Actuarial Society of South Africa

EXAMINATION SOLUTIONS AND EXAMINER COMMENTS

OCTOBER 2022

Subject A213 — Contingencies

QUESTION 1

Explain the terms 'unit fund' and 'non-unit fund' in the context of a unit-linked life assurance contract and the various items that make up the non-unit fund.

[Total 4]

SOLUTION:

The unit fund is the amount held in units on behalf of the policyholder at any time.
It may not necessarily be the amount that the policyholder is entitled to at that time. For example, if the policy is surrendered, the policyholder may receive only a proportion of the full bid value of the units, and on death there may be a guaranteed minimum sum assured which means that more than the unit fund value might be paid.
On death, maturity or surrender, the units held will be used to pay the benefit. Any excess/ shortfall in the unit fund will give rise to a positive/negative cashflow in the non-unit fund.

The amount of money in the non-unit fund is the net result of the life office's (non-unit) cashflows.
These cashflows arise from the following sources:

- premium less cost of allocation, ie the difference between the premium paid by the policyholder and the amount invested in the unit fund on the policyholder's behalf
- expenses incurred by the life office
- interest earned/charged on the non-unit fund
- management charges taken from the unit fund
- extra death or maturity costs (if the benefit payable on death or maturity is greater than the value of the units held at the time of death or maturity)
- profit on surrender (if the benefit payable on surrender is less than the value of the units held at the time of surrender).

EXAMINER COMMENTS:

Students scored well in this bookwork question.

Average performance: 70% - 80%

QUESTION 2

Derive the expected present value of a benefit of R500 000 payable immediately on death in respect of a life currently aged 65 exactly, if death occurs within the next year.

Assume the force of mortality is constant between consecutive integer ages.

Basis:
Mortality AM92 Ultimate
Interest rate 4.8% per annum.

[Total 5]

SOLUTION:

$$500,000\,\bar{A}^{1}_{65:1\rceil}$$

$$= 500,000\int_0^1 e^{-\delta_{0.048}\cdot t}\,{}_t p_{65}\,\mu_{65+t}\,dt$$

Assuming μ is constant for $0<t<1$:

$$\mu = -\ln\left(p_{65}\right) = -\ln\left(1-0.01424\right)$$

$$= 0.014342$$

$${}_t p_{65} = e^{-\mu t} = e^{-0.01435t}$$

$$\delta_{0.048} = \ln\left(1.048\right) = 0.046884$$

Now, $\bar{A}^{1}_{65:1\rceil}$

$$= \int_0^1 e^{-0.04688t}\cdot e^{-0.01435t}\cdot 0.01435dt$$

$$= \frac{-0.01435}{\left(0.046888+0.01435\right)}\cdot\left[e^{-(0.04688+0.01435)t}\right]_0^1$$

$$= -0.23429\cdot\left(0.94061-1\right)$$

$$= 0.01392$$

$$\text{Thus EPV } = 500,000\left(0.01392\right)$$

$$= 6,957.53$$

Note that its incorrect to use the force of mortality age 65 from table which is not uniform over the year.

EXAMINER COMMENTS:

Many students lost marks for using the force of mortality assumption from the tables which is not correct. Student should be guided by the number of marks available for a question in forming a view of the complexity of a question.

Average performance: 50% - 60%

QUESTION 3

A life insurance company issues an immediate annuity to a retiring individual aged 63 exactly. The following terms regarding the annuity payment were agreed upon:

- The annuity is payable annually in advance.
- The annuity payments are made for as long as the life is alive but guaranteed for a minimum period of 7 years.

The purchase price is R2 000 000.

Calculate the annual annuity payment amount.

Basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Expenses	None

[Total 5]

SOLUTION:

The EPV of an annuity of R1 is

$\ddot{a}_{\overline{7}|} + {}_{7}|\ddot{a}_{63}$

Now

$$\ddot{a}_{\overline{7}|} = \frac{1-v^7}{d} = \frac{1-0.75992}{0.03846}$$

$$= 6.24214$$

and

$${}_{7}|\ddot{a}_{63} = v^7\ {}_{7}p_{63}\ddot{a}_{70}$$

$${}_{7}p_{63} = \frac{l_{70}}{l_{63}} = \frac{8054.0544}{9037.3973} = 0.89119$$

$$\ddot{a}_{70} = 10.375$$

Thus ${}_{7}|\ddot{a}_{63} = (1.04)^{-7} \cdot (0.89119) \cdot (10.375)$

$$= 7.02629$$

Hence annual annuity $= 2,000,000 / (7.02629 + 6.24214)$

$$= 150,733.73$$

EXAMINER COMMENTS:

Students scored well in this question with average performance: 80% - 90%.

QUESTION 4

A life insurance company issues an annuity contract to two lives in return for a single premium. The annuity payment of R200,000 per annum is payable annually in advance while at least one of the lives is alive. The first life is aged 60 exactly and the second life is aged 58 exactly at the inception of the policy.

a) Write down an expression for the future loss random variable at the outset for this policy.

[3]

b) Calculate the single premium.

[4]

Basis:

Mortality	PFA92C20 for the first life
	PMA92C20 for the second life
Interest	4% per annum
Expenses	None

c) State what the impact on the premium would be and explain your reasoning if:

(i) An interest rate of 6% per annum were used.
(ii) PMA92C20 were used for both lives.

No further calculations are necessary.

[4]

[Total 11]

SOLUTION:

a:

$$200{,}000\ddot{a}_{\overline{\max\left(K^{F}_{60}+1,K^{M}_{58}+1\right)}|} - P$$

Where P is the single premium

b:

$$P = 200{,}000\left(\ddot{a}_{\substack{F\\60}} + \ddot{a}_{\substack{M\\58}} - \ddot{a}_{\substack{F\ M\\60:58}}\right)$$

$$\ddot{a}_{\substack{F\\60}} = 16.652$$

$$\ddot{a}_{\substack{M\\58}} = 16.356$$

$$\ddot{a}_{\substack{F\ M\\60:58}} = 14.549$$

$$\text{Hence } P = 200{,}000\left(16.652 + 16.356 - 14.549\right)$$

$$= 3{,}691{,}800$$

c (i):

- Premium will be lower.
- The EPV of all annuities would decrease.
- The reduction in the EPV of the two individual annuities will outweigh the reduction in the joint live annuity which is being subtracted (as both individual annuities will reduce, and the magnitude of each individual annuity is larger than the joint annuity).

- Intuitively, discounting a future stream of positive payments at a higher interest rate yields a lower NPV.

c (ii):

- Premium will be lower.
- The EPV of the second life will remain unchanged.
- The EPV of both the individual annuity for the first life and the joint life will decrease.
- The EPV of the individual annuity for the first life will decrease more than the EPV of the joint life (which is being deducted). This is because the impact on ${}_tp_x$ will always be greater than the impact on ${}_tp_x \cdot {}_tp_y$ (given ${}_tp_y$ remains unchanged).

- Male mortality generally exceeds female mortality and therefore the expected lifetime of a male is lower than female, hence the annuity would be expected to be paid for a shorter period of time hence have lower NPV. Hence premium would be lower.

EXAMINER COMMENTS:

In part a) many students gave an expression for the EPV and not a random variable and therefore lost easy marks.

Average performance: 70% - 80%

QUESTION 5

a) In the context of with-profit policies, describe the super compound method of adding bonuses.

[3]

b) Suggest a reason why a life insurance company might use the super compound method of adding bonuses as opposed to the compound method, all else being equal.

[1]

A life aged 40 exactly purchases a 20-year with-profit endowment assurance policy from a life insurer. The sum assured of R800 000 plus declared reversionary bonuses are payable at maturity or at the end of the year of death if earlier. Level premiums are payable quarterly in advance whilst the policyholder is alive.

Assume that future simple reversionary bonuses will be declared at the rate of 5% per annum vesting at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year in which death occurred).

c) Calculate the expected present value of all benefits from this policy.

[6]

d) Calculate the expected present value of all expenses and commission for this policy.

[3]

e) Calculate the quarterly premium.

[4]

Basis:

Mortality	AM92 Select
Interest	6% per annum
Initial expenses	R2 000 at inception
Initial commission	20% of the gross annual premium
Renewal expenses	R500 per annum payable annually in advance commencing at the start of the second policy year
Claims expenses	R1 000 on death R1 200 on maturity

The life insurer sells a range of different products.

f) Explain whether the premium would have been larger, the same or smaller than in (e) above if the insurer now assumes lower volumes of new business sales for this product.

[4]

[Total 21]

SOLUTION:

a

The super compound bonus method is a method of allocating bonuses under which two bonus rates are declared each year. The first rate, usually the lower, is applied to the basic sum assured and the second rate is applied to the bonuses already declared.

b

The sum assured and bonuses increase more slowly than under other methods for the same ultimate benefit, enabling the office to retain surplus for longer and thereby providing greater investment freedom.

c

EPV of Benefits =

$$760,000A_{[40]:\overline{20|}} + 40,000(IA)_{[40]:\overline{20|}}^{1} + 840,000v^{20} \cdot {}_{20}p_{[40]}$$

$$A_{[40]:\overline{20|}} = 0.32076$$

$$(IA)_{[40]:\overline{20|}}^{1} = (IA)_{[40]} - v^{20} \cdot {}_{20}p_{[40]}\left((IA)_{60} + 20A_{60}\right)$$

$$= 3.85489 - v^{20} \cdot {}_{20}p_{[40]}\left(5.46572 + 20\left(0.32692\right)\right)$$

$$= 0.32734$$

EPV of Benefits $= 503,714.82164$

d

EPV of Expenses =

$$2,000 + 0.2(4P) + 500\left(\ddot{a}_{[40]:\overline{20|}} - 1\right) + 1,000A_{[40]:\overline{20|}} + 200v^{20} \cdot {}_{20}p_{[40]}$$

$$= 0.8P + 7879.53225$$

e

EPV of Premiums $= 4P\ddot{a}_{[40]\overline{20|}}^{(4)}$

$$= 4P\left[\ddot{a}_{[40]:\overline{20|}} - \frac{3}{8}\left(1 - v^{20} \cdot {}_{20}p_{[40]}\right)\right]$$

Now

$$\ddot{a}_{[40]:\overline{20|}} = 12.000$$

$${}_{20}p_{[40]} = \frac{l_{60}}{l_{[40]}} = \frac{9287.2164}{9854.3036} = 0.94245$$

Hence EPV $= 46.94079P$

Using the equation of value:

EPV of Premiums = EPV of Benefits + Expenses

$$P = 11,087.68$$

f

- Assume maintenance expenses and claims expenses are determined based on the whole in force book of the company, hence lower new business sales have a negligible impact (at least in the short to medium term).
- Acquisition expenses consist of a variable and fixed (overhead) component.
- Lower assumed new business volumes would have affected acquisition expenses per policy as follow:
  - variable: no impact
  - fixed: higher due to less policies sold to cover fixed/overheads expenses
- Higher initial expenses would have resulted in higher premium.

EXAMINER COMMENTS:

Performance for part a) was below expectation for a simple bookwork question with very few students scoring well in part b.

Average performance: 50% - 60%

QUESTION 6

A life insurance company specialising in retirement products issues a reversionary annuity contract. The terms of the contract are as follow:

- An annuity of R100 000 per annum is payable annually to a female life, whilst alive, following the death of a male life.
- Annuity payments will commence at the end of the year of death of the male life.
- Premiums are to be paid monthly in advance until the annuity commences or the contract ceases, if earlier.

Calculate the monthly premium required in respect of contract where the female life is age 65 exactly and the male life is age 68 exactly at inception.

Basis:

Mortality	PFA92C20 for the female
	PMA92C20 for the male
Interest	4% per annum
Expenses	0.5% of each premium payment
	7.5% of each annuity payment

[Total 6]

SOLUTION:

$$\text{EPV of Benefits } = 100,000a_{68|65}$$

$$\text{Now } a_{68|65} = \left(a_{65} - a_{68:65}\right)$$

$$= \left(14.871 - 1\right) - \left(11.112 - 1\right)$$

$$= 3.759$$

$$100,000a_{68|65} = 375900$$

$$\text{EPV of Premiums } =$$

$$12P\ddot{a}^{(12)}_{68:65} = 12P\left(\ddot{a}_{68:65} - \frac{11}{24}\right)$$

$$= 127.844P$$

$$\text{EPV of Expenses } = 0.005 \cdot 12P\ddot{a}^{(12)}_{68:65} + 0.075a_{68:65}$$

Equation of value:

$$\left(0.995\right)\left(127.844\right)P = \left(1.075\right)\left(375,900\right)$$

$$\text{Hence } P = 3{,}176.71 \text{ per month}$$

EXAMINER COMMENTS:

Performance on this question varied a lot with well prepared students scoring well but others very poorly.

Average performance: 50% - 60%

QUESTION 7

A life insurance company issues the following policies:

- 20-year term assurances with a sum assured of R500 000 where the death benefit is payable at the end of the year of death
- 20-year pure endowment assurances with a sum assured of R200 000
- 8-year temporary immediate annuities with an annual benefit payable in arrear of R100 000

On 1 January 2017, the company sold the following policies:

Product	Policyholder age exact	Number of Policies
Term Assurance	40	100
Pure Endowment Assurance	40	200
Temporary Immediate Annuity	57	50

The premiums in respect of the term assurance and pure endowment policies, are payable annually in advance, whereas a single premium is payable in respect of the temporary immediate annuity policies.

a) Determine the gross premium payable for the two types of assurance policies. [3]

The company calculates reserves for all three products using the prospective reserving methodology. The calculated reserves at the end of 2019 are as follows:

Product	Reserve
Term Assurance	2 585.25
Pure Endowment Assurance	20 021.27
Temporary Immediate Annuity	355 000.00

b) Calculate the death strain at risk for each type of policy during 2019 using these reserves. [3]

The claim records of the company reflect the following number of deaths for each full calendar year:

Product	2017	2018	2019	2020
Term Assurance	7	8	3	18
Pure Endowment Assurance	2	9	7	27
Temporary Immediate Annuity	4	6	7	14

c) Calculate the total mortality profit or loss to the insurer in the year 2019. [11]

Basis:
Interest 4% per annum
Mortality AM92 Ultimate

d) Suggest a possible reason why there is a profit or loss to the insurer. [1]

[Total 18]

SOLUTION:

Term Assurance

$$P_{TA} = \frac{500,000 \cdot A^{\ 1}_{40:\overline{20}|}}{\ddot{a}_{40:\overline{20}|}}$$

$$A^{\ 1}_{40:\overline{20}|} = A_{40:\overline{20}|} - v^{20}\ {}_{20}p_{40}$$

$${}_{20}p_{40} = \frac{l_{60}}{l_{40}} = \frac{9287.216}{9856.286} = 0.94226$$

$$A_{40:\overline{20}|} = 0.46433$$

So $A^{\ 1}_{40:\overline{20}|} = 0.03429$

$$\ddot{a}_{40:\overline{20}|} = 13.927$$

$$P_{TA} = 1,231.06$$

Pure Endowment

$$P_{PE} = \frac{200,000 \cdot v^{20}\ {}_{20}p_{40}}{\ddot{a}_{40:\overline{20}|}}$$

$$= 6175.56$$

Prospective Reserves at end of 2019

Pure Endowment

$${}_3V_{PE} = 20,021.27$$

Term Assurance

$${}_3V_{TA} = 2585.25$$

Annuity

$${}_3V_{A} = 355,000$$

b)

Death Strain at Risk

Term Assurance $= S - {}_3V_{TE}$

$= 500,000 - 2585.25$

$= 497,414.75$

Pure Endowment $= 0 - 20,021.27$

$= -20,021.27$

Annuity $= -(355,000 + 100,000)$

$= -455,000$

c)

Mortality Profits

Mortality Profits = Expected Death Strain - Actual Death Strain

Term Assurance

$\text{ADS} = (3)(497,414.75)$

$= 1,492,244.25$

$\text{EDS} = (100-7-8)\cdot q_{42}\,(497,414.75)$

$q_{42} = 0.001104$

$\text{EDS} = 46,677.40$

$\text{MP} = 46,677.40 - 1,492,244.25$

$= -1,445,566.85$ i.e. a loss

Pure Endowment

$\text{ADS} = (7)(-20,021.27)$

$= -140,148.88$

$\text{EDS} = (200-2-9)\cdot q_{42}\,(-20,021.27)$

$= -4,177.56$

$\text{MP} = -4,177.56 + 140,148.88$

$= 135,971.33$

Annuity

$\text{ADS} = (7)(-455,000)$

$= -3,185,000$

$\text{EDS} = (50-4-6)\cdot q_{59}\,(-455,000)$

$q_{59} = 0.00714$

$\text{EDS} = -129,948$

$\text{MP} = -129,948 + 3,185,000$

$= 3,055,052$

$\text{Total Profit} = 1,745,456.47$

d

Overall, there were more deaths than expected with the release of reserves on the annuity far exceeding the losses on the term assurance.

EXAMINER COMMENTS:

Answer layout was important in this question as students with clearly laid out answers made it easier to identify where an error was made in order to give partial credit for workings.

Average performance: 50% - 60%

QUESTION 8

a) Show that $\ddot{a}_x - \ddot{a}_{x:\overline{n}|} = v^n \cdot {}_n p_x \cdot \ddot{a}_{x+n}$. [3]

A life insurance company issues identical deferred annuities to 50 individuals aged 60 exactly. An annuity of R8 000 per annum is payable continuously from an individual's 65th birthday, if still alive at that time, and for life thereafter.

b) Calculate the expected present value of this book of business at outset.

Basis:

Mortality	AM92 Ultimate
Interest	6% per annum

[3]

c) Calculate the variance of the present value of this book of business at outset, using the same basis as in part (b). [12]

[Total 18]

SOLUTION:

a

$$\ddot{a}_x - \ddot{a}_{x:\overline{n}|} = \sum_{j=0}^{\infty} v^j \cdot {}_j p_x - \sum_{j=0}^{n-1} v^j \cdot {}_j p_x$$

$$= \sum_{j=n}^{\infty} v^j \cdot {}_j p_x$$

Set $k = j - n$

$$= \sum_{k=0}^{\infty} v^{k+n} \cdot {}_{k+n} p_x$$

$$= v^n \cdot {}_n p_x \cdot \sum_{k=0}^{\infty} v^j \cdot {}_j p_{x+n}$$

$$= v^n \cdot {}_n p_x \cdot \ddot{a}_{x+n}$$

b

$$\text{EPV} = 8000 \cdot v^5 \cdot {}_5p_{60} \cdot \bar{a}_{65}$$

$${}_5p_{60} = \frac{8821.2612}{9287.2164}$$

$$= 0.94983$$

$$\bar{a}_{65} = \ddot{a}_{65} - 0.5$$

$$= 10.569 - 0.5$$

$$= 10.069$$

$$\text{EPV of one Policy} = 57{,}173.15$$

$$\text{EPV of 50 Policy} = 50\left(57{,}173.15\right)$$

$$= 2{,}858{,}657.48$$

c

$$\text{var}(X) = E(X^2) - [E(X)]^2$$

$$\text{Now } E(X^2) = \int_5^{\infty} {}_t p_{60} \cdot \mu_{60+t} \left(v^5 \bar{a}_{\overline{t-5}|}\right)^2 dt$$

$$\text{Let } r = t - 5$$

$$E(X^2) = \int_0^{\infty} {}_{r+5} p_{60} \cdot \mu_{60+r+5} \left(v^5 \bar{a}_{\overline{r}|}\right)^2 dr$$

$$= \int_0^{\infty} {}_5 p_{60} \cdot {}_r p_{65} \cdot \mu_{65+r} \cdot v^{10} \left(\frac{1-v^r}{\delta}\right)^2 dr$$

$$= \frac{v^{10} \cdot {}_5 p_{60}}{\delta^2} \int_0^{\infty} {}_r p_{65} \cdot \mu_{65+r} \left(1 - 2v^r + v^{2r}\right) dr$$

$$= \frac{v^{10} \cdot {}_5 p_{60}}{\delta^2} \left[1 - 2\bar{A}_{65} + {}^2\bar{A}_{65}\right]$$

Now

$$\bar{A}_{65} = (1.06)^{0.5} \left(1 - d\ddot{a}_{65}\right)$$

$$= (1.06)^{0.5} \left[1 - \left(\frac{0.06}{1.06}\right)(10.569)\right]$$

$$= 0.41363$$

$${}^2\bar{A}_{65} = (1.06)\left({}^2A_{65}\right)$$

$$= (1.06)(0.19985)$$

$$= 0.21184$$

$$E(X^2) = \frac{(1.06)^{-10}(0.94983)}{\ln(1.06)^2} \cdot [1 - (2)(0.41363) + 0.21184]$$

$$= 60.07537$$

$$\text{So var}(X) = 60.07537 - (7.14664)^2$$

$$= 9.00086$$

$$\text{Total Variance } = 50(8000)^2 (9.00086)$$

$$= 28,802,738,219.30$$

EXAMINER COMMENTS:

Part c was poorly answered with many students not appreciating the need to take a first principles approach.

Average performance: 30% - 40%

QUESTION 9

A life company sells 20-year with-profit endowment assurances. The death benefit is payable at the end of the year of death after the bonus amount for the current year has been added.

The basic sum assured is R1 500 000 and compound bonuses of 1.9231% are added to the sum assured at the end of the year. Level premiums of R5 500 are payable monthly for a 40-year-old with the first monthly premium being due immediately.

Basis:

Mortality	AM92 Ultimate
Interest rate	6% per annum
Initial expense	45% of the first year's total premiums at the start of the policy
Renewable expense	3% of all annual premiums, including the first year, payable at the start of each year

i) Calculate the gross premium prospective reserve for a policyholder aged 40 at inception, just before the start of the 16th year of the policy, assuming that the bonuses have been declared in line with the intended rates. [9]

ii) State under what conditions the prospective and retrospective gross premium reserves would be equal for a without-profits policy. [3]

[Total 12]

SOLUTION:

i)

The expected present value of the future benefits at the start of the 16th policy year is

$EPV = 1\ 500\ 000[(1.01923)^{16}v\frac{d_{55}}{l_{55}} + (1.01923)^{17}v^2\frac{d_{56}}{l_{55}} + \ldots\ldots\ldots]$

$EPV = 1\ 500\ 000 * (1.01923)^{15}\ [(1.01923)v\frac{d_{55}}{l_{55}} + (1.01923)^2v^2\frac{d_{56}}{l_{55}} + \ldots\ldots..]$

$EPV = 1\ 500\ 000 * (1.01923)^{15}\ (v_* \frac{d_{55}}{l_{55}} + v_*^2 \frac{d_{56}}{l_{55}} + \ldots\ldots..]$

$EPV = 1\ 500\ 000 * (1.01923)^{15} A^{4\%}_{55:5}$

Where $v_* = \frac{1.01923}{1.06} = \frac{1}{1.04}$ as before

The expected present value of expenses at the start of the 16th policy year is

$EPV(Expenses) = (0.03)(12P)a_{\ddot{55:5}}$

The expected present value of the future premiums at start of 16th policy year is

$EPV(Premium) = P\ a_{\ddot{55:5}}^{(12)}$

$A^{4\%}_{55:5} = A_{55} - v^5\frac{l_{60}}{l_{55}}A_{60} + v^5\frac{l_{60}}{l_{55}}$

$= (0.3895) - (1.04)^{-5} * \frac{9\ 287.2164}{9\ 557.8179} * (0.4564) + (1.04)^{-5}\frac{9\ 287.2164}{9\ 557.8179}$

$= 0.82364976$

$a_{\ddot{55:5}} = 4.423$ at 6%

$a_{\ddot{55:5}}^{(12)} = a_{55:5} - \frac{11}{24}(1 - v^5\frac{l_{60}}{l_{55}})$ at 6%

$= 4.423 - \frac{11}{24} * (1 - 1.06^{-20} * \frac{9\ 287.2164}{9\ 557.8179})$

$= 4.297463305$

The expected present value of the future benefit

$tV = 1\ 500\ 000 * (1.01923)^{15} * 0.82364976 + (0.03)(12 * 5500)(4.423) - (12 * 5500 * 4.297463305)$

tV = R 1 369 181.73

ii)

- Gross premium has been calculated using the equivalence principle
- The basis used to calculate the prospective and retrospective reserves are the same
- And the basis is the same as used when calculating the gross premium

EXAMINER COMMENTS:

Time pressure became evident for some students at this stage. Proper exam planning is critical for students.

Average performance: 50% - 60%

COMPUTER BASED ASSESSMENT

EXAMINER COMMENTS:

QUESTION 1:

Students scored well in most parts. The reserving aspect was generally done poorly.

Average performance: 50% - 60%

QUESTION 2:

Well done by most students.

Average performance: 80% - 90%

