i - \phi, r_M - \phi\} = \text{cov}\{R_i - R_F - \phi, R_M - R_F - \phi\};$$

where ϕ is the rate of inflation. Since (unlike ϕ) R_F is a constant known at the start of the year, this gives:

$$\begin{aligned} \text{cov}\{r_i - \phi, r_M - \phi\} &= \text{cov}\{R_i - \phi, R_M - \phi\} \\ &= \text{cov}\{R_i, R_M\} - \text{cov}\{R_i, \phi\} - \text{cov}\{R_M, \phi\} + \text{var}\{\phi\}. \end{aligned}$$

This will generally be different from:

$$\text{cov}\{r_i, r_M\} = \text{cov}\{R_i, R_M\}$$

and the betas will differ accordingly. Thus the issue of real versus nominal returns is not made redundant by the use of excess returns. Nevertheless, the discussion may be simplified by using excess returns. Except where otherwise necessary, the discussion in this paper therefore focuses on excess returns.

2.9.3 In economic terms, as acknowledged or implied by various authors (e.g. Friend & Blume, 1970; Thomson & Gott, 2009) the use of real returns is also heuristically preferable, as preferences must ultimately be expressed in terms of consumer goods and services. However, most tests of the CAPM are applied in nominal terms. This issue is further discussed in section 4.2 below.

2.9.4 Most tests of the CAPM use relatively short intervals. Actuarial applications may require the modelling of returns on assets many years into the future. For some applications (e.g. Thomson, 2005; Thomson, unpublished), even yearly intervals necessitate computationally demanding programming. The use of shorter time intervals for such projections would become prohibitive. Both Thomson (1996) and Wilkie (1995) argue that, for the purposes of long-term actuarial modelling, annual intervals are appropriate.

2.9.5 Another consequence of actuarial interests in long-term modelling relates to the use of explanatory variables other than those that are integral to the modelling process or to the output required (i.e. of 'exogenous' variables). For the purposes of modelling such as that envisaged in ¶2.9.1, such variables tend to become irrelevant after the first few projection periods. (Thomson, 1996: 770) Their explanatory value is therefore eroded over time and considerations of parsimony and computational efficiency suggest that they are better omitted. Tests over annual intervals may show that the effects of exogenous variables conditional on information at the start of each year

are significant. However, if the CAPM is used as the basis of pricing liabilities in future years conditional on information at that time, then such information is largely dependent on additional noise terms and exogenous variables are merely a source of additional noise. Furthermore, many standard tests of the CAPM are premised on the statement, implicit in the CAPM, that no other factors than beta explain expected excess returns. A weaker statement of the CAPM is that, while other factors than the means and variances of returns may have influenced average excess returns *ex post* over short intervals, they do not influence expected excess returns *ex ante* over annual intervals. For these reasons, studies such as those reported in ¶¶2.4.1, 2.4.2 and 2.8.2 may be considered irrelevant.

2.9.6 The CAPM expresses the systematic risk of a security relative to a comprehensive ‘market portfolio’, which includes not just traded financial assets such as equities and bonds, but also fixed property, consumer durables and human capital (Fama & French, 2004). The market portfolio, which is central to testing the CAPM, is unobservable in practice. As Roll (op. cit.) stated, there is practically no way that any test can be conducted with reference to the actual market portfolio. A proxy for the market portfolio therefore needs to be selected. Roll (op. cit.) warned that the use of an incorrect index as a market proxy can lead to invalid results. As mentioned in ¶2.8.1, Bowie & Bradfield (op. cit.) have reiterated this in the South African context. It may therefore be argued that the results obtained in this study are inconclusive as a result of the use of an inappropriate market proxy. Once again, however, one can weaken the CAPM to state that, while the effect of unobservable components of the market portfolio may have influenced average excess returns *ex post* over short intervals, they do not influence expected excess returns *ex ante* over annual intervals.

2.9.7 Davidson (op. cit.) argued that the JSE Securities Exchange should be viewed as a single homogeneous market and that the FTSE/JSE All-Share Index is a suitable market proxy to be used in the CAPM. However, Nyirenda (unpublished) commented that the JSE Securities Exchange is a segmented market and the estimation of systematic risk had to take into account this segmentation by using appropriate market proxies. He argued that the FTSE/JSE All-Share Index is not an appropriate market proxy for the modelling of a particular sector (in that case gold mining), for which purpose a sectoral index provided a better proxy.

2.9.8 Whereas many tests of the CAPM treat the market portfolio as comprising equities only, actuarial applications will generally require at least the inclusion of bonds (e.g. Thomson & Gott, op. cit.).

3. DATA DESCRIPTION

3.1 DATA SOURCES

Data for this study were obtained from I-Net Bridge. They comprised quarterly total-return indices from 30 June 1995 to 30 June 2009 for ten sectoral indices on the JSE Securities Exchange, the FTSE/JSE All-Share Index and, from its inception on 31 March 2000, the STEFI Composite Index. For the periods prior to the inception of the STEFI Composite Index, chain-linked values of the Ginsberg, Malan & Carson Money-market Index were used. Further details of the data are given below.

3.2 THE MARKET PORTFOLIO

3.2.1 As stated in ¶2.9.6, returns on the comprehensive market portfolio are impossible to measure. A proxy for the market portfolio therefore needed to be selected. In this study, the FTSE/JSE All-Share Index (FTSE/JSE ALSI) was used as a market proxy. The code M is used for this index below.

3.2.2 As indicated in ¶2.9.8, actuarial applications will generally require at least the inclusion of bonds. This is not only true of the market portfolio itself, but also of tests of the securities market line.

3.3 THE RISK-FREE INDEX

Certain money-market instruments are considered to be almost risk-free because of the high security and low variability associated with these instruments. As indicated above, the STEFI Composite Index was chosen as an appropriate proxy for the risk-free asset. That index was available only from 31 March 2000; for earlier dates the Ginsberg, Malan & Carson Money-market Index was used. The code F is used for the risk-free index below.

3.4 INDIVIDUAL SECTORAL INDICES

The ten sectoral indices with the highest market capitalisations were selected. These were as follows (the codes shown in brackets are used below):

- Basic materials (Bm);
- Chemicals (Ch);
- Consumer goods (Cg);
- Consumer services (Cs);
- Financials (Fi);
- Food & drug retailers (Fd);
- Health care (Hc);
- Industrials (In);
- Mining (Mi); and
- Telecommunications (Te).

4. METHOD

4.1 FORCE OF RETURN

4.1.1 The CAPM is generally expressed in terms of expected rates of return. However, for this study, the returns on the market portfolio, the money-market index and each individual sectoral index were expressed as forces of return. This approach was used since rates of return depend on the length of the interval over which the return is calculated, producing different expected returns for different intervals. On the other hand, forces of return overcome this as they are additive and easy to aggregate; hence they produce linear expected returns. It may, in effect, be assumed that, in continuous time during the interval $t \in (0,1)$, each investor maintains constant exposure m_i to security i , so that the market portfolio remains constant and the force of return at time t is:

$$\delta_t = \sum_i m_i \delta_{it} ;$$

where:

δ_{it} is the force of return on security i at time t ; and

$$\sum_i m_i = 1 ;$$

and the aggregate force of return over the interval is:

$$\begin{aligned} R_M &= \int_0^1 \delta_{Mt} dt \\ &= \int_0^1 \sum_i m_i \delta_{it} dt \\ &= \sum_i m_i \int_0^1 \delta_{it} dt \\ &= \sum_i m_i R_i ; \end{aligned}$$

where:

δ_{Mt} is the force of return on the market portfolio at time t ; and

R_i is the aggregate force of return on security i during the interval.

Unless otherwise stated, ‘returns’ referred to below are therefore forces of return. The CAPM is a single-period model, and in this application investors make portfolio allocation decisions only at year-ends. Between year-ends they are effectively assumed to maintain constant proportions in the respective securities. For example, if the CAPM is to be used to determine a benchmark portfolio for the purposes of a mandate to investment managers during the forthcoming year, it would be reasonable to assume that the tracking of the mandate would require continuous rebalancing to it during the year.

4.1.2 Let T_{it} denote the value of total return index i at time t , where:

$$i \in \{M, F, Bm, Ch, Cg, Cs, Fi, Fd, Hc, In, Mi, Te\} .$$

Then we define the return on that index over a unit interval $(t-1, t)$ as:

$$R_{it} = \ln \left(\frac{T_{it}}{T_{i,t-1}} \right) .$$

Units of time are defined below.

4.2 NOMINAL RETURNS

As observed in ¶¶2.9.2 and 2.9.3, while it would be preferable to measure returns in real terms, most tests of the CAPM, being more concerned about applications to trading, use relatively short intervals. Because of the difficulty of measuring real returns over such intervals—and because of the relative certainty of the level of inflation over such intervals—tests are usually applied to nominal returns. Those problems do not apply to long-term modelling. Conversely, the longer the term of the model, the greater

is the uncertainty about levels of inflation. For the sake of comparability, and for use with nominal liabilities, this research focused on nominal returns.

4.3 TIME INTERVALS

4.3.1 As indicated in ¶¶1.4 and 2.9.4, it is considered that, in general, annual intervals are appropriate for actuarial applications.

4.3.2 For the purposes of this study it was decided to use quarterly intervals for the determination of betas and annual intervals for tests of the CAPM. The use of quarterly intervals for the determination of betas may appear inconsistent. The reason for the use of shorter intervals is that, while it may be assumed that decisions are made annually, it may also be assumed that they are made on the basis of information available at shorter intervals. Because of the linearity discussed in ¶4.1.1, there is no bias in the use of betas at more frequent intervals. The method used is explained in more detail below.

4.4 INDIVIDUAL SECTORAL INDICES

4.4.1 In order to test the CAPM, it is necessary to translate the *ex-ante* parameters of an equilibrium model into *ex-post* realisations. For that purpose it is necessary to assume the validity of some return-generating function. For any *ex-ante* model and *ex-post* realisations there is almost certainly some generating function that will link those realisations with the model (Blume & Husic, op. cit.). The return-generating process in equation (2) may be used to test the CAPM.

4.4.2 The investigation was first carried out using ten one-year periods, each ending on 30 June from 2000 to 2009. Let $Y(q)$ denote the q th quarter of calendar year Y and let $[Y]$ denote the one-year period comprising the sequence of quarters:

$$(Y - 1 (3), Y - 1 (4), Y (1), Y (2)) \text{ for } Y = 2000, \dots, 2009.$$

4.4.3 For each sectoral index i (cf. section 3.4) for each period $[Y]$, beta was estimated as:

$$\hat{\beta}_{i[Y]} = \frac{\hat{\sigma}_{iM[Y]}}{\hat{\sigma}_{MM[Y]}}; \tag{8}$$

where:

$$\hat{\sigma}_{ij[Y]} = \frac{4}{15} \sum_{u=Y-5(3)}^{Y-1(2)} \left(R_{iu} - \frac{1}{4} \hat{\mu}_{i[Y]} \right) \left(R_{ju} - \frac{1}{4} \hat{\mu}_{j[Y]} \right); \text{ and} \tag{9}$$

$$\hat{\mu}_{i[Y]} = \frac{4}{16} \sum_{u=Y-5(3)}^{Y-1(2)} R_{iu}. \tag{10}$$

In equations (9) and (10), the numerator 4 annualises the quarterly estimates. In the case of equation (9), the implicit assumption is that:

$$\text{cov}(R_{iu}, R_{it}) = 0 \text{ for } u \neq t.$$

4.4.4 For each sectoral index i for each period $[Y]$, the excess return, was determined using equation (3) as:

$$r_{i[Y]} = \sum_{u=Y-1(3)}^{Y(2)} R_{iu} - R_{Fu}. \tag{11}$$

The error term was found, following equation (2), as:

$$\varepsilon_{i[Y]} = r_{i[Y]} - \hat{\beta}_{i[Y]} r_{M[Y]} \tag{12}$$

4.4.5 A further investigation was carried out using all periods combined. For this purpose, for each sectoral index i , the prior beta was estimated as:

$$\hat{\beta}_i = \hat{\beta}_{i[2000]} ; \tag{13}$$

i.e. using equation (8) for the first prior period. For each sectoral index i the mean excess annual return, was determined as:

$$\bar{r}_i = \frac{1}{10} \sum_{Y=2000}^{2009} r_{i[Y]} . \tag{14}$$

The error term was found as:

$$\varepsilon_i = \bar{r}_i - \hat{\beta}_i r_M . \tag{15}$$

4.4.6 If the REH holds, then it is not necessary to use prior betas. As explained by Blume & Husic (op. cit.), if *ex-ante* values of beta differ from *ex-post* values at the start of a period, then *ex-post* estimates derived from values during the period “may more accurately mirror investors’ *ex-ante* expectations.” If both *ex-ante* betas and *ex-post* in-period sample betas are unbiased estimates of population betas for the respective indices, then the in-period sample betas are unbiased estimates of the *ex-ante* betas. The in-period sample betas may then be used to test the joint hypothesis that both the CAPM and the REH hold. Such a test is useless as an operational test: it does not test whether the CAPM works, because in-period sample betas are not available *ex ante*. However, for the purpose of testing whether the CAPM can be used in long-term models it is relevant, since such a model can generate unbiased *ex-ante* betas.

4.4.7 Another investigation was therefore carried out using all periods combined. For this purpose, for each sectoral index i , the in-period beta was estimated as:

$$\hat{\beta}_i = \frac{\hat{\sigma}_{iM}}{\hat{\sigma}_{MM}} ; \tag{16}$$

where:

$$\hat{\sigma}_{ij} = \frac{4}{39} \sum_{u=1999(3)}^{2009(2)} \left(R_{iu} - \frac{1}{4} \hat{\mu}_i \right) \left(R_{ju} - \frac{1}{4} \hat{\mu}_j \right) ; \text{ and} \tag{17}$$

$$\hat{\mu}_i = \frac{4}{40} \sum_{u=1999(3)}^{2009(2)} R_{iu} . \tag{18}$$

The error term was found as:

$$\varepsilon_i = \bar{r}_i - \hat{\beta}_i \bar{r}_M . \tag{19}$$

4.5 PORTFOLIOS OF SECTORAL INDICES

4.5.1 As noted in ¶1.6 and in section 2.7, the precision of the beta estimates may be improved by grouping the selected sectoral indices into portfolios according to their beta. In general, researchers have constructed their portfolios out of individual equities. While sectoral returns themselves may be correlated to different extents by exogenous variables, the measurement-error problem is considerably reduced by the longer time intervals used in this research than in typical tests of the CAPM. On the other hand, the discrepancy between *ex-ante* and *ex-post* expectations would arguably be less variable between sectoral indices than between individual equities: the latter would include differences that would be averaged out in the former. Thus, while the reason given in the literature considered for grouping into portfolios is less relevant to this paper, the dependence of the tests on the REH is reduced.

4.5.2 For each period [Y] the sectoral indices were divided into four groups $I_{1[Y]}, \dots, I_{4[Y]}$ according to their beta estimates $\hat{\beta}_{i[Y]}$. The three sectoral indices with the lowest beta estimates for that period were placed into $I_{1[Y]}$, the two with the next lowest into $I_{2[Y]}$, the two with the next lowest into $I_{3[Y]}$ and the three with the highest into $I_{4[Y]}$. This process led to four portfolios for each of the ten annual periods.

4.5.3 It was assumed that, during each period [Y], in each portfolio $I_{p[Y]}$ ($p=1,2,3,4$), equal amounts were invested in each sectoral index in that portfolio, and that the portfolio was continuously rebalanced accordingly. The excess return on portfolio $I_{p[Y]}$ during period [Y] was therefore:

$$r_{p[Y]} = \frac{\sum_{i \in I_{p[Y]}} r_{i[Y]}}{k_p}, \tag{20}$$

where k_p is the number of sectoral indices included in that portfolio.

4.5.4 Let:

$$\hat{\beta}_{p[Y]} = \frac{\sum_{i \in I_{p[Y]}} \hat{\beta}_{i[Y]}}{k_p}, \tag{21}$$

The error term was found as:

$$\varepsilon_{p[Y]} = r_{p[Y]} - \hat{\beta}_{p[Y]} r_{M[Y]} \tag{22}$$

4.5.5 As for sectoral indices, further investigations were carried out using all periods combined. For this purpose, the sectoral indices were divided into four groups $I_{1[2000]}, \dots, I_{4[2000]}$ according to their beta estimates $\hat{\beta}_{i[2000]}$ as in ¶4.5.2. For each portfolio $I_{p[2000]}$ ($p = 1, 2, 3, 4$), the prior beta was estimated as:

$$\hat{\beta}_p^{(p)} = \hat{\beta}_{p[2000]}. \tag{23}$$

The superscript (p) denotes prior betas. For each portfolio $I_{p[2000]}$ the excess annual return, was determined as:

$$\bar{r}_p^{(p)} = \frac{1}{10} \sum_{Y=2000}^{2009} r_{p[Y]}^{(p)}; \tag{24}$$

where:

$$r_{p[Y]}^{(p)} = \frac{\sum_{i \in I_{p[2000]}} r_{i[Y]}}{k_p}.$$

The error term was found as:

$$\varepsilon_p^{(p)} = \bar{r}_p^{(p)} - \hat{\beta}_p^{(p)} \bar{r}_M. \tag{25}$$

4.5.6 Again, tests were also made on in-period betas. For this purpose the sectoral indices were divided into four groups I_1, \dots, I_4 according to their beta estimates:

$$\hat{\beta}_p^{(i)} = \frac{\hat{\sigma}_{pM}^{(i)}}{\hat{\sigma}_{MM}^{(i)}}; \tag{26}$$

where:

$$\hat{\sigma}_{pM}^{(i)} = \frac{1}{k_p} \sum_{i \in I_p} \hat{\sigma}_{iM}^{(i)}. \tag{27}$$

The superscript (i) denotes in-period betas. For each portfolio I_p the excess annual return, was determined as:

$$\bar{r}_p^{(i)} = \frac{\sum_{i \in I_p} \bar{r}_i}{k_p}.$$

The error term was found as:

$$\varepsilon_p^{(i)} = \bar{r}_p^{(i)} - \hat{\beta}_p^{(i)} \bar{r}_M. \tag{28}$$

4.6 EXOGENOUS VARIABLES

4.6.1 While the variances and covariances of the variables to be modelled are endogenous to the requirements of long-term actuarial modelling, other explanatory variables would be exogenous, and would require separate modelling. As observed in section 2.4, many authors have investigated the effects of exogenous variables.

4.6.2 However, as observed in ¶2.9.5, these effects add little value to long-term modelling. Furthermore, they are diluted by the aggregation of equities into sectors and portfolios. The effects of exogenous variables have therefore been ignored.

4.6.3 As explained below, however, (cf. sections 6.4.2 and 6.5.2) the effects of nonlinearity were tested.

5. PRELIMINARY OBSERVATIONS

5.1 SECTORAL INDICES

5.1.1 Figure 1 plots the excess returns on the sectoral indices against beta for each period. Values for $\hat{\beta}_{F[Y]} = 0$ (i.e. for the risk-free asset) and for $\hat{\beta}_{M[Y]} = 1$ (i.e. for the market portfolio) are included and the straight line through them (i.e. the securities market line for the standard version of the CAPM) is drawn. Apart from periods [2005], [2006] and [2007], there is little evidence of linearity. In periods [2000] and [2001] the relationship is disturbed by outliers. In periods [2003] and [2009], where the excess return on the market portfolio is negative, the excess returns are widely scattered. (The

phenomenon of negative excess returns on the market portfolio from time to time does not conflict with the CAPM; the latter refers to expected excess returns.)

Figure 1a: Excess return versus beta: [2000]

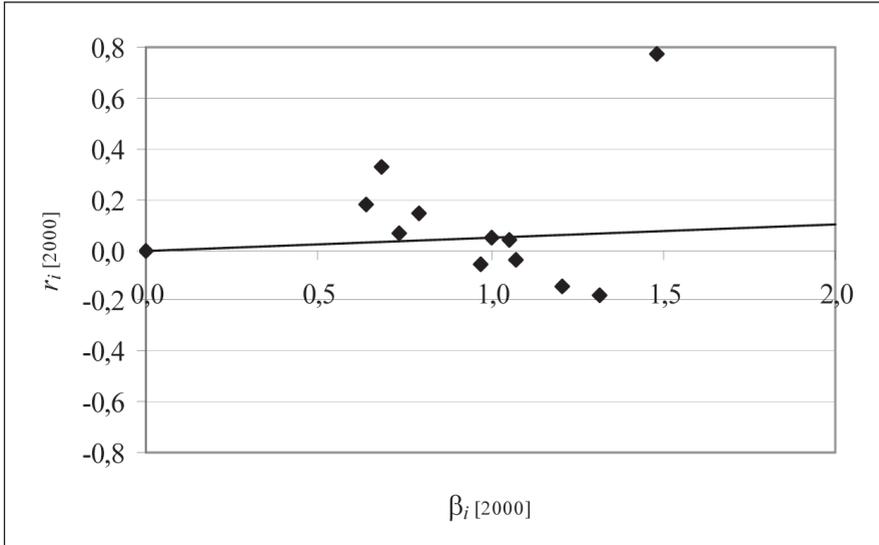


Figure 1b: Excess return versus beta: [2001]

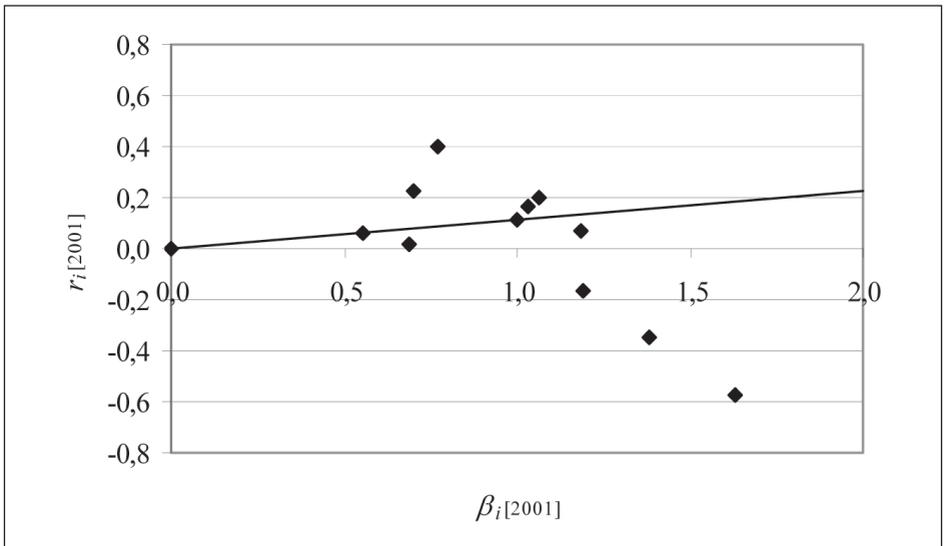


Figure 1c: Excess return versus beta: [2002]

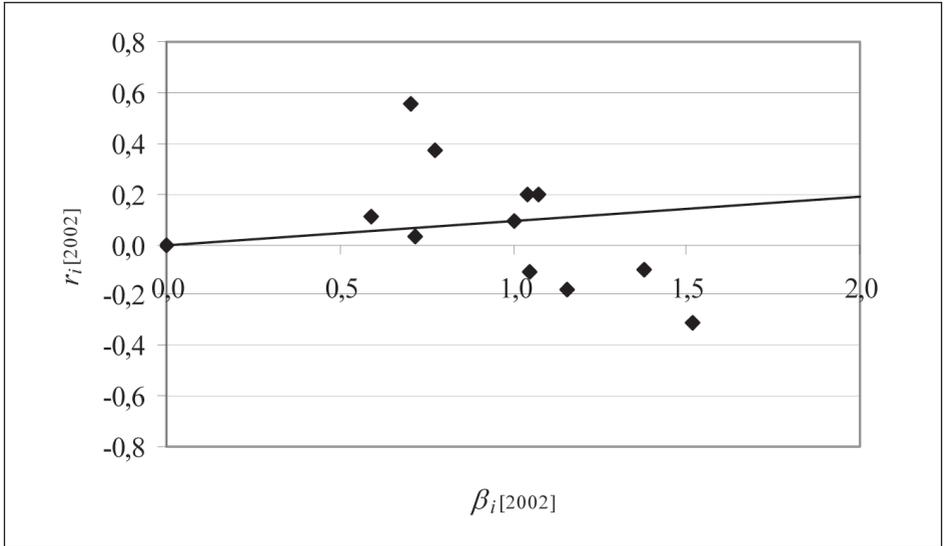


Figure 1d: Excess return versus beta: [2003]

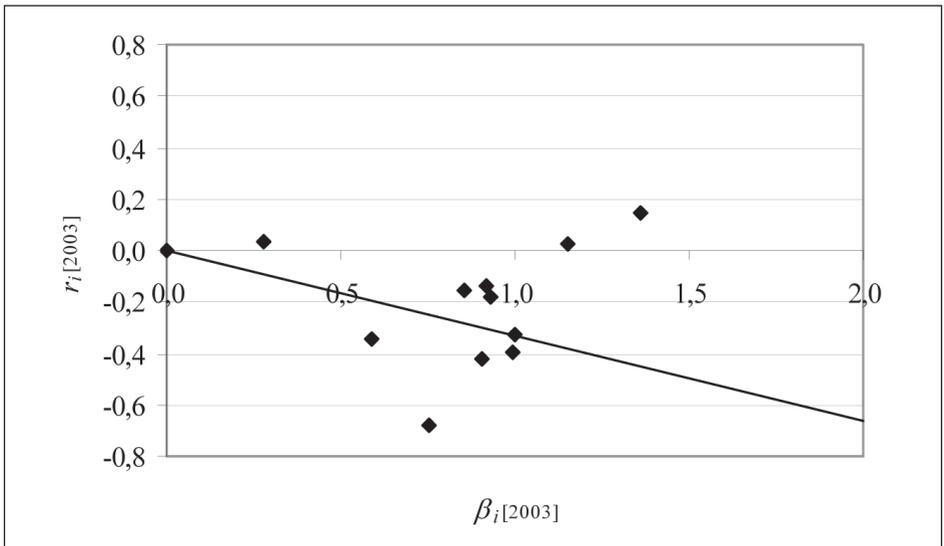


Figure 1e: Excess return versus beta: [2004]

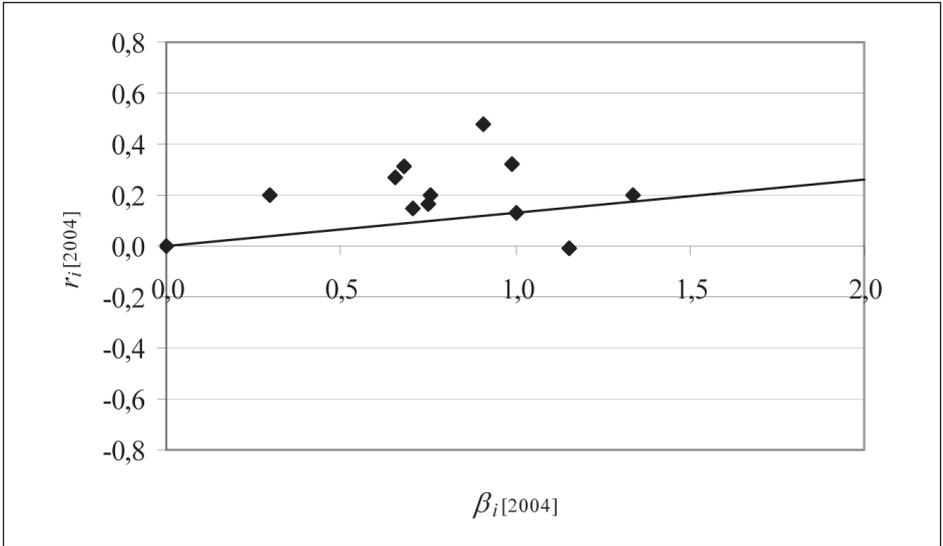


Figure 1f: Excess return versus beta: [2005]

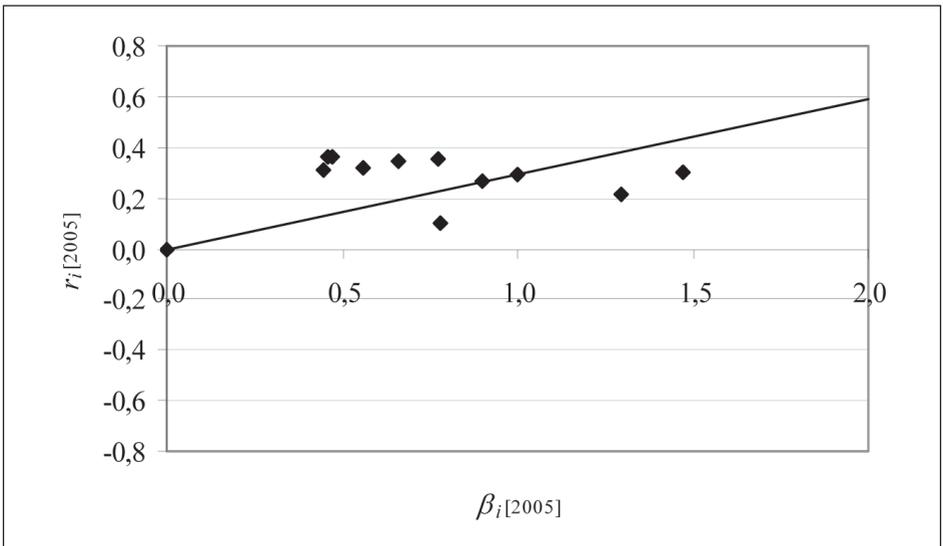


Figure 1g: Excess return versus beta: [2006]

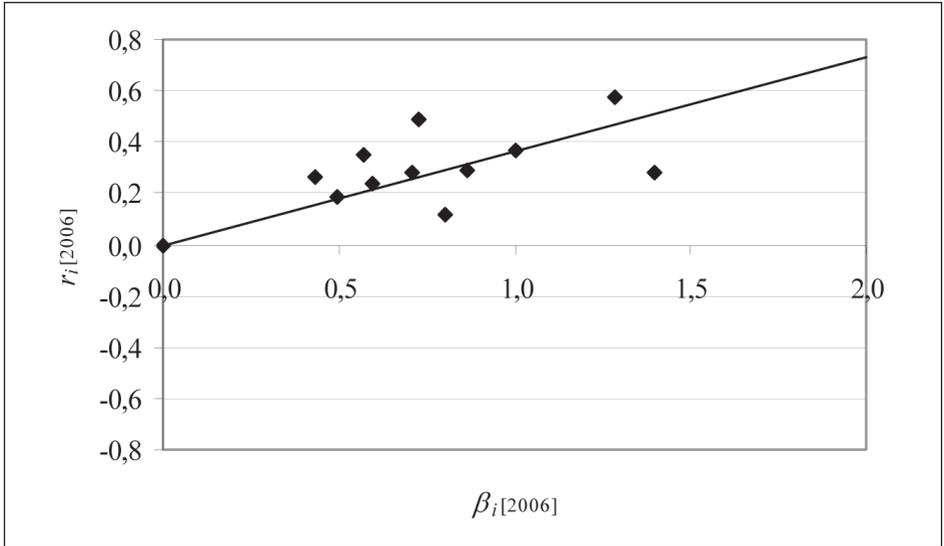


Figure 1h: Excess return versus beta: [2007]

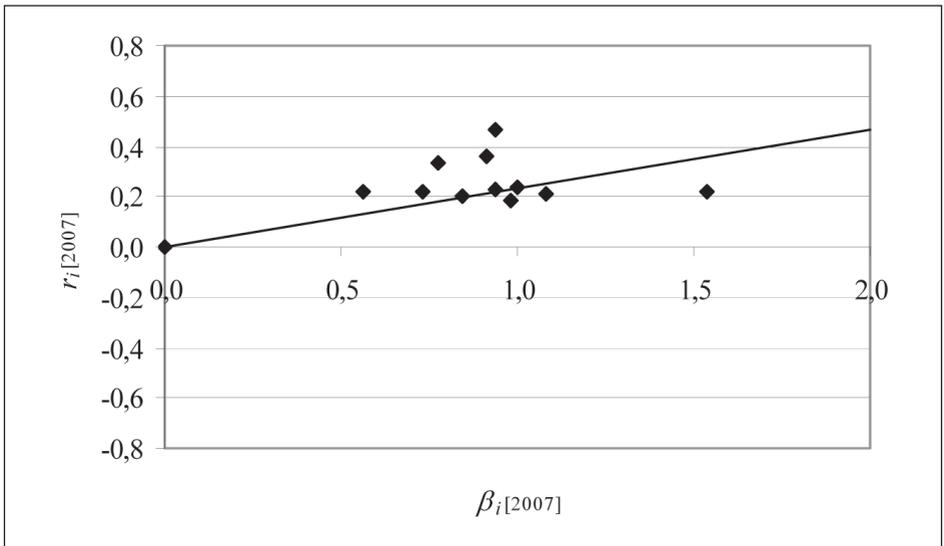


Figure 1i: Excess return versus beta: [2008]

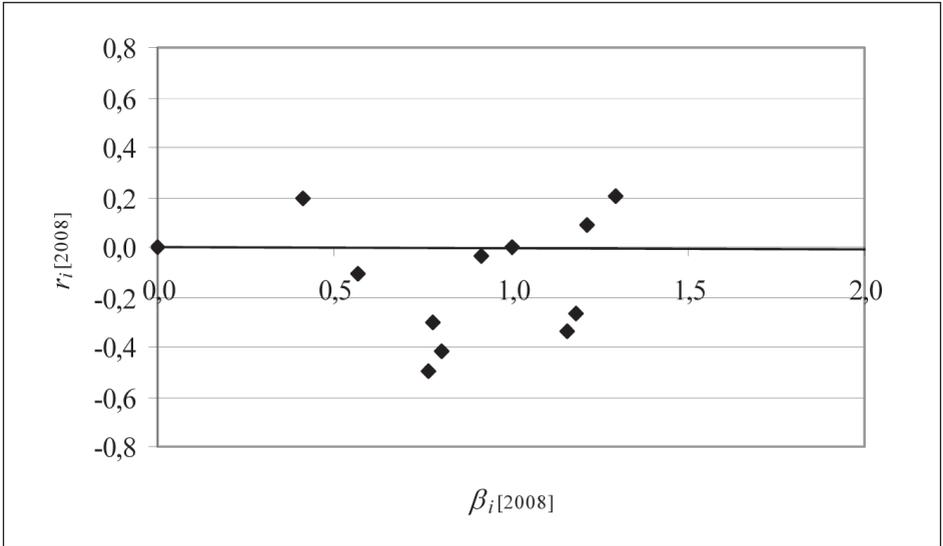
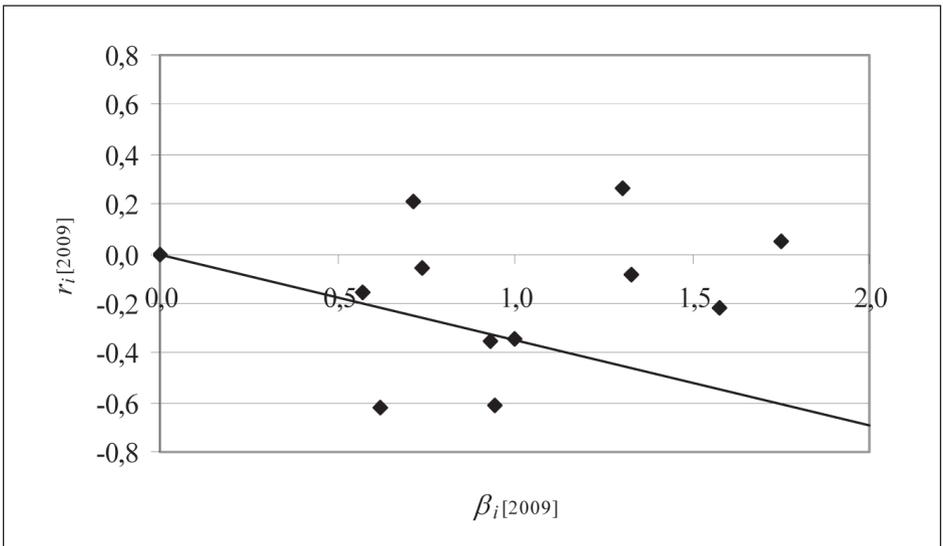


Figure 1j: Excess return versus beta: [2009]



5.1.2 Figure 2 plots the excess returns on the sectoral indices against beta for all periods combined using prior betas as in equations (13) and (14). Note that the scale of this figure differs from that of Figure 1. Here the effects of outliers and wide scattering are strongly diluted and the relationship appears more linear.

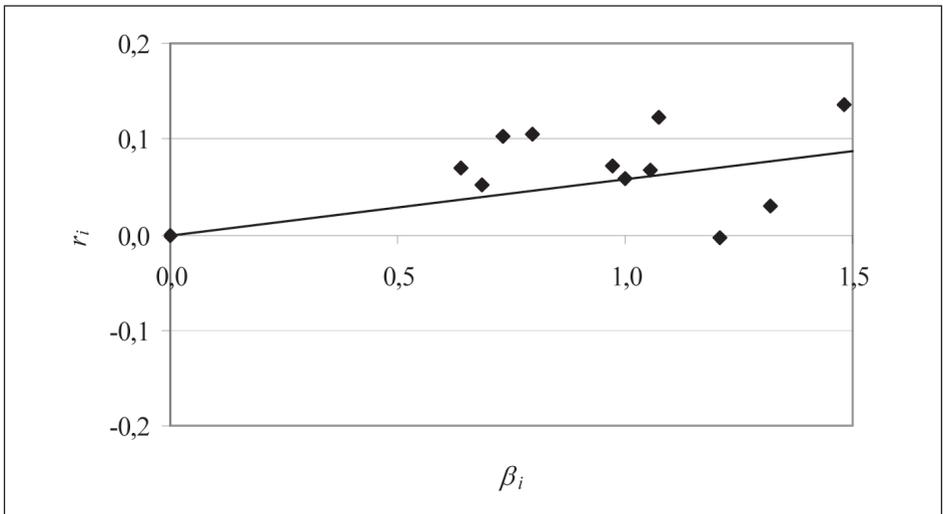
5.1.3 Figure 3 plots the excess returns on the sectoral indices against beta for all periods combined using in-period betas as in equations (14) and (16). In comparison with Figure 2, the scatter here is wider for lower betas and the relationship appears less linear. The distribution of betas is counter-intuitive as the market portfolio, largely comprising a weighted average of the sectoral indices, should have a similarly weighted beta. Because of the large weighting in mining, that sector, which is represented by the point (1,28; 0,105), offsets all other sectors in the market proxy. This is notably different from the observations in Figure 2, where mining has a prior beta of 0,79.

5.1.4 As mentioned in ¶2.2.2, the use of the CAPM presupposes that, *ex post*, betas are more stable over time than mean excess returns on the market portfolio. Because these two variables are multiplied together in the CAPM formula for the securities market line, the variability of each must be determined relative to its magnitude. For the data used, the relative variability of beta for each sector was determined as:

$$v(\beta_i) = \frac{\sqrt{\frac{1}{9} \sum_{t=2000}^{2009} (\hat{\beta}_{i[t]} - \hat{\beta}_i)^2}}{\hat{\beta}_i}.$$

Similarly, the relative variability of the mean excess return on the market portfolio was determined as:

Figure 2: Excess return versus beta: sectoral indices, all periods combined, prior betas



$$v(r_M) = \frac{\sqrt{\frac{1}{9} \sum_{Y=2000}^{2009} (r_{M[Y]} - \bar{r}_M)^2}}{\bar{r}_M}$$

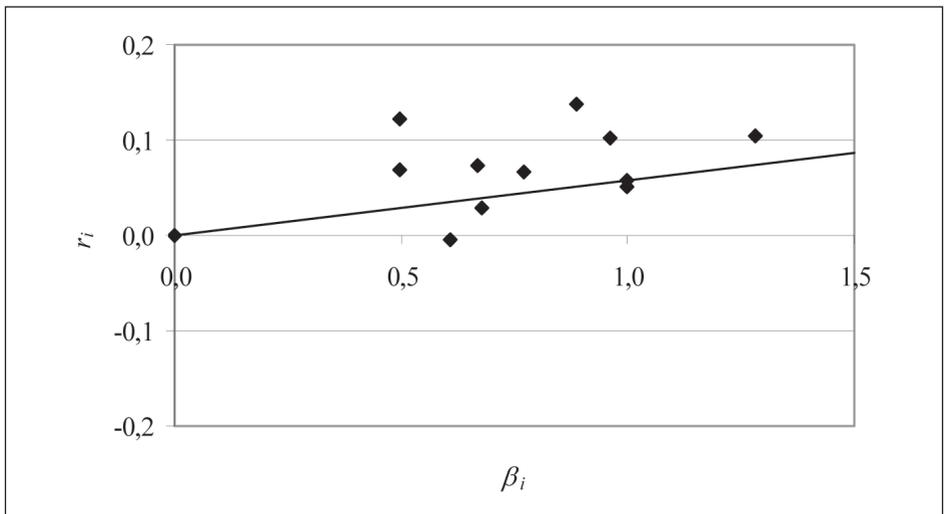
The results are shown in Table 1. Also shown is the average variability of beta over all sectors, viz.:

$$v(\bar{\beta}) = \frac{1}{10} \sum_i v(\beta_i)$$

Table 1: Relative variability of beta and mean historic excess return

Sector (<i>i</i>)	$v(\beta_i)$	Sector (<i>i</i>)	$v(\beta_i)$
Bm	0,334	In	0,458
Ch	0,422	Mi	0,270
Cg	0,392	Te	0,496
Cs	0,868	average $\overline{v(\beta)}$	0,539
Fi	0,721		
Fd	0,886		$v(r_M)$
Hc	0,544	M	4,079

Figure 3: Excess return versus beta: sectoral indices, all periods combined, in-period betas



5.1.5 From Table 1 it may be seen that the relative variability of beta of every sector is much lower than that of the excess return on the market portfolio.

5.1.6 Figure 4 shows a histogram of $\epsilon_{i[Y]}$ across all i and $[Y]$. In that figure the upper limit of each interval is shown on the horizontal axis. The distribution apparently deviates from the normal and its mean appears to be greater than 0.

5.2 PORTFOLIOS OF SECTORAL INDICES

5.2.1 Figures 5 and 6 plot the excess returns on the portfolios against beta for all periods: in the former prior betas are used and in the latter, in-period betas. Values for $\hat{\beta}_{F[Y]} = 0$ (i.e. for the risk-free asset) and for $\hat{\beta}_{M[Y]} = 1$ (i.e. for the market portfolio) are included, together with the corresponding securities market line. These figures suggest that, as found by other authors (cf. ¶2.3.6), the securities market line is flatter than that of the standard CAPM; it would be more consistent with the zero-beta version. Figure 6 shows a general reduction in in-period betas relative to prior betas. This follows from the effect of the mining industry discussed in ¶5.1.3.

5.2.2 As in ¶¶5.1.4–5, the relative variability of beta—in this case for each portfolio—was compared with that of the excess return on the market portfolio. The results are shown in Table 2.

Figure 4: Histogram of the sample distribution of $\epsilon_{i[Y]}$

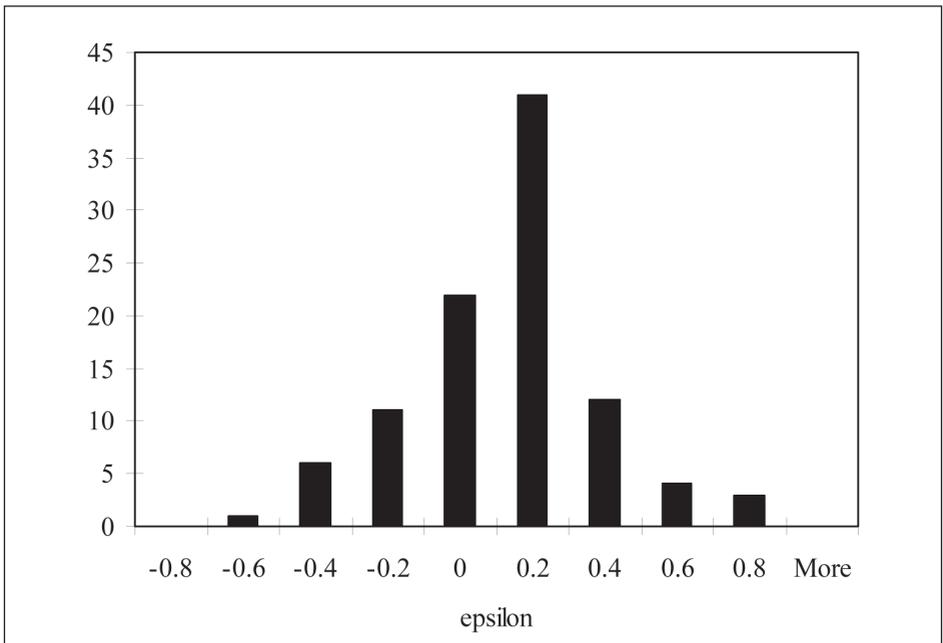


Figure 5: Excess return versus beta: portfolios, all periods combined, prior betas

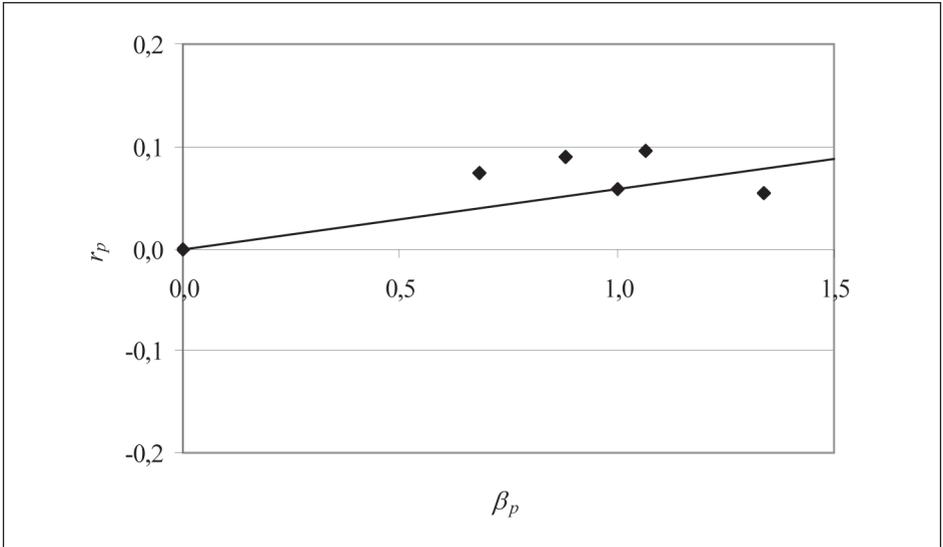


Figure 6: Excess return versus beta: portfolios, all periods combined, in-period betas

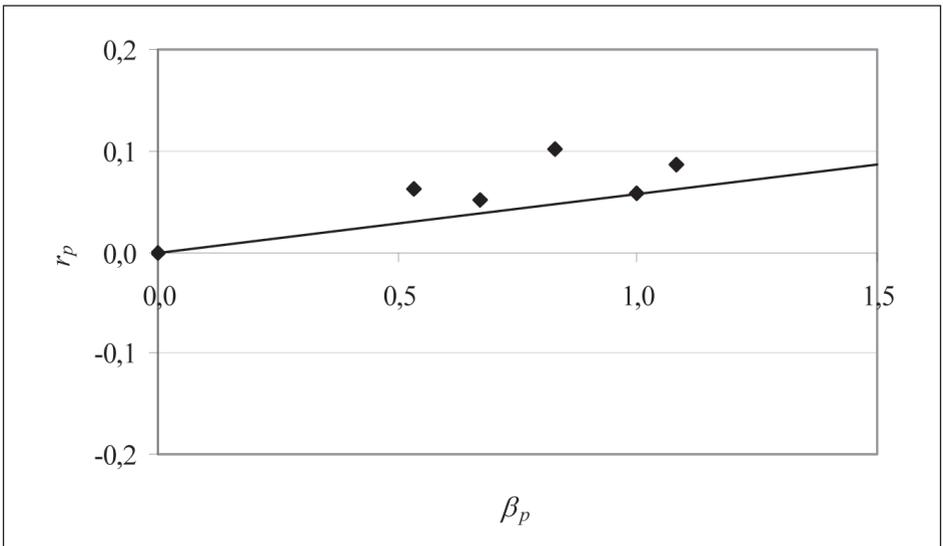


Table 2: Relative variability of beta and mean historic excess return

Portfolio (p)	$v(\beta_p)$
1	0,196
2	0,265
3	0,238
4	0,228
average $\overline{v(\beta)}$	0,232
	$v(r_M)$
M	4,079

5.2.3 From Table 2 it may be seen that the relative variability of beta of every sector is much lower than that of the excess return on the market portfolio. The difference is even greater in this case than in that of the sectors. This is because the sectors have been sorted into portfolios with homogeneous betas.

6. EMPIRICAL TESTS

6.1 A PARAMETRIC TEST OF THE SECURITIES MARKET LINE USING PRIOR BETAS

6.1.1 The first empirical test refers to the capital market line as expressed in equation (2). The hypothesis tested is that $\varepsilon_{i[Y]}$, as derived from equation (12), is normally distributed with a mean of 0. The test statistic used was:

$$S = \sum_{Y=2000}^{2009} S_{[Y]} ; \tag{29}$$

where:

$$S_{[Y]} = \varepsilon'_{[Y]} V^{-1}_{[Y]} \varepsilon_{[Y]} ; \tag{30}$$

$$\varepsilon_{[Y]} = \begin{pmatrix} \varepsilon_{Bm[Y]} \\ \vdots \\ \varepsilon_{Te[Y]} \end{pmatrix} ;$$

$$V = \begin{pmatrix} \hat{\sigma}_{BmBm[Y]} & \cdots & \hat{\sigma}_{BmTe[Y]} \\ \vdots & & \vdots \\ \hat{\sigma}_{TeBm[Y]} & \cdots & \hat{\sigma}_{TeTe[Y]} \end{pmatrix} .$$

6.1.2 The statistic $S_{[Y]}$, being the quadratic form of a multivariate normal, has a χ^2 distribution. There are ten components, and approximately one degree of freedom is lost because the relationship between r_{M_t} and r_{it} is approximately linear. This is because:

$$r_{M[Y]} \approx \sum_{i=Bm}^{Te} a_i r_{i[Y]} ;$$

where:

$$\sum_{i=Bm}^{Te} a_i \approx 1 .$$

The approximations arise from the fact that, while the indices selected comprise the bulk of the constituents of the ALSI, they are not comprehensive. The test statistic S therefore has a χ^2 distribution with approximately 90 degrees of freedom.

6.1.3 It was found that the value of S was 1108,53, as shown in Table 3.

Table 3: Computation of the test statistic S

Period [Y]	$S_{[Y]}$
[2000]	356,33
[2001]	27,99
[2002]	19,99
[2003]	27,79
[2004]	14,71
[2005]	4,88
[2006]	13,46
[2007]	5,76
[2008]	484,54
[2009]	153,08
Total	1108,53

The standard normal approximation to this result is:

$$z = \sqrt{2\chi^2} - \sqrt{2n-1} ;$$

where n is the number of degrees of freedom. This gives $z = 33,71$, which is extremely significant. Most of the value comes from [2000], [2008] and [2009]. In [2000] the extremity came from telecommunications, as well as consumer services, which was strongly correlated with telecommunications. In that year the excess return on telecommunications was 77,25%, while the excess return on the market portfolio was only 4,92%. In [2008] the extremities came from chemicals, health care and consumer services, on which the excess returns were -30,56%, -50,03% and -34,16% respectively,

while the excess return on the market was $-0,36\%$. [2009] was the year of the global financial crisis; the extremities came from a relatively wide range of sectors.

6.2 A PARAMETRIC TEST OF THE SECURITIES MARKET LINE USING IN-PERIOD BETAS

6.2.1 The test in section 6.1 makes the questionable assumption that investors' *ex-ante* assumptions at any time are based on *ex-post* observations, without any cognisance of prospects at the time. At the other extreme, let us suppose that both *ex-ante* assumptions at the start of a period and estimates based on *ex-post* observations during that period are unbiased estimates of the underlying values during that period, and therefore that the latter are unbiased estimates of the former. While this does not imply perfect foresight, it does imply greater correspondence than might reasonably be expected. Nevertheless, as explained in ¶4.4.6, for the purpose of testing whether the CAPM can be used in long-term models it is relevant, since such a model can generate unbiased *ex-ante* betas.

6.2.2 The method used for this test follows Shanken (1985). Because the covariances and betas are estimated in-period, the quadratic form does not follow a multivariate χ^2 distribution. Instead, with Shanken's (op. cit.) notation, the regression statistic

$$Q_c = T\mathbf{e}'\hat{\Sigma}^{-1}\mathbf{e}$$

follows a Hotelling's T^2 with $N - 2$ and $T - 2$ degrees of freedom, where:

N is the number of portfolios (in this case 4);

T is the length of the time series (in this case 10);

$$\mathbf{e} = \hat{\boldsymbol{\mu}} - \hat{\mathbf{X}}'\hat{\boldsymbol{\Gamma}}_c;$$

$$\hat{\boldsymbol{\mu}} = \begin{pmatrix} \hat{\mu}_1 \\ \vdots \\ \hat{\mu}_4 \end{pmatrix};$$

$$\hat{\mu}_p = \frac{1}{10} \sum_{u=1999(3)}^{2009(2)} R_{pu};$$

$$\hat{\mathbf{X}} = \begin{pmatrix} 1 & \hat{\beta}_1 \\ 1 & \vdots \\ 1 & \hat{\beta}_4 \end{pmatrix};$$

$$\hat{\beta}_p = \frac{\hat{\sigma}_{pM}}{\hat{\sigma}_{MM}};$$

$$\hat{\sigma}_{pM} = \frac{1}{39} \sum_{u=1999(3)}^{2009(2)} \left(R_{pu} - \frac{1}{4} \hat{\mu}_p \right) \left(R_{Mu} - \frac{1}{4} \hat{\mu}_M \right);$$

$$\Gamma_C = \begin{pmatrix} \hat{\gamma}_0 \\ \hat{\gamma}_1 \end{pmatrix} = \left(\hat{X}' \hat{\Sigma}^{-1} \hat{X} \right)^{-1} \hat{X}' \hat{\Sigma}^{-1} \hat{\mu};$$

$$\hat{\Sigma} = \begin{pmatrix} \hat{\sigma}_{\varepsilon 11} & \cdots & \hat{\sigma}_{\varepsilon 14} \\ \vdots & \ddots & \vdots \\ \hat{\sigma}_{\varepsilon 41} & \cdots & \hat{\sigma}_{\varepsilon 44} \end{pmatrix};$$

$$\hat{\sigma}_{\varepsilon pq} = \frac{1}{8} \sum_{Y=2000}^{2009} \left(\varepsilon_{p[Y]} - \bar{\varepsilon}_p \right) \left(\varepsilon_{q[Y]} - \bar{\varepsilon}_q \right);$$

$$\bar{\varepsilon}_p = \frac{1}{10} \sum_{Y=2000}^{2009} \varepsilon_{p[Y]};$$

and R_{pu} is the return on portfolio p during quarter u .

6.2.3 The value of Q_C was found to be 0,293. After adjustment for bias, we obtain:

$$Q_C^A = \frac{Q_C}{\left(1 + \frac{\hat{\gamma}_1^2}{\hat{\sigma}_{MM}} \right)} = 0,290.$$

As noted in Shanken (op. cit.),

$$F = T^2 \frac{m - n + 1}{mn}$$

follows the F distribution with n and $m - n + 1$ degrees of freedom, where T^2 has n and m degrees of freedom. Here $n = 2$ and $m = 8$, giving:

$$F=0,127$$

with 2 and 7 degrees of freedom. The corresponding p -value is 0,117, so on the basis of this test the CAPM cannot be rejected.

6.3 A NON-PARAMETRIC TEST OF THE SECURITIES MARKET LINE

6.3.1 The above test assumes that $\varepsilon_{i[Y]}$ is normally distributed. No such assumption is made in the statement of the CAPM. As observed in ¶5.1.6, the distribution of $\varepsilon_{i[Y]}$ does not appear to be normal. The test may therefore merely be rejecting that assumption, not the CAPM assumptions themselves.

6.3.2 Of the 100 values of $\varepsilon_{i[Y]}$, 40 are negative and 60 positive. Also, the median absolute value is 0,1510. The observed values of $\varepsilon_{i[Y]}$ may be grouped according to whether $\varepsilon_{i[Y]} \leq 0$ and according to whether $|\varepsilon_{i[Y]}| \leq 0,1510$. If

$$E\{\varepsilon_{i|Y}\} = 0$$

and the distribution of $\varepsilon_{i|Y} \leq 0$ is symmetric, then the median will be 0 and, for both non-positive and positive values, the expected numbers of absolute values greater than the median absolute value ($e_{-,>}$ and $e_{+,>}$ respectively) will be equal to the expected number of absolute values less than or equal to the median absolute value ($e_{-, \leq}$ and $e_{+, \leq}$ respectively). Table 4 shows the observed numbers o_j of values in each group $j \in \{-, \leq; -, >; +, \leq; +, >\}$, with the corresponding expected numbers e_j in brackets.

Table 4: Grouping of $\varepsilon_{i|Y}$ relative to 0 and of $|\varepsilon_{i|Y}|$ relative to its median value

	$\varepsilon_{i Y} \leq 0$	$\varepsilon_{i Y} > 0$	Total
$ \varepsilon_{i Y} \leq 0,150$	18 (20)	32 (30)	50
$ \varepsilon_{i Y} > 0,150$	22 (20)	28 (30)	50
Total	40	60	100

The statistic:

$$G = \sum_j \frac{(o_j - e_j)^2}{e_j}$$

has a χ^2 distribution with 1 degree of freedom. At the 5% level, the critical value of χ^2_2 is 5,991. The value of G is 0,667. The hypothesis can therefore not be rejected.

6.3.3 On this test we may therefore accept that the mean of $\varepsilon_{i|Y}$ is zero, and we cannot reject the CAPM.

6.4 TESTS OF LINEARITY IN BETA USING SECTORAL INDICES

6.4.1 A TEST OF THE EXPLANATORY POWER OF THE CAPM

6.4.1.1 Using the same method as that used by Michailidis et al. (op. cit.) in their study of the Greek stock market to test whether the CAPM explains excess rates of return, the excess return for each index was regressed for each period against the corresponding beta estimate. The relationship examined is equation (7), expressed in terms of estimated betas as:

$$r_{it} = \gamma_{r0} + \gamma_{r1} \hat{\beta}_{it} + \varepsilon_{it}$$

The results of the analysis for each period, and for all periods combined, are given in Table 5.

6.4.1.2 First, as indicated in Table 5, the values of R^2 for the tests for most of the periods are low, indicating that most of the risk is unsystematic. For all periods combined, the values are even lower than the average of the periods. As explained in ¶2.3.3, the standard CAPM predicts that $\gamma_{r0} = 0$ is zero; the null hypothesis is that $\gamma_{r0} = 0$ and the alternative hypothesis is that $\gamma_{r0} \neq 0$. The CAPM is rejected if the null hypothesis is rejected. With the exceptions of periods [2001], [2002], [2005] and [2007], the estimated

Table 5: Summary of regression analysis of excess returns on sectoral indices

Period	R^2	Estimate	Value	t -value	p -value
[2000]	0,012	$\gamma_{r,0}$	-0,017	-0,048	0,963
		$\gamma_{r,1}$	0,108	0,317	0,759
[2001]	0,593	$\gamma_{r,0}$	0,674	3,275	0,011
		$\gamma_{r,1}$	-0,656	-3,411	0,009
[2002]	0,503	$\gamma_{r,0}$	0,690	3,068	0,015
		$\gamma_{r,1}$	-0,615	-2,845	0,022
[2003]	0,260	$\gamma_{r,0}$	-0,390	-1,450	0,185
		$\gamma_{r,1}$	0,207	0,705	0,501
[2004]	0,019	$\gamma_{r,0}$	0,278	2,049	0,075
		$\gamma_{r,1}$	-0,062	-0,395	0,703
[2005]	0,150	$\gamma_{r,0}$	0,365	5,591	0,001
		$\gamma_{r,1}$	-0,091	-1,186	0,270
[2006]	0,160	$\gamma_{r,0}$	0,173	1,487	0,175
		$\gamma_{r,1}$	0,170	1,236	0,252
[2007]	0,018	$\gamma_{r,0}$	0,312	2,573	0,033
		$\gamma_{r,1}$	-0,048	-0,378	0,716
[2008]	0,003	$\gamma_{r,0}$	-0,193	-0,664	0,526
		$\gamma_{r,1}$	0,050	0,163	0,874
[2009]	0,114	$\gamma_{r,0}$	-0,424	-1,516	0,168
		$\gamma_{r,1}$	0,252	1,016	0,339
all: prior betas	0,001	$\gamma_{r,0}$	0,079	1,426	0,192
		$\gamma_{r,1}$	-0,003	-0,064	0,950
all: in-period betas	0,088	$\gamma_{r,0}$	0,035	0,732	0,485
		$\gamma_{r,1}$	0,051	0,406	0,406

values of $\gamma_{r,0}$ are not significantly different from zero, as the p -values (using a two-tailed test at 5%) are greater than 2,5%. One would not necessarily expect all periods to be insignificant at the 5% level as the probability of that is $0,95^{10} = 0,6$. But the probability of having at least four significant is about 0,1%, which is significant. Nevertheless, both for prior and for in-period betas, the results for all periods combined are not significant. The zero-beta version of the CAPM predicts that $\gamma_{r,0} > 0$. For this version of the CAPM the null hypothesis is that $\gamma_{r,0} = 0$ and the alternative hypothesis is that $\gamma_{r,0} < 0$. Again, the CAPM is rejected if the null hypothesis is rejected. Since the p -values for periods with negative values of $\gamma_{r,0}$ are all greater than 5% (using a one-tailed test), the null hypothesis is not rejected for any of the periods. For all periods combined the values are positive and relatively reasonable.

6.4.1.3 Both forms of the CAPM predict that $\gamma_{r1} > 0$. The null hypothesis is that $\gamma_{r1} = 0$ and the alternative hypothesis is that $\gamma_{r1} < 0$. The CAPM is rejected if the null hypothesis is rejected. The null hypothesis is rejected only for periods [2001] and [2002]. The probability that at least two periods will be significant at the 5% level is about 9%, so these results are inconclusive. This is confirmed by the all-periods tests. The zero-beta form of the CAPM also predicts that $\gamma_{r1} > \gamma_{r0}$. This prediction is not tested here, though it appears that there may be significant differences in [2005] and [2007]. However, the amount of systematic risk in those years is relatively low. Neither of the results for the all-period tests is significant; for prior betas the slope is slightly negative, suggesting a relatively flat securities market line as observed in ¶5.1.3.

6.4.1.4 On the basis of this test, we cannot reject the hypothesis either for the standard form of the CAPM or for the zero-beta form for all periods combined, though for a significant number of years we could do so for the standard form. We note, however, that, for most periods, most of the risk of each index is unsystematic.

6.4.2 A TEST FOR NONLINEARITY

6.4.2.1 As observed in ¶5.1, the plots of $r_{i[Y]}$ display little evidence of linearity. Following Michailidis et al. (op. cit.) (cf. ¶2.3.10 above), in order to test for nonlinearity between the excess returns on the sectoral indices and their beta estimates, the relationship examined, for each period, is:

$$r_{i[Y]} = \gamma_{r0} + \gamma_{r1}\hat{\beta}_{i[Y]} + \gamma_{r2}\hat{\beta}_{i[Y]}^2 + \varepsilon_{i[Y]}. \tag{31}$$

The results of the analysis for each period, and for all periods combined, are given in Table 6. The estimate values, *t*-values and *p*-values are those for γ_{r2} only; other coefficients are not relevant to this test.

Table 6: Summary of regression analysis of nonlinearity for sectoral indices

Period	R^2	$\hat{\gamma}_{r2}$	<i>t</i> -value	<i>p</i> -value
[2000]	0,622	3,119	3,358	0,012
[2001]	0,825	-1,267	-3,045	0,019
[2002]	0,528	-0,500	-0,614	0,559
[2003]	0,579	1 570	2,942	0,022
[2004]	0,098	-0,343	-0,782	0,460
[2005]	0,333	0,348	1,387	0,208
[2006]	0,164	-0,090	-0,164	0,875
[2007]	0,065	-0,227	-0,593	0,572
[2008]	0,579	0,039	0,042	0,967
all: prior betas	0,064	0,156	0,687	0,514
all: in-period betas	0,113	0,108	0,447	0,669

6.4.2.2 As expected, the value of R^2 increases for each period relative to that shown in Table 5. For the relationship between the return and the beta estimate to be linear, one would expect that $\gamma_{r2} = 0$. This is the null hypothesis; the alternative hypothesis is that $\gamma_{r2} \neq 0$. As indicated in Table 6, γ_{r2} is significantly non-zero in [2000], [2001] and [2003]. Again, one would not necessarily expect all periods to be insignificant at the 5% level. But the probability of having at least 3 is about 1%. However, for all periods combined, the results are not significant.

6.5 PERIOD-BY-PERIOD TESTS USING PORTFOLIOS OF SECTORAL INDICES

6.5.1 A TEST OF THE EXPLANATORY POWER OF THE CAPM

6.5.1.1 Using the same method as in section 6.4.1, the return for each portfolio, for each period $[Y]$, was regressed against the corresponding beta values. The relationship examined is the following:

$$r_{p[Y]} = \gamma_{r0} + \gamma_{r1} \hat{\beta}_{p[Y]} + \varepsilon_{p[Y]}. \quad (32)$$

The results of the analysis for each period, and for all periods combined, are given in Table 7.

6.5.1.2 As shown in Table 7, the values of R^2 for the tests for most of the periods are low, indicating that most of the risk is unsystematic. However, for all periods except [2001] and [2002], the value of R^2 has increased relative to Table 5. For all periods combined the values have increased remarkably. None of the estimated values of γ_{r0} is significantly different from zero; the p -values are all greater than 2,5%. Therefore, we cannot reject the null hypothesis that $\gamma_{r0} = 0$ against the alternative hypothesis that $\gamma_{r0} \neq 0$. The zero-beta version of the CAPM predicts that $\gamma_{r0} > 0$. Since the p -values for periods with negative values of γ_{r0} are all greater than 5% (using a one-tailed test), we cannot reject the null hypothesis that $\gamma_{r0} = 0$ against the alternative hypothesis that $\gamma_{r0} < 0$ for any of the periods.

6.5.1.3 As noted in section 6.4.1, both forms of the CAPM predict that $\gamma_{r1} > 0$. Again, the null hypothesis is that $\gamma_{r1} = 0$ and the alternative hypothesis is that $\gamma_{r1} < 0$. The null hypothesis is not rejected for any period. Again, for the zero-beta form of the CAPM, it is not tested here that $\gamma_{r1} > \gamma_{r0}$, though it appears that there may be significant differences in [2002] and (as for the sectoral indices) for [2005]. For all periods combined the slope is positive, but not significant.

6.5.1.4 On the basis of this test, we cannot reject the hypothesis either for the standard form of the CAPM or for the zero-beta form.

6.5.2 A TEST FOR NONLINEARITY

6.5.2.1 The same procedure as in section 6.4.2 was used to test for nonlinearity between the excess returns on the portfolios and their beta estimates. The relationship examined, for each period, is:

$$r_{p[Y]} = \gamma_{r0} + \gamma_{r1} \hat{\beta}_{p[Y]} + \gamma_{r2} \hat{\beta}_{p[Y]}^2 + \varepsilon_{p[Y]}. \quad (33)$$

The results of the analysis for each period, and for all periods combined, are given in Table 8. As in Table 6, the estimate values, t -values and p -values are those for $\gamma_{r,2}$ only.

6.5.2.2 As expected, the value of R^2 increases for each period over that shown in Table 7. This is the null hypothesis; the alternative hypothesis is that $\gamma_{r,2} \neq 0$. As indicated in Table 8, $\gamma_{r,2}$ is not significantly different from zero for any period. (The test being two-tailed, the critical value is 2,5%.) The probability of having at least 1 significant value is about 40%. For all periods combined the results are not significant. Hence, we cannot reject the hypothesis that the relationship between return and beta is linear.

Table 7: Summary of regression analysis of excess return on portfolios

Period	R^2	Estimate	Value	t -value	p -value
[2000]	0,022	$\gamma_{r,0}$	0,123	0,541	0,643
		$\gamma_{r,1}$	-0,047	-0,210	0,853
[2001]	0,532	$\gamma_{r,0}$	0,684	1,541	0,263
		$\gamma_{r,1}$	-0,633	-1,506	0,271
[2002]	0,112	$\gamma_{r,0}$	0,770	3,340	0,079
		$\gamma_{r,1}$	-0,692	-3,046	0,093
[2003]	0,811	$\gamma_{r,0}$	-0,574	-4,492	0,046
		$\gamma_{r,1}$	0,413	2,928	0,100
[2004]	0,151	$\gamma_{r,0}$	0,319	2,062	0,175
		$\gamma_{r,1}$	-0,109	-0,596	0,612
[2005]	0,431	$\gamma_{r,0}$	0,378	5,047	0,037
		$\gamma_{r,1}$	-0,113	-1,232	0,343
[2006]	0,932	$\gamma_{r,0}$	0,162	5,741	0,029
		$\gamma_{r,1}$	0,180	5,226	0,035
[2007]	0,263	$\gamma_{r,0}$	0,401	2,650	0,118
		$\gamma_{r,1}$	-0,136	-0,844	0,488
[2008]	0,105	$\gamma_{r,0}$	-0,393	-0,854	0,483
		$\gamma_{r,1}$	0,240	0,485	0,676
[2009]	0,384	$\gamma_{r,0}$	-0,376	-1,871	0,202
		$\gamma_{r,1}$	0,203	1,116	0,380
all: prior betas	0,417	$\gamma_{r,0}$	0,022	0,474	0,682
		$\gamma_{r,1}$	0,054	1,196	0,354
all: in-period betas	0,415	$\gamma_{r,0}$	0,026	0,601	0,609
		$\gamma_{r,1}$	0,064	1,190	0,356

Table 8: Summary of regression analysis of nonlinearity for portfolios

Period	R^2	$\hat{\gamma}_2$	t -value	p -value
[2000]	0,998	1,556	22,410	0,028
[2001]	0,999	-2,539	-21,570	0,030
[2002]	0,922	-1,088	-1,132	0,461
[2003]	0,854	0,448	0,547	0,681
[2004]	0,228	-0,397	-0,316	0,805
[2005]	0,779	0,480	1,255	0,428
[2006]	0,962	0,174	0,900	0,533
[2007]	0,908	-1,281	-2,640	0,230
[2008]	0,823	3,383	2,014	0,293
[2009]	0,384	-0,028	-0,028	0,982
all: prior betas	0,446	-0,071	-0,230	0,856
all: in-period betas	0,489	-0,165	0,380	0,769

7. SUMMARY AND CONCLUSION

7.1 In section 5.1, it appeared that, in most of the twelve-month periods from 2000 to 2009 there was little *prima-facie* evidence of linearity in the relationship between excess returns on sectoral indices and the betas of those indices. In two periods the relationship was disturbed by outliers and in two others, when the excess return was negative, the excess returns were widely scattered. For all periods combined the relationship appeared more linear, particularly for prior betas. The residuals did not appear to be normally distributed, nor to have a mean of zero.

7.2 In section 5.2 it appeared that, for prior betas, (as found by other authors) the relationship of excess returns to beta was flatter than predicted by the standard CAPM.

7.3 In section 6.1 a test using prior estimates of betas and of expected returns showed that, on the assumption that the error term in the return-generating process is normally distributed, the hypothesis that its mean was 0 could be rejected. However, that test made the questionable assumption that investors' *ex-ante* assumptions at any time are based on *ex-post* observations, without any cognisance of prospects at the time.

7.4 In section 6.2, at the other extreme, a test using in-period estimates of betas and expected returns showed that, on the assumption that the error term in the return-generating process is normally distributed, the hypothesis that its mean was 0 could not be rejected. The use of in-period estimates implies that those estimates are correct. In fact there may be biases in their estimation. However, as explained in ¶4.4.6, for the purpose

of testing whether the CAPM can be used in long-term models, a test based on in-period sample betas is relevant, since such a model can generate unbiased *ex-ante* betas.

7.5 Clearly, the reality lies between the assumptions of section 6.1 and those of section 6.2. An investor will not necessarily rely entirely on historical returns in estimating expected returns and betas. Nor, on the other hand, can it be assumed that the in-period sample represents a sample from investors' prior expectations. Without information as to how investors form expectations, it is not possible to reject the CAPM categorically. The results are therefore inconclusive, but considered together, they give no adequate grounds for rejection of the CAPM.

7.6 The tests in sections 6.1 and 6.2 assume that the residual in the securities market line is normally distributed. In the statement of the CAPM, however, no assumption of the normality of the residual was assumed. A non-parametric test reported in section 6.3 was unable to reject the hypothesis that its mean was zero.

7.7 The tests in sections 6.1 to 6.3 all test the residuals of the securities market line, calculated on the assumption that the expected returns are unbiased. As shown in ¶¶5.1.4–5, while sectoral betas were quite stable, average excess returns on the market portfolio varied quite widely over time. It is therefore preferable to test the relationship:

$$E\{r_{it}\} = \gamma_0 + \gamma_1 \hat{\beta}_{it}. \quad (34)$$

This is done in sections 6.4 and 6.5. Whereas the tests in section 6.1 to 6.3 use *ex-post* estimations of the expected return on the market, the tests reported in section 6.4 and 6.5 use in-period values, which are proxies for unbiased *ex-ante* estimations. The latter must therefore be regarded as preferable tests, except in that they necessarily imply that the residuals are normally distributed.

7.8 As explained in ¶4.5.1, the tests in section 6.4, being based on sectoral indices, may be strongly dependent on the REH. The tests in section 6.5 were based on portfolios of indices.

7.9 The tests in sections 6.4 and 6.5 showed that, with the exception of the explanatory test on portfolios in section 6.5.1, the CAPM could be rejected for certain periods. In general, the number of years for which it could be rejected was not significant. The two exceptions were the test of the standard CAPM on sectoral indices and the test for nonlinearity on sectoral indices. For all periods combined it was not possible to reject either the standard version of the CAPM or the zero-beta version. It may be that, in some years, market participants' expectations are more biased than in others; that of itself would not invalidate the CAPM, only the REH. From the tests of the explanatory power of beta in section 6.4.1 it may be noted that, for sectoral indices for all periods combined, most of the risk was unsystematic. However, for portfolios, R^2 was almost

50%. Also, the estimates of γ_{r_0} and γ_{r_1} were generally positive. The exception was for sectoral indices with prior betas, where they were slightly negative. It appears, therefore, that equation (7) cannot be reliably used as a basis of estimation of γ_{r_0} and γ_{r_1} with prior betas. However, as discussed in ¶¶7.4–5, the emphasis must be placed on in-period tests rather than on tests using prior betas.

7.10 The market portfolio, which is central to the testing of the CAPM, is unobservable in practice. Hence, a market proxy was required. The FTSE/JSE ALSI was used as a market proxy. Roll (op. cit.) and Bowie & Bradfield (op. cit.) warned that the choice of the wrong market proxy would reduce the predictive ability of the CAPM. It is possible, therefore, that, because of an inappropriate market proxy, the results obtained in this study are misstated.

7.11 In addition to this, the CAPM's empirical problem may be a result of its many simplifying assumptions and difficulties in implementing valid tests of the model. In light of these problems, it is clear that the results obtained are not entirely conclusive.

7.12 As explained in section 4.2, for the sake of comparability, and for use with nominal liabilities, this research focused on nominal returns. For actuarial applications it would be helpful to reconsider the CAPM in South Africa in real terms. This is left to further research.

7.13 As mentioned in sections 2.9 and 3.2, actuarial applications will generally require at least the inclusion of bonds in the market portfolio. This paper is therefore to be seen as a preliminary exercise: it needs to be extended to include bonds and other assets in which long-term financial institutions may invest, both in the composition of the market portfolio and in tests of the securities market line. This, too, is left to further research.

7.14 This study showed that while, on the assumption that the residuals of the return-generating function are normally distributed, the CAPM could be rejected for certain periods, the use of that model for long-term actuarial modelling in the South African market can be reasonably justified. While the zero-beta version better supports the relatively flat securities market line observed both in the literature and in this study than the standard version, the standard version is not generally rejected. For the purposes of actuarial modelling, the standard version has advantages of parsimony, which must be weighed up by the practitioner against the zero-beta version's advantages of fidelity.

7.15 So, to return to the motto of this paper, "Announcements of the 'death' of beta seem premature." (Black, 1972)

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