ACTUARIAL SOCIETY
OF SOUTH AFRICA

WRITTEN EXAMINATION

OCTOBER 2023

MEMORANDUM

Subject A213 — Contingencies
Intermediate Technical

General comments:

Overall, the performance of the paper was lower than previous sessions. Students continued to struggle with theoretical bookwork questions where answers were often incomplete. Students are encouraged to show their workings clearly and to show the calculation results of intermediate steps as this enhances the ability of the marker to give partial credit where due.

QUESTION 1

An insurer would apply a basis more cautious than best-estimate to

- allow a contingency margin to ensure a high probability that the premiums plus interest income meet the cost of benefits, allowing for random variation
- ensure a high probability of making profit
- allow for uncertainty in the estimates themselves
- meet a regulatory requirement

Other valid points are accepted.

Examiner comments:

Question performance: 60-65%

Students performed satisfactorily in this question.

QUESTION 2

(i) Under with-profit policies, the contract between policyholders and the company is such that, whatever surplus arises on the underlying fund, is to be shared between policyholders and shareholders in a predetermined manner. As the surplus arises over future years, the same is shared with policyholders in the form of bonuses and hence the need of bonuses under with-profit policies

- to reward policyholders for sharing in the risk
- to make a product attractive and attract new customers
- to protect the benefit against the eroding effect of inflation

Award marks for all reasonable points

(ii) Terminal bonuses could be used as a strategy to build lower guarantees over the lifetime of with-profit policies

For example, if the proportion of regular bonuses and terminal bonuses is skewed towards terminal bonuses, then it would mean lower guarantees to policyholders hence the insurer would have greater freedom in terms of choosing the investment assets.

By doing this they can take the risk of investing in real assets like equities and property which can lead to better final returns to policyholders, which would be distributed through higher terminal bonuses.

Award marks for all reasonable points

Examiner comments:

Question performance: 45-50%

Performance for this question was below expectations for a standard bookwork question.

QUESTION 3

$$ {}_3V = 2\,000\ 800 \times T\bar{A}_{58:58:7} - 4\,000 \times 0.97 \times \ddot{a}^{(12)}_{58:58:\overline{7}|} $$

Where

$$ \ddot{a}_{58:58:\overline{7}|} = \ddot{a}_{58:58} - \frac{l^m_{65}}{l^m_{58}} \times \frac{l^f_{65}}{l^f_{58}} \times v^7 \times \ddot{a}_{65:65} $$

$$ = 14.891 - \frac{9647.797}{9864.803} \times \frac{9703.708}{9881.764} \times 1.04^{-7} \times 11.958 $$

$$ = 6.16394 $$

$$ \ddot{a}^{(12)}_{58:58:\overline{7}|} = \ddot{a}_{58:58:\overline{7}|} - \frac{11}{24} \times (1 - \frac{l^m_{65}}{l^m_{58}} \times \frac{l^f_{65}}{l^f_{58}} \times v^7) $$

$$ = 6.16394 - \frac{11}{24}((1 - \frac{9647.797}{9864.903} \times \frac{9703.708}{9881.764} \times 1.04^{-7}) $$

$$ = 6.040099 $$

$$ T\bar{A}_{58:58:7} = 1.04^{0.5} \times (1 - d\ddot{a}_{58:58:\overline{7}|} - \frac{l^m_{65}}{l^m_{58}} \times \frac{l^f_{65}}{l^f_{58}} \times v^7) $$

$$ = 1.04^{0.5}(1 - 0.0384615 \times 6.16394 - 0.72981) $$

$$ = 1.04^{0.5} \times 0.0331161 $$

$$ = 0.03377194 $$

$$ {}_3V = 2\ 000\ 800 \times 0.03377194 - 4\ 000 \times 0.97 \times 6.040099 $$

$$ = 67\ 570.90 - 23\ 435.584 $$

$$ = 44\ 135.32 $$

Examiner comments:

Question performance: 65-70%

Students performed well in this question.

QUESTION 4

(i) $0.95P \times \ddot{a}_{[35]:\overline{25|}} = 100\,000A_{[35]:\overline{25|}} + 0.55P$

$0.95P \times 13.392 = 100\,000 \times 0.24198 + 0.55P$

$P = 1\ 987.94$

(ii) Reserve per policyin force at 31.12.2022

$_{11}V = 100\,000A_{46:\overline{14|}} \times 0.95P\ddot{a}_{46:\overline{14|}}$

$= 100\,000 \times 0.45028 - 0.95 \times 1987.94 \times 9.712$

$= 26\ 686.47$

Retrospective approach also accepted

(iii) Surrender value at 31.12.2022 is $_{11}V^{Retro}$

$$_{11}V^{Retro} = (1.04)^{11} \times \frac{l_{[35]}}{l_{46}} \times (0.95 \times P \times \ddot{a}_{[35]:\overline{11|}} - 0.55P - 100000(TA)_{[35]:\overline{11|}})\ @4\%$$

where

$l_{[35]} = 9\ 892.9151$ $\quad \ddot{a}_{[35]} = 21.006$ $\quad \ddot{a}_{46} = 18.563$ $\quad v_{4\%}^{11} = 0.649580932$

$l_{46} = 9\ 786.9534$ $\quad A_{[35]} = 0.19207$ $\quad A_{46} = 0.28605$

$$\ddot{a}_{[35]:\overline{11|}} = \ddot{a}_{[35]} - v_{4\%}^{11}\frac{l_{46}}{l_{[35[}}\ddot{a}_{46}$$

$$= 21.006 - 0.649580932 \times \frac{9786.9534}{9892.9151} \times 18.563$$

$$= 9.076982634$$

$$(TA)_{[35]:\overline{11|}} = A_{[35]} - v_{4\%}^{11}\frac{l_{46}}{l_{[35[}}A_{46}$$

$$= 0.19207 - 0.649580932 \times \frac{9786.9534}{9892.9151} \times 0.28605$$

$$= 0.008247589$$

therefore

$$_{11}V^{Retro} = (1.04)^{11} \times \frac{9\,892.9151}{9\,786.9534} \times (0.95 \times 1\ 987.94 \times 9.076982634 - 0.55 \times 1\ 987.94$$

$$-100000 \times 0.008247589)$$

$$= 23\ 690.62$$

(iv) The retrospective and prospective reserve at 31.12.2022 are different. This is due to the different discount rates being used. The retrospective reserve using a 4% discount rate will yield a lower value. Had the same discount rate been used, then the value would have been the same as in (ii) above.

(v) Number of in - force policies at start of 2022 = 50 000 000 / 100 000 = 500

Number of policyholders that died in 2022 = 400 000 / 100 000 = 4

Number of policies surrendered in 2022 = 2 000 000 / 100 000 = 20

Number of policies at end of 2022 = 500 - 4 - 20 = 476

Total funds available as at 31.12.2022 before setting up reserves and claim payments

$A = (500 \times (P + {}_{10}V) - 150\,000) \times (1.085) = 13\ 604\ 335.35$

Total claims paid in 2022

$B = 400\,000 + (20 \times 23\ 690.62) = 873\ 812.39$

Total reserves at 31.12.2022

$C = 476 \times {}_{11}V = 476 \times 26\,686.47 = 12\ 702\,760.21$

$\text{Profit for 2022} = A - B - C = 13\ 604\ 335.35 - 873\ 812.39 - 12\ 702\ 760.21$

$= 27\ 763.75$

(vi) The two main sources of *profit* are interest and surrenders:

- A profit is made when the investment returns earned on the reserves are higher than what was assumed
- The assets backing the reserves earned a return of 8.5% p.a. but it was assumed that the reserves would earn 6% pa.
- The margin earned above the assumed return leads to an investment profit

- No surrenders were allowed for in the reserving basis.
- Higher surrenders would lead to a profit
- ….as the benefit paid was based on a reserve calculated on a lower discount rate (4%)
- …but the reserve held was calculated at 6% but earned 8.5%.

Examiner comments:

Question performance: 40-45%

Students struggled with this question especially from part iii onwards. A common issue was using the incorrect time at which the reserve should be calculated. Many students did not attempt part v and vi.

QUESTION 5

Solution

(i) EPV of benefits

$750000(q_{45} v^{0.5} + {}_{1|} q_{45} (1+b)v^{1.5} + ... + {}_{19|} q_{45} (1+b)^{19} v^{19.5})$

where $b = 0.06$

$= 750000 \frac{(1.06)^{0.5}}{(1,06)} \times (q_{45} v(1.06) + {}_{1|} q_{45} (1.06)^2 v^2 + ... + {}_{19|} q_{45} (1.06)^{20} v^{20})$

$= 750000 \times (1.06)^{-0.5} \times (TA)_{45:\overline{20|}} \quad @i' \text{ where } i' = \frac{1.06}{1.06} - 1 = 0\%$

$= 750000 \times (1.06)^{-0.5} \times (A_{45} - v^{20} \times \frac{l_{65}}{l_{45}} \times A_{65}) \quad @i'$

but

$A_{45} = A_{65} = v^{20} = 1 \text{ for } i' = 0\%$

$l_{45} = 9\ 801.3123 \text{ and } l_{65} = 8\ 821.2612$

$= 750000 \times (1.06)^{-0.5} \times (1 - 1 \times \frac{8\ 821.2612}{9\ 801.3123} \times 1)$

$= 750000 \times (1.06)^{-0.5} \times 0.099991825$

$= 72\ \ 840.48$

(ii) Let P denote the monthly premium

EPV Premiums $= 12P\ddot{a}^{(12)}_{45:\overline{20|}} \quad @6\%$

$= 12P \times (\ddot{a}_{45:\overline{20|}} - \frac{11}{24} \times (1 - v^{20} \times \frac{l_{65}}{l_{45}}))$

$= 12P * (11.884 - \frac{11}{24} \times (1 - \frac{8821.2612}{9801.3123} \times 1.06^{-20}))$

$= 12P \times 11.554287$

$= 138.6514P$

EPV of expenses at 6% unless stated otherwise

$$= 0.5P + 4000 + 0.05\times 12\left(P\ddot{a}^{(12)}_{45:\overline{20|}} - \frac{1}{12}\right) + 3000\times 1.06^{0.5}\times (TA)_{45:\overline{20|}} + 750(\ddot{a}^{0\%}_{45:\overline{20|}} - 1)$$

where

$$(TA)_{45:\overline{20|}} = (A_{45} - v^{20}\times\frac{l_{65}}{l_{45}}\times A_{65})$$

$$= 0.15943 - \frac{8821.2612}{9801.3123}\times 1.06^{-20}\times 0.40177$$

$$= 0.0466826$$

$$\ddot{a}^{0\%}_{45:\overline{20|}} - 1 = \frac{(l_{46} + l_{47} + ... + l_{64})}{l_{45}}$$

$$= e_{45} - \frac{l_{64}}{l_{45}}e_{64}$$

$$= 34.271 - \frac{8934.8771}{9801.3123}\times 17.421$$

$$= 18.390015$$

$$EPVexpenses = 0.5P + 4000 + 0.05\times\left(138.6514P - \frac{1}{12}\right) + 3000\times 1.06^{0.5}\times 0.0466826 + 750\times 18.390015$$

$$= 7.42840333P + 17936.69929$$

$$138.6514P = 72\,840.48 + 12.33257P + 17\ 936.69929$$

$$P(138.6514 - 7.42840333) = 72\,840.48 + 17\ 936.69929$$

$$P = 90\,777.17929 / 131.223$$

$$P = 691.78$$

(iv) Premiums would be received later and hence are invested over a shorter period
The present value of premiums would be lower (total premium income is lower due to discounting)
In addition, we would expect to receive fewer premiums (e.g. a policyholder can die in the first month and then we will not receive any premium)
A higher premium would thus be payable

Award marks for all reasonable points

Examiner comments:

Question performance: 50-55%

Performance in this question was average. Here student will well formatted and set out answers, where the key steps are clear, performed better than others.

QUESTION 6

Second moment for first benefit $E\left[X^2\right]$

$$= (250\,000)^2 \times ({}^2A_{45} - v^{15}\frac{l_{60}}{l_{45}}\,{}^2A_{60}) \text{ at i = 8.16\%}$$

$$= (250\,000)^2 \times (0.09458 - 0.30832 \times \frac{9\,287.2164}{9\,801.3123} \times 0.23723)$$

$$= (250\,000)^2 \times 0.02527401$$

Second moment for second benefit

$$= (400\,000)^2 \times \left[v^{15}\frac{l_{60}}{l_{45}}\,{}^2A_{60} + v^{30}_{4\%}\left(1 - \frac{l_{60}}{l_{45}}\right)\right] \text{ at i = 8.16\%}$$

$$= (400\,000)^2 \times 0.30832 \times \frac{9\,287.2164}{9\,801.3123} \times 0.23723 + 0.30832\left(1 - \frac{9\,287.2164}{9\,801.3123}\right)$$

$$= (400\,000)^2\left[0.069305989 + 0.01617192\right]$$

$$\text{Second moment} = (250\ 000^2 \times 0.02527401) + (400\ 000^2 \times 0.08547791)$$

$$= 1,525609 \times 10^{10}$$

$$\text{Stand deviation} = \sqrt{(1,525609 \times 10^{10} - (105\ 032.04)^2)}$$

$$= 64\ 995.09$$

Examiner comments:

Question performance: 40-45%

Performance in this question was disappointing. Many students did not attempt the question and most who did, did not allow for the second benefit correctly.

QUESTION 7

(i) $(aq)_x^j = \frac{\mu_x^j}{\mu_x^d + \mu_x^w} * (1 - e^{-(\mu_x^d + \mu_x^w)})$

assuming that forces of decrement are constant at all years

$(aq)_x^d = \frac{0.01}{(0.01 + 0.05)} \times (1 - e^{-0,06}) = 0.0097059$

$(aq)_x^w = \frac{0.05}{(0.01 + 0.05)} \times (1 - e^{-0,06}) = 0.048530$

(ii)

$(ap)_x = 1 - (aq)_x^d - (aq)_x^w = 0.941765$

$$\text{EPV death} = 1000\,000((aq)_x^d v_{9\%} + (ap)_x (aq)_{x+1}^d v_{9\%}^2)$$
$$= 1000\,000(0.0097059 \times (1.09^{-1}) + 0.941765 \times 0.0097059 \times (1.09^{-2}))$$
$$= 1000\,000(0.016598035)$$
$$= 16\ 598.04$$

$$\text{EPV maturity} = 250\,000 \times (ap)_x \times (ap)_{x+1} \times v_{9\%}^2$$
$$= 250\,000 \times 0.941765 \times 0.941765 \times (1.09^{-2})$$
$$= 186\ 625.80$$

$$\text{EPV benefits} = 16\,598.04 + 186\,625.80 = 203\ 223.8$$

for completeness (Question strictly required quantification of benefits only)

$$\textit{EPV premiums} = 110\,000[1 + (ap)_x \times v_{9\%}]$$
$$= 110\,000 \times 1.864004159$$
$$= 205\ 040.46$$

$$NPV = 205\ 040.46 - 186\ 625.80 - 16\ 958.04$$
$$= 1\ 816.63$$

Examiner comments:

Question performance: 45-50%

Performance in this question was disappointing. Part i was done well. Many students tried to follow an overcomplicated method in part ii.

END

