Actuarial Society of South Africa

EXAMINATION

SEMESTER 2 2021
EXAMINER COMMENTS

Subject A213 — Contingencies

WRITTEN EXAMINATION

Generally, students would have scored better marks where the following exam techniques have been applied:

- Label each question clearly. Some students marked their page numbers as well as question numbers next to each other which caused confusion at times.
- Start each question on a new page.
- Use more space rather than less and make it clear how you break up key parts of a question. This makes it easier to see your workings and to understand what was done to give principal marks for incorrect solutions.
- Show the results of your intermediate steps especially any values of actuarial functions.
- Student should try to avoid loading duplicated pages and also load the pages in the correct order to reduce confusion.

Overall, the results from this semester's exams were an improvement from the previous which was encouraging.

QUESTION 1

This question was standard bookwork in which many students scored well.

Average mark: 60-65%

Let X denote the present value of this benefit, then:

$$X = \begin{cases} 0 & if \quad K_x < m \quad or \quad K_x \geq m+n \\ v^{K_x+1} & if \quad m \leq K_x < m+n \end{cases}$$

Where K_x is the curtate future lifetime of a life aged x. K_x represents the complete number of years a life currently aged x survives for, before dying.

QUESTION 2

This question was a simple proof even though some student tried to overcomplicate it somewhat. The best question across the paper.

Average mark: 70-75%

$$\bar{a}_{x:\overline{n|}} = \bar{a}_x - v^n \cdot {}_n p_x \cdot \bar{a}_{x+n}$$

$$\approx \ddot{a}_x - \frac{1}{2} - v^n \cdot {}_n p_x \left(\bar{a}_{x+n} - \frac{1}{2} \right)$$

since $$\bar{a}_x \approx \ddot{a}_x - \frac{1}{2}$$

$$= \bar{a}_x - v^n \cdot {}_n p_x \cdot \bar{a}_{x+n} - \frac{1}{2}\left(1 - v^n \cdot {}_n p_x\right)$$

but $\ddot{a}_x - v^n \cdot {}_n p_x \cdot \ddot{a}_{x+n} = \ddot{a}_{x+\overline{n|}}$

so $$\bar{a}_{x:\overline{n|}} = \ddot{a}_{x+\overline{n|}} - \frac{1}{2}\left(1 - v^n \cdot {}_n p_x\right)$$

QUESTION 3

A straightforward question testing basic application. Most students did well. Some students lost marks for stopping at the variance calculation.

Average mark: 70-75%

Expected Present Value:

$$EPV = 100\,000 \cdot A^{@4\%}_{[35]:\overline{10}|}$$

Now $$A_{[35]:\overline{10}|} = A^{\ \ 1}_{[35]:\overline{10}|} + A_{[35]:\overline{10}|}^{\quad\ \ 1}$$

$$= A_{[35]} - v^{10} \cdot {}_{10}p_{[35]} \cdot A_{45} + v^{10} \cdot {}_{10}p_{[35]}$$

$${}_{10}p_{[35]} = \frac{l_{45}}{l_{[35]}} = \frac{9801.3123}{9892.9151}$$

$$= 0.990741$$

$$A_{[35]} = 0.19207$$

$$A_{45} = 0.27605$$

So $$EPV = 100\,000(0.19207 - 0.18476 + 0.66931)$$

$$= R\,67\,661.61$$

Standard Deviation

$$Stdev = \sqrt{Var}$$

Now Variance $$= 100\,000^2\left({}^2A_{[35]:\overline{10}|} - \left(A_{[35]:\overline{10}|}\right)^2 \right)$$

with $${}^2A_{[35]:\overline{10}|} = {}^2A^{\ \ 1}_{[35]:\overline{10}|} + {}^2A_{[35]:\overline{10}|}^{\quad\ \ 1}$$

$$= {}^2A_{[35]} - \left(v^2\right)^{10} \cdot {}_{10}p_{[35]} \cdot {}^2A_{45} + \left(v^2\right)^{10} \cdot {}_{10}p_{[35]}$$

$${}^2A_{[35]} = 0.04861$$

$${}^2A_{45} = 0.09458$$

So $${}^2A_{[35]:\overline{10}|} = (0.04861 - 0.04277 + 0.45216)$$

$$= 0.45801$$

Hence the S*tdev* $$= 100\,000 \cdot \sqrt{0.45801 - 0.67662^2}$$

$$= R\,1\,401.03$$

QUESTION 4

This question was done well by the well prepared students but some scored very poorly. Students with laid out answers, showing the values of the actuarial functions separated, scored better marks.

Average mark: 45-50%

$$EPV = 200\,000\left(\bar{a}_{\overline{50:55}} - \bar{a}_{50:55}\right) + 200\,000 \cdot \bar{a}_{\overline{3|}} \cdot \bar{A}_{\overline{50:55}}$$

Now $$\bar{a}_{\overline{50:55}} = \bar{a}^{m}_{50} + \bar{a}^{f}_{50} - \bar{a}_{50:55}$$
$$= (18.843 - 0.5) + (18.210 - 0.5) - (16.909 - 0.5)$$
$$= 19.644$$

$$\bar{A}_{\overline{50:55}} = 1 - \delta \cdot \bar{a}_{\overline{50:55}}$$
$$= 1 - \ln(1.04)(19.644)$$
$$= 0.22955$$

Therefore $$EPV = 200\,000 \cdot \frac{1 - v^3}{\ln(1.04)} \cdot 0.22955$$
$$= R\,776\,935$$

QUESTION 5

Student performance on this question was disappointing seeing that it is a straightforward application of the work. Well laid out answers scored better.

Average mark: 55-60%

i. EPV of premiums

$$12 \cdot P\ddot{a}^{(12)}_{45:\overline{20|}}$$

where $$\ddot{a}^{(12)}_{45:\overline{20|}} = \ddot{a}^{(12)}_{45:\overline{20|}} - \frac{11}{24}\left(1 - v^{20} \cdot {}_{20}p_{45}\right)$$
$$= 13.780 - \frac{11}{24}\left(1 - v^{20} \cdot {}_{20}p_{45}\right)$$
$$= 13.780 - \frac{11}{24}\left(1 - 0.41075\right)$$
$$= 13.50993$$

thus $$12 \cdot P\ddot{a}^{(12)}_{45:\overline{20|}} = 162.1191P$$

ii. EPV of benefits

$$350\,000\cdot \bar{A}^{\;1}_{45:\overline{20|}} + 50\,000\left(I\bar{A}\right)^{\;1}_{45:\overline{20|}}$$

$$\bar{A}^{\;1}_{45:\overline{20|}} = \bar{A}_{45} - v^{20}\cdot {}_{20}p_{45}\cdot \bar{A}_{65}$$

$$=\left(1.04\right)^{0.5}\left(A_{45} - v^{20}\cdot {}_{20}p_{45}\cdot A_{65}\right)$$

$$=\left(1.04\right)^{0.5}\left(0.05923\right)$$

$$=0.0604$$

$$\left(I\bar{A}\right)^{\;1}_{45:\overline{20|}} = I\bar{A}_{45} - v^{20}\cdot {}_{20}p_{45}\left(20\bar{A}_{65} + I\bar{A}_{65}\right)$$

$$=\left(1.04\right)^{0.5}\left(I\bar{A}_{45} - v^{20}\cdot {}_{20}p_{45}\left(20A_{65} + IA_{65}\right)\right)$$

$$=\left(1.04\right)^{0.5}\left(8.33628 - v^{20}\cdot {}_{20}p_{45}\left(20\left(0.52786\right) + 7.89442\right)\right)$$

$$=0.77224$$

Thus EPV of benefits

$$= R\,59\,753.05$$

iii. EPV of expenses

$$1000 + 0.25\cdot 12P + 120\left(\ddot{a}^{@6\%}_{45:\overline{20|}} - 1\right) + 0.05\cdot 12P\cdot \ddot{a}^{(12)}_{45:\overline{20|}} - 0.05P + 300\cdot \bar{A}^{@6\%\;1}_{45:\overline{20|}}$$

$$\ddot{a}^{@6\%}_{45:\overline{20|}} = 11.884$$

$$\bar{A}^{@6\%\;1}_{45:\overline{20|}} = \left(1.06\right)^{0.5}\cdot A^{@6\%\;1}_{45:\overline{20|}}$$

$$=\left(1.06\right)^{0.5}\left(0.32731 - v^{20}\cdot {}_{20}p^{@6\%}_{45}\right)$$

$$0.190476$$

EPV of expenses

$$= 2363.223 + 11.05596P$$

iv. Equation of value $162.1191P = 59753.05 + 2363.223 + 11.05596P$ thus $P = R\,441.194$

QUESTION 6

This question was one of the hardest question across the paper. In part i many students did not give an equation for the profit over a year but rather the profit over the lifetime of the policy. Many students were not able to do the mortality profit calculation at a portfolio level. Good exam technique was shown by some who attempted part iv even though other parts were done incorrectly, and thereby scoring method marks.

Average mark: 30-35%

i. Profit $=\left({}_tV+G-e\right)\left(1+i\right)-q_{x+t}\cdot\left(S+f\right)-\left(1-q_{x+t}\right)\cdot{}_{t+1}V$ where

${}_tV$	- Gross premium reserve at time t
$G-e$	- Premium less expenses paid at t
$q_{x+t}\cdot\left(S+f\right)$	- Expected claims plus expense paid at $t+1$
${}_{t+1}V$	- Gross premium reserve at time $t+1$

ii.

$$EDS=\sum_{All\ Policies} q_{55}\left[S-{}_{56}V\right]$$

$$=\sum_{All\ Policies} q_{55}\left[S-\left(S\left(A_{56:\overline{9}|}+v^9\cdot{}_9p_{56}\right)-P\cdot\ddot{a}_{56:\overline{9}|}\right)\right]$$

$$=q_{55}\left[\left(\sum S\right)-\left(\sum S\right)\left(A_{56:\overline{9}|}+v^9\cdot{}_9p_{56}\right)+\left(\sum P\right)\ddot{a}_{56:\overline{9}|}\right]$$

$$=q_{55}\left[\left(10\,000\,000\right)-\left(10\,000\,000\right)\left(A_{56:\overline{9}|}+v^9\cdot{}_9p_{56}\right)+\left(600\,000\right)\ddot{a}_{56:\overline{9}|}\right]$$

Now $A_{56:\overline{9}|}=0.60160$

$\ddot{a}_{56:\overline{9}|}=7.038$

$q_{55}=0.004469$

Hence $EDS=R\,12\,153.13$

iii.

$$ADS=\sum_{Claims}\left[S-{}_{t+1}V\right]$$

$$=\sum_{Claims}\left[S-\left(S\left(A_{56:\overline{9}|}+v^9\cdot{}_9p_{56}\right)-P\cdot\ddot{a}_{56:\overline{9}|}\right)\right]$$

$$=\sum_{Claims}S-\left(\sum_{Claims}S\right)\left(A_{56:\overline{9}|}+v^9\cdot{}_9p_{56}\right)+\sum_{Claims}P\cdot\ddot{a}_{56:\overline{9}|}$$

$$=1\,000\,000-\left(1\,000\,000\right)\left(A_{56:\overline{9}|}+v^9\cdot{}_9p_{56}\right)+(55\,000).\ddot{a}_{56:\overline{9}|}$$

$$=R\,236\,752.86$$

iv.

$$\text{Mortality Loss} = EDS - ADS$$
$$= R\,224\,599.73$$

The insurer has made a mortality loss.

QUESTION 7

This question was testing bookwork in which many students did not attempt.

Average mark: 20-25%

i.
Unitised (accumulating) with-profits contracts

- Many companies that sell AWP administer the contract in unitised form (called unitised with-profits (UWP)).
- The policyholder is allocated units, and the fund value at any time for any policy is equal to the number of units held multiplied by the current price (or value) of each unit at that time.

In this way, UWP operates in a very similar way to unit-linked contracts.

ii.
A key difference is the way the unit price is calculated. Two example possibilities are:
Method (1)
the unit price allows for guaranteed bonus interest increases only; the discretionary bonus is credited to the policy by awarding additional (bonus) units from time to time
Method (2) the unit price allows for both guaranteed and bonus interest increases.
In both cases, it would be normal for unit prices to be changing on an effectively continuous basis (eg daily). The company would declare its regular bonus interest rate in advance, so that interest would accrue to policies at the equivalent daily rate.

QUESTION 8

This question was done well by well prepared students but many struggled. Part i was standard bookwork with part ii standard application. In part iii many students felt the answer should be the same because the basis for the prospective and retrospective reserves are the same, failing to appreciate that the basis is different to the pricing basis which means that the answer would not be the same.

Average mark: 40-45%

i. The accumulated value allowing for interest and survivorship of the premiums received to date
less
the accumulated value allowing for interest and survivorship of the benefits and expenses paid to date.

ii.

$$ {}_{20}V^{Retro} = \frac{(1+i)^{20}}{{}_{20}p_{30}} \left[0.97 \cdot 12P \cdot \ddot{a}^{(12)}_{30:\overline{20}|} - 0.17P - 1200 - 500500 \cdot A^{1}_{30:\overline{20}|} \right] $$

Now,

$$ \begin{aligned} A^{1}_{30:\overline{20}|} &= A_{30} - v^{20} \cdot {}_{20}p_{30} \cdot A_{50} \\ &= 0.07328 - v^{20} \cdot {}_{20}p_{30} \cdot 0.20508 \\ &= 0.010708 \end{aligned} $$

$$ \begin{aligned} \ddot{a}^{(12)}_{30:\overline{20}|} &= \ddot{a}_{30:\overline{20}|} - \frac{11}{24}\left(1 - v^{20} \cdot {}_{20}p_{30}\right) \\ &= \ddot{a}_{30} - v^{20} \cdot {}_{20}p_{30} \cdot \ddot{a}_{50} - \frac{11}{24}\left(1 - v^{20} \cdot {}_{20}p_{30}\right) \\ &= 16.372 - v^{20} \cdot {}_{20}p_{30}(14.044) - \frac{11}{24}\left(1 - v^{20} \cdot {}_{20}p_{30}\right) \\ &= 11.76856 \end{aligned} $$

Thus ${}_{20}V^{Retro} = R\,202\,709.64$

iii. The prospective reserve would have been smaller. At 4% both would be the same but

$$ V^{\text{Re}tro}_{@6\%} > V^{\text{Re}tro}_{@4\%} = V^{\text{Pr}o}_{@4\%} > V^{\text{Pr}o}_{@6\%} $$

since retrospective reserves are accumulating premiums in excess of claims and expenses and higher interest leads to higher reserves but prospective reserves are meeting the excess of future claims over future premiums and higher interest leads to lower reserves.

QUESTION 9

Students lost marks in part i mainly because they only referred to the policy and not the function and therefore did not say that it is the "expected present value". The majority did well in part ii a) with many struggling with part ii b). Part iii was done poorly.

Average mark: 55-60%

i. The expected present value of a continuous assurance for a sum assured of 20 000 calculated at a force of interest on 2 lives aged *x* and *y* whereby the sum is paid on the death of *y* only if life aged *y* dies after life aged *x*.

ii. a)

$$ {}_t p_{40} = e^{-\int_0^t 0.05 ds} = e^{-0.05t} $$

and

$$ {}_t p_{50} = e^{-0.06t} $$

also $\delta = \ln(1.083287) = 0.08$

$$ 20\,000\bar{A}^{\;\;2}_{50:40} = 20\,000\int_0^\infty v^t \cdot {}_t p_{40}\left(1 - {}_t p_{50}\right)\cdot \mu_{40+t} dt $$

$$ = 20\,000\int_0^\infty e^{-0.08t} \cdot e^{-0.05t}\left(1 - e^{-0.06t}\right)\cdot 0.05 dt $$

$$ = 20\,000(0.05)\int_0^\infty \left(e^{-0.13t} - e^{-0.19t}\right)dt $$

$$ = 1000\left[\frac{-e^{-0.13t}}{0.13} + \frac{e^{-0.19t}}{0.19}\right]_0^\infty $$

$$ = 1000\left[\frac{1}{0.13} - \frac{1}{0.19}\right] $$

$$ = R\,2429.15 $$

b) For the premium we require $\bar{a}_{\overline{50:40}}$

$$
\begin{aligned}
\bar{a}_{\overline{50:40}} &= \int_0^\infty v^t \left[1-\left(1-{}_t p_{40}\right)\left(1-{}_t p_{50}\right)\right]dt \\
&= \int_0^\infty e^{-0.08t} \left[1-\left(1-e^{-0.05t}\right)\left(1-e^{-0.06t}\right)\right]dt \\
&= \int_0^\infty e^{-0.08t} \left[e^{-0.05t} + e^{-0.06t} - e^{-0.11t}\right]dt \\
&= \int_0^\infty \left(e^{-0.13t} + e^{-0.14t} - e^{-0.19t}\right)dt \\
&= \left[\frac{e^{-0.13t}}{0.13} + \frac{e^{-0.14t}}{0.14} - \frac{e^{-0.19t}}{0.19}\right]_0^\infty \\
&= \frac{1}{0.13} + \frac{1}{0.14} - \frac{1}{0.19} \\
&= 9.572
\end{aligned}
$$

Therefore $P = R\ 253.78$

iii.

If the life age 40 dies first the policy ceases without benefit yet the premium is expected to be maintained by the life aged 50 so long as they survive. There is no incentive to continue.

The sensible option would be to establish the premium paying period as ceasing on the death of the life aged 40.

A single premium is possible as an alternative if affordable.