Actuarial Society of South Africa

WRITTEN EXAMINATION

SEPTEMBER 2020 SAMPLE SOLUTIONS

Subject A213 — Contingencies

Generally, students would have scored better marks where the following exam technique has been applied. Some improvement has been observed since Semester 1 of 2020 but student can do more refinement in this regard.

- Label each question clearly. Some students marked their page numbers as well as question numbers which caused confusion at times.
- Start each question on a new page.
- Use more space than less and make it clear how you break up key parts of a question. This makes it easier to see your workings and to understand what was done in order to give principle marks for incorrect solutions.
- Some students lost marks because the electronic scans were not clear.

QUESTION 1

A standard bookwork question in part i) which students did surprisingly poorly. Students generally performed better with the calculation part in ii) but many still lost out on easy marks.

Average mark: 40-50%

i) Assuming a U.D.D.

$${}_{s}q_{x} = s.q_{x}$$

So

$$\begin{aligned}
{}_{t-s}q_{x+s} &= \left(1 - {}_{t-s}p_{x+s}\right) \\
&= \left(1 - \frac{{}_{t}p_{x}}{{}_{s}p_{x}}\right) \\
&= \left[1 - \frac{1 - {}_{t}q_{x}}{1 - {}_{s}q_{x}}\right] \\
&= \left[1 - \frac{1 - t \cdot q_{x}}{1 - s \cdot q_{x}}\right] \\
&= \frac{(t-s) \cdot q_{x}}{(1 - s \cdot q_{x})}
\end{aligned}$$

ii)

$$\begin{aligned}
{}_{0.5|}q_{6} &= {}_{0.5}p_{6} \cdot q_{6.5} \\
&= {}_{0.5}p_{6} \cdot (1 - p_{6.5}) \\
&= {}_{0.5}p_{6} . [1 - {}_{0.5}p_{6.5} \cdot {}_{0.5}p_{7}] \\
&= {}_{0.5}p_{6} - p_{6} \cdot {}_{0.5}p_{7} \\
&= [1 - {}_{0.5}q_{6}] - p_{6} \cdot [1 - {}_{0.5}q_{7}] \\
&= [1 - 0.5 \cdot 0.07] - (1 - 0.07)(1 - 0.5 \cdot 0.08) \\
&= 0.0722
\end{aligned}$$

Or

$$\begin{aligned}
{}_{0.5|}q_{6} &= {}_{0.5}p_{6} \cdot q_{6.5} + p_{6} \cdot {}_{0.5}q_{7} \\
&= (1 - {}_{0.5}q_{6})({}_{0.5}q_{6.5}) + (1 - q_{7})({}_{0.5}q_{7}) \\
&= (1 - 0.5 \cdot 0.07)\left(\frac{0.5 \cdot 0.07}{1 - 0.5 \cdot 0.07}\right) + (1 - 0.07)(0.5 \cdot 0.08) \\
&= 0.0722
\end{aligned}$$

QUESTION 2

A standard application question in which many students did not do well. Some maybe took too long on this question so time management on this question was key. The calculation of the standard deviation was particularly poorly done.

Average mark: 30-40%

(i)

$$\bar{a}_{x:\overline{n}|} = E\left[\bar{a}_{\overline{\min[T_x,n]}|}\right]$$

$$= E\left[\frac{1-v^{\min[T_x,n]}}{\delta}\right]$$

$$= \frac{1}{\delta}\left(1-\bar{A}_{x:\overline{n}|}\right)$$

Hence $\bar{A}_{x:\overline{n}|} = 1-\delta\bar{a}_{x:\overline{n}|}$

where $T_x =$ complete future lifetime of a life aged x

(ii)

(a)

$$\delta = \ln 1.05$$

$$EPV = \int_0^{10} v^t \cdot {}_tp_{25}\,dt$$

$$= \int_0^5 v^t \cdot {}_tp_{25}\,dt + v^5 \cdot {}_5p_{25}\int_0^5 v^t \cdot {}_tp_{30}\,dt$$

$$= \int_0^5 v^t e^{-\mu_1 \cdot t}\,dt + v^5 \cdot {}_5p_{25}\int_0^5 v^t e^{-\mu_2 \cdot t}\,dt$$

$$= \int_0^5 e^{-(\delta+\mu_1)\cdot t}\,dt + v^5 \cdot {}_5p_{25}\int_0^5 e^{-(\delta+\mu_2)\cdot t}\,dt$$

$$= \frac{1}{(\delta+\mu_1)}\left[1-e^{-(\delta+\mu_1)\cdot 5}\right] + \frac{v^5 \cdot {}_5p_{25}}{(\delta+\mu_2)}\left[1-e^{-(\delta+\mu_2)\cdot 5}\right]$$

$$= 4.3841 + 3.3422$$

$$= 7.7263$$

So

$$EPV = 365,000(7.7263)$$
$$= 2,820,093.07$$

(b)

$$Var = \frac{{}^2\bar{A}_{25:10} - \left(\bar{A}_{25:10}\right)^2}{\delta^2}$$

But we know

$$\bar{A}_{25:10} = 1 - \delta \cdot \bar{a}_{25:10}$$
$$= 1 - \ln(1.05) \cdot 7.7263$$
$$= 0.6230$$

Let i^* the interest rate for ${}^2\bar{A}_{25:10}$

Then $i^* = (1+i)^2 = 0.1025$

Now

$$\bar{A}_{25:10} = \int_0^{10} v^t \cdot {}_t p_{25} \cdot \mu_{25+t} dt + v^{10} \cdot {}_{10} p_{25}$$
$$= \int_0^5 v^t \cdot {}_t p_{25} \cdot \mu_1 dt + v^5 \cdot {}_5 p_{25} \int_0^5 v^t \cdot {}_t p_{25} \cdot \mu_2 dt + v^{10} \cdot {}_{10} p_{25}$$
$$= \mu_1 \int_0^5 v^t e^{-\mu_1 \cdot t} dt + v^5 \cdot {}_5 p_{25} \cdot \mu_2 \int_0^5 v^t e^{-\mu_2 \cdot t} dt + v^{10} \cdot {}_{10} p_{25}$$
$$= \mu_1 \int_0^5 e^{-(\delta+\mu_1)\cdot t} dt + v^5 \cdot {}_5 p_{25} \cdot \mu_2 \int_0^5 e^{-(\delta+\mu_2)\cdot t} dt + v^{10} \cdot {}_{10} p_{25}$$
$$= \frac{\mu_1}{(\delta+\mu_1)}\left[1 - e^{-(\delta+\mu_1)\cdot 5}\right] + \frac{v^5 \cdot {}_5 p_{25} \cdot \mu_2}{(\delta+\mu_2)}\left[1 - e^{-(\delta+\mu_2)\cdot 5}\right] + v^{10} \cdot e^{-\mu_1 \cdot 5} \cdot e^{-\mu_2 \cdot 5}$$
$$= 0.0196 + 0.0140 + 0.3567$$
$$0.3903$$

Hence

$$Stdev = 365,000 \cdot \frac{\sqrt{0.3903 - 0.6230^2}}{\ln 1.05}$$
$$= 6,600,565.58$$

QUESTION 3

A question that tested whether students were able to apply their skills in a wider context. Students generally struggled and overcomplicated the question. Many did not make any attempt at this question

Average mark: 40-50%

$$
\begin{aligned}
{}_5p_{35} &= \frac{l_{40}}{l_{35}} \\
&= \frac{9,856.2863}{9,894.4299} \\
&= 0.9961
\end{aligned}
$$

$$
\begin{aligned}
{}_5p_{30} &= \frac{l_{35}}{l_{40}} \\
&= \frac{9,894.4299}{9,925.2094} \\
&= 0.9969
\end{aligned}
$$

There are four possible outcomes:

1. Both Survive

$$
\begin{aligned}
EPV &= v^5 \cdot {}_5p_{35} \cdot {}_5p_{30} \cdot \frac{100,000}{3} \\
&= 25,936.17
\end{aligned}
$$

2. Only the director survives

$$
\begin{aligned}
EPV &= v^5 \cdot {}_5p_{35} \cdot {}_5q_{30} \cdot \frac{100,000}{2} \\
&= 121.02
\end{aligned}
$$

3. Only the game ranger survives

$$
\begin{aligned}
EPV &= v^5 \cdot {}_5q_{35} \cdot {}_5p_{30} \cdot \frac{100,000}{2} \\
&= 150.56
\end{aligned}
$$

4. Neither survives

$$
\begin{aligned}
EPV &= v^5 \cdot {}_5q_{35} \cdot {}_5q_{30} \cdot 100,000 \\
&= 0.94
\end{aligned}
$$

Hence the total

$$
\begin{aligned}
EPV &= 1 + 2 + 3 + 4 \\
&= 26{,}208.69
\end{aligned}
$$

QUESTION 4

A mostly standard type question which most students did well. Answer layout could have been improved by many students but generally the best question of the paper.

Average mark: 60-70%

EPV benefits

$$510{,}000 A^{\,1}_{[40]:\overline{20}|} - 10{,}000 (IA)^{\,1}_{[40]:\overline{20}|} \ @\, 6\%\, p.a.$$

$$= 510{,}000\left[A_{[40]} - v^{20}\,{}_{20}p_{[40]} A_{60}\right] - 10{,}000\left[IA_{[40]} - v^{20}\,{}_{20}p_{[40]} \cdot \left(20A_{60} + (IA)_{60}\right)\right]$$

$$= 510{,}000 A_{[40]} - 100{,}00 IA_{[40]} + v^{20}\,{}_{20}p_{[40]}\left[10{,}000 (IA)_{60} - 310{,}000 A_{60}\right]$$

Now

$${}_{20}p_{[40]} = \frac{l_{60}}{l_{[40]}}$$

$$= \frac{9{,}287.2164}{9{,}854.3036}$$

$$= 0.94245$$

Now

$$EPV = 510{,}000(0.12296) - 10{,}000(3.85489) + v^{20}\,{}_{20}p_{40}\left(10{,}000(5.46572) - 310{,}000(0.32692)\right)$$

$$= 10{,}440.91$$

EPV of premiums

$$P \cdot \ddot{a}_{[40]:\overline{20}|} @\, 6\%$$

$$= 12P$$

EPV of expenses

$$500 + 0.2P + 200 a^{@0\%}_{[40]:\overline{19}|} + 0.01 P a^{@6\%}_{[40]:\overline{19}|} + 475 A^{@6\%\ 1}_{[40]:\overline{20}|} + 25 (IA)^{@6\%\ 1}_{[40]:20}$$

$$= 500 + 0.2P + 200\left(e_{[40]} - {}_{19}p_{[40]} \cdot e_{59}\right) + 0.01P \cdot (12 - 1) + 475 \cdot \left(0.32076 + 25 (IA)^{\,1}_{[40]:\overline{20}|} - v^{2}\,{}_{20}p_{[40]}\right)$$

$$= 0.31P + 3{,}750.43$$

Hence the premium is R1, 213.97.

QUESTION 5

Many students did well on this question which was a reasonably standard death strain at risk question. Students who provided a good layout to their results and provided sufficient details in their calculations scored well even in cases where smaller (initial) calculation errors were made. Part iv) has been a higher order skills question which required students to present well-reasoned solutions. Many students did not attempt this part.

Average mark: 45-55%

i) Benefits are paid until death i.e. an uncertain date, however the only premiums received is at outset.
If the premium that was not required to pay benefits early in the contract were spent by the company, on dividends for example, then later in the contract, if the policyholder was still alive, the company may not be able to find funds to pay the benefits.

ii) Term assurance

$$P = 100{,}000\frac{A^{\;1}_{50:\overline{10}|}}{\ddot{a}_{50:\overline{10}|}} @\,4\%$$

$$= 100{,}000\left[\frac{A_{50:\overline{10}|} - v^{10}\,{}_{10}p_{50}}{\ddot{a}_{50:\overline{10}|}}\right]$$

$$= 100{,}000\frac{\left[0.68024 - v^{10}\dfrac{9{,}287.2164}{9{,}712.0728}\right]}{8.314}$$

$$= 411.70$$

Reserves at end of second year:

$${}_2V^{TA} = 100{,}000\cdot A^{\;1}_{52:\overline{8}|} - P\,\ddot{a}_{52:\overline{8}|}$$

$$= 100{,}000\left(A_{52:\overline{8}|} - v^{8}\,{}_{8}p_{52}\right) - P\,\ddot{a}_{52:\overline{8}|}$$

$$= 100{,}000\left(0.73424 - v^{8}\cdot\frac{9{,}287.2164}{9{,}660.5021}\right) - 6.910P$$

$$= 333.57$$

$${}_2V^{AN} = 12\cdot 10{,}000\,\ddot{a}^{(12)}_{52:\overline{8}|}$$

$$= 120{,}000\cdot\left[\ddot{a}_{52:\overline{8}|} - \frac{11}{24}\cdot v^{8}\,{}_{8}p_{52}\right]$$

$$= 790{,}564.92$$

Now DSAR

$$TA = 100000 - {}_2V = 9,966.43$$
$$AN = -{}_2V = -790,564.92$$

iii) Mortality Profit/Loss = *EDS-ADS*

Term Assurance:

$$EDS = (2,000 - 30) \cdot q_{51} \cdot DSAR$$
$$= 1,970 \cdot (0.002809) \cdot DSAR$$
$$= 551,527.12$$

$$ADS = 35 \cdot DSAR$$
$$= 3,488,452.39$$

Hence mortality loss of 2,936,798.00

Annuity:

$$EDS = (1,000 - 50) \cdot q_{51} \cdot DSAR$$
$$= -2,109,662.01$$

$$ADS = 60 \cdot DSAR$$
$$= -47,433,895.07$$

Hence profit of 45,324,233.05

iv) a. Initial expenses would result in a higher premium, a lower reserve and a higher *DSAR*.

Hence a larger mortality loss.

b. Select mortality would result in a lower premium, a higher reserve and a lower *DSAR*.

Hence a smaller mortality loss.

[

QUESTION 6

Well prepared students scored well on this question which tested application and higher order skills. One of the questions which students generally struggled with even though part i) was a straightforward joint life question.

Average mark: 30-40%

i)

$$
\begin{aligned}
EPV &= 120{,}000 \cdot a_{\overline{68:70}} \\
&= 120{,}000\left(a_{68}^{m} + a_{70}^{f} - a_{68:70}\right) \\
&= 120{,}000\left(11.412 + 11.934 - 9.27\right) \\
&= 120{,}000 \cdot 14.076 \\
&= 1{,}689{,}120
\end{aligned}
$$

ii) The office losses money if PV of payments > 900,000

i.e. $120{,}000 a_{\overline{n}|} > 900{,}000$ or $a_{\overline{n}|} > 7.5$

At 4%, $a_{\overline{9}|} = 7.44$ and $a_{\overline{10}|} = 8.11$ so the office will make a loss if it makes the 10th payment.

It therefore makes a profit if both lives have died before this time, with probability

$$
\begin{aligned}
{}_{10}q_{\overline{68:70}} &= {}_{10}q_{68}^{m} \cdot {}_{10}q_{70}^{f} \\
&= \left(1 - \frac{l_{78}^{m}}{l_{68}^{m}}\right)\left(1 - \frac{l_{80}^{f}}{l_{70}^{f}}\right) \\
&= \left(1 - \frac{7{,}615.818}{9{,}440.717}\right)\left(1 - \frac{7{,}724.737}{9{,}392.621}\right) \\
&= 0.03433
\end{aligned}
$$

QUESTION 7

A longer type question testing application of standard methodology. As for other longer type questions, answer layout and providing enough detail assisted students.

Average mark: 40-50%

i)

$$\bar{A}^{1}_{xy} + \bar{A}_{x\overset{1}{y}} = \int_{t=0}^{\infty} v^t\, {}_tp_{xy}\mu_{x+t}dt + \int_{t=0}^{\infty} v^t\, {}_tp_{xy}\mu_{y+t}dt$$

$$= \int_{t=0}^{\infty} v^t\, {}_tp_{xy}\left(\mu_{x+t} + \mu_{y+t}\right)dt$$

$$= \int_{t=0}^{\infty} v^t\, {}_tp_{xy}\mu_{x+t:y+t}dt$$

$$= \bar{A}_{xy}$$

ii) If the non-smoker dies in 15 years:

-R800,000 is paid if the smoker is still alive;

-R200,000 is paid if the smoker is dead

If the smoker dies in 15 years:

-R400,000 is paid if the non-smoker is still alive

-R400,000 is paid if the non-smoker is dead

Hence

$$EPV = \int_0^{15} {}_tp_{NS}\mu_{NS}\left(800,000\, {}_tp_{SM} + 200,000\, {}_tq_{SM}\right)\cdot e^{-\delta t}dt + 400,000\int_0^{15} {}_tp_{SM}\mu_{SM}\cdot e^{-\delta t}dt$$

$${}_tp_{SM} = e^{-0.04t}$$

$${}_tp_{NS} = e^{-0.03t}$$

$$EPV = \int_0^{15} e^{-0.03t}(0.03)\cdot\left[800,000e^{-0.04t} + 200,000\left(1-e^{-0.04t}\right)\right]e^{-0.07t}dt + 400,000\int_0^{15}\left(e^{-0.04t}\right)(0.04)e^{-0.07t}dt$$

$$= 18,000\int_0^{15} e^{-0.14t}dt + 6,000\int_0^{15} e^{-0.1t}dt + 16,000\int_0^{15} e^{-0.11t}dt$$

$$= \frac{18,000}{-0.14}\left[e^{(-0.14)(15)} - 1\right] + \frac{6,000}{-0.1}\left[e^{(-0.1)(15)} - 1\right] + \frac{16,000}{-0.11}\left[e^{(-0.11)(15)} - 1\right]$$

$$= 112,827 + 46,612 + 117,520$$

$$= 276,959.23$$