ACTUARIAL SOCIETY OF SOUTH AFRICA

EXAMINATION SOLUTIONS AND EXAMINER COMMENTS

S1 2023 WRITTEN EXAM

Subject A213 — Contingencies

General comments:

Overall, the performance of the paper was in line with previous sessions. Students continued to struggle with theoretical bookwork questions where answers were often incomplete. Students are encouraged to show their workings clearly and to show the calculation results of intermediate steps as this enhances the ability of the marker to give partial credit where due.

QUESTION 1

i)

$EPV =$

$500000\int_0^{25} e^{-0.05}\left\{e^{-0.02t}\left(1-e^{-0.03t}\right)\cdot 0.02+e^{-0.03t}\left(1-e^{-0.02t}\right)\cdot 0.03\right\}dt$

$=500000\int_0^{25} 0.02e^{-0.07t}+0.03e^{-0.08t}-0.05e^{-0.1t}dt$

$=500000\left\{\left[\frac{-0.02e^{-0.07t}}{0.07}\right]_0^{25}+\left[\frac{-0.03e^{-0.08t}}{0.08}\right]_0^{25}-\left[\frac{-0.05e^{-0.1t}}{0.1}\right]_0^{25}\right\}$

$=50675$

ii)

$EPV = e^{-1.25}\left(400000e^{-0.5}e^{-0.75}+200000e^{-0.5}\left(1-e^{-0.75}\right)+200000e^{-0.75}\left(1-e^{-0.5}\right)\right)$

$=200000e^{-1.75}+200000e^{-2}$

$=34754.8+27067.2$

$=61822$

iii)
Profit
$=0.05\times 500000$

$=25000$

Value of Premiums=
$P\int_0^{25} e^{-0.05t}\left\{e^{-0.02t}\left(1-e^{-0.03t}\right)+e^{-0.03t}\left(1-e^{-0.02t}\right)+e^{-0.05t}\right\}dt$
$=13.432P$

$$P=\frac{50675+61822+25000}{13.432}=R10236.52$$

EXAMINER COMMENTS:

Question performance: 60-65%

Students performed satisfactorily in this question.

QUESTION 2:

$$(aq)_x^{\alpha} = \int_0^1 {}_t(ap)_{30}\,(a\mu)_{30+t}^{\alpha}\,dt$$

$${}_t(ap)_{30} = {}_tp_{30}^{\alpha} \times {}_tp_{30}^{\beta} = \left(\frac{30-t}{30}\right)^3 \quad 0 \le t \le 30$$

$$(a\mu)_{30+t}^{\alpha} = \mu_{30+t}^{\alpha} = -\frac{\partial}{\partial t}\ln {}_tp_{30}^{\alpha}$$

$$= -\frac{\partial}{\partial t}\left[\ln(30-t) - \ln 30\right]$$

$$= \frac{1}{30-t}$$

hence

$$(aq)_x^{\alpha} = \int_0^1 \frac{(30-t)^2}{30^3}\,dt = \frac{1}{30^3}\int_0^1 900 - 60t + t^2 dt = \frac{1}{30^3}\left[900t - 30t^2 + \frac{1}{3}t^3\right]_0^1 = 0.032235$$

EXAMINER COMMENTS:

Question performance: 45-50%

A more challenging question for some students. Students tended to either score very well or poorly in this question.

QUESTION 3:

i)

We need the reserve at time 3. First calculate the annual premium.

$$P\ddot{a}_{50:\overline{15}|} = 500000A^1_{50:\overline{15}|}$$

$$\ddot{a}_{50:\overline{15}|} = 11.253$$

$$A^1_{50:\overline{15}|} = A_{50:\overline{15}|} - \frac{D_{65}}{D_{50}} = 0.56719 - \frac{689.23}{1366.61} = 0.06285$$

$$\therefore P = 2792.59$$

hence

$${}_3V = 500000A^1_{53:\overline{12}|} - 2792.59\ddot{a}_{53:\overline{12}|}$$

$$= 500000\left(0.63460 - \frac{689.23}{1204.65}\right) - 2792.59 \times 9.5$$

$$= 4699.75$$

Therefore,

$$DSAR = S - {}_3V = 500000 - 4699.75 = 495300.25$$

$$\forall policies = (9000 - 24) \times 495300.25$$

$$= 4\ 445\ 815\ 044$$

ii)
Expected number of deaths are (9000-24) x 0.003152=28.29
During 2023, we also expect 12 deaths

As this is an assurance, we expect less deaths than what was allowed for in the reserving and hence we expect a mortality profit to arise.

EXAMINER COMMENTS:

Question performance: 60-65%

Students performed well in this question with some struggling specifically with part ii and unable to interpret the DSAR accurately.

QUESTION 4:

i)

$$V^{retro} = 1.06^{20}\frac{l_{30}}{l_{50}}\left[12\times 2000\times 0.975\ddot{a}^{(12)i\%}_{30:\overline{20|}} + 0.025(2000) - 1000000A^{1@i\%}_{30:\overline{20|}} - 480\left(\ddot{a}^{(12)j\%}_{30:\overline{20|}} - 1\right) - 2000A^{1@j\%}_{30:\overline{20|}}\right]$$

$$@6\%: \ddot{a}^{(12)j\%}_{30:\overline{20|}} = \ddot{a}_{30:\overline{20|}} - \frac{11}{24}\left(1 - v^{20}\frac{l_{50}}{l_{30}}\right)$$

$$= \ddot{a}_{30} - v^{20}\frac{l_{50}}{l_{30}}\ddot{a}_{50} - \frac{11}{24}\left(1 - v^{20}\frac{l_{50}}{l_{30}}\right)$$

$$\ddot{a}_{30} = 16.372$$

$$\ddot{a}_{50} = 14.044$$

$$\therefore \ddot{a}^{(12)6\%}_{30:\overline{20|}} = 11.76855$$

$$\frac{l_{30}}{l_{50}} = \frac{9925.2094}{9712.0728} = 1.021945532$$

$$A^{1@6\%}_{30:\overline{20|}} = A_{30} - v^{20}\frac{l_{50}}{l_{30}}A_{50} = 0.07328 - v^{20}\frac{l_{50}}{l_{30}}(0.020508) = 0.010708256$$

$$A^{1@4\%}_{30:\overline{20|}} = 0.013271822$$

$$\ddot{a}_{30:\overline{20|}}@4\% = \ddot{a}_{30} - v^{20}\frac{l_{50}}{l_{30}}\ddot{a}_{50} = 21.834 - v^{20}\frac{l_{50}}{l_{30}}(17.444)$$

$$= 14.04374735$$

Now

$$V^{retro} = 1.06^{20}(1.021945532)\begin{bmatrix}12(2000)(0.975)(11.76855) + 0.025(2000) \\ -1000000(0.0107083) - 480(13.0437435) - 2000(0.0132718)\end{bmatrix}$$

$$= 870178.8678$$

ii)

Now allowing for the paid up alteration expense of R2500, we need to solve for:

$$870178.8678-2500=(PU)A_{50:\overline{15}|}^{@6\%}+480(1.0192308)^{20}\ddot{a}_{50:\overline{15}|}^{@4\%}+2000(1.0192308)^{20}A_{50:\overline{15}|}^{@4\%}$$

with

$$A_{50:\overline{15}|}^{@6\%}=0.43181$$

$$\ddot{a}_{50:\overline{15}|}^{@4\%}=11.253$$

$$A_{50:\overline{15}|}^{@4\%}=0.56719$$

hence

$$Paid-up-value=R1993034.938$$

iii)

- The paid-up value is almost double the original sum assured which seems counter intuitive and therefore arguably not reasonable.
- This is a result of the pricing basis being more conservative than the paid-up basis being used here.
- In practice the company would therefore have to review the paid-up basis to ensure that the paid-up value would ideally be less than the original sum assured.

EXAMINER COMMENTS:

Question performance: 34-40%

Students struggled specifically with part ii and iii even though the concept of a “paid-up value” has been clearly defined in the paper and therefore tested students ability to apply their learnings to a different concept than what they might have been used to.

QUESTION 5:

i) "Profit vector" gives the expected profit at the end of each policy year per policy in force at the beginning of that policy year.

ii) "Profit signature" refers to the pattern of expected profits over time for a policy or portfolio of insurance policies, only assuming that the policy is in force at the start of the policy and not in force at the start of each policy year as per the profit vector.

iii)

- "Non-unitised accumulating with-profits contract" refers to a type of insurance contract that provides policyholders with the potential for investment returns in addition to the cover provided by the policy.

- Under an accumulating with-profits (AWP) contract, the basic benefit takes the form of an accumulating fund of premiums. If the accumulating fund at time t is denoted by Ft, the simplest form of an AWP contract follows the following recursive formula:
- $F_t=(F_{t-1}+P)(1+b_t)$
- This assumes that annual premiums of P are payable at the start of each year and bt is the annual bonus interest declared for year t.

- The bonus rate is a discretionary amount determined by the insurance company each year.

- The bonus will reflect both the returns achieved on the underlying assets over the period plus any additional profits made on the contract in this time. As it is discretionary, it does not exactly reflect these amounts, and in practice the insurer tends to smooth out the variations in returns and profits achieved from year to year to produce a bonus interest rate that is more stable over time than the underlying asset returns.

- A key feature of the regular bonus interest is that it cannot be negative, whereas for certain asset types (eg equity portfolios) actual returns can be negative.

- Sometimes a minimum bonus rate can be guaranteed.

iv) "Terminal bonus" refers to a bonus that is paid to policyholders of with-profit insurance policies at the end of the policy term. Terminal bonuses are used by insurance companies to reward policyholders for their loyalty and to provide an additional return on their investment. The terminal bonus is determined based on the performance of the insurance company's investments and the level of profits earned over the policy term. Terminal bonuses offer benefits to both the policyholder and the insurance company. For the policyholder, they provide an additional source of income at the end of the policy term, which can help to boost their retirement savings or other benefits. For the insurance company, terminal bonuses help to attract and retain policyholders, as they provide an added incentive for policyholders to keep their policies in force.

EXAMINER COMMENTS:

Question performance: 45-50%

Unfortunately, in line with past sessions, students do not know their bookwork well and therefore lost out on easy marks.

QUESTION 6:

$$EPV = 400\,000\left(\bar{a}_{\overline{50:55}} - \bar{a}_{50:55}\right) + 400\,000 \cdot \bar{a}_{\overline{3|}} \cdot \bar{A}_{\overline{50:55}}$$

Now $\bar{a}_{\overline{50:55}} = \bar{a}^{m}_{50} + \bar{a}^{f}_{50} - \bar{a}_{50:55}$

$$= (18.843 - 0.5) + (18.210 - 0.5) - (16.909 - 0.5)$$

$$= 19.644$$

$$\bar{A}_{\overline{50:55}} = 1 - \delta \cdot \bar{a}_{\overline{50:55}}$$

$$= 1 - \ln(1.04)(19.644)$$

$$= 0.22955$$

Therefore $EPV = 400\,000 \cdot \left[19.644 - (16.909 - 0.5) + \dfrac{1 - v^3}{\ln(1.04)} \cdot 0.22955\right]$

$$= R1553870$$

EXAMINER COMMENTS:

Question performance: 50-55%

In line with past experience, students often perform slightly worse with these types of questions.

QUESTION 7:

Deferred annuity:

$$EPV = 60000\,{}_{25|}\bar{a}_{65} = 65000\frac{D_{65}}{D_{[40]}}\bar{a}_{65}$$

$$\frac{D_{65}}{D_{[40]}} = \frac{689.23}{2052.54} = 0.33579$$

$$\bar{a}_{65} = \ddot{a}_{65} - 0.5 = 13.666 - 0.5 = 13.166$$

$$EPV = 60000(0.33579)(13.166) = 265263.60$$

Death benefit:

$EPV = 18000\left(IA\right)^{1}_{[40]:\overline{25}|}$

$= 18000\left[\left(IA\right)_{[40]} - \frac{D_{65}}{D_{[40]}}\left[\left(IA\right)_{65} + 25A_{65}\right]\right] = 18000\left[7.95835 - 0.33579\left(7.89442 + 25(0.52786)\right)\right]$

$= 18000\left(0.87615\right)$

$= 15770.72$

Hence total EPV = R265 263.60 + R15 770.72 = R281 034.32

EXAMINER COMMENTS:

Question performance: 55-60%

Many students particularly struggled with the term-increasing assurance function aspect.

QUESTION 8:

i)

$EPV = 100000v^{2}\,{}_{2}p_{63.25}$

${}_{2}p_{63.25} = {}_{0.75}\,p_{63.25} \times p_{64} \times {}_{0.25}\,p_{65}$

$p_{64} = 1 - q_{64} = 1 - 0.02199 = 0.97801$

Under UDD:

${}_{0.25}\,p_{65} = 1 - 0.25q_{65} = 1 - 0.25(0.02447) = 0.99388$

and

${}_{0.75}\,p_{63.25} = \frac{p_{63}}{{}_{0.25}\,p_{63}} = \frac{p_{63}}{1 - 0.25q_{63}} = \frac{0.98035}{1 - 0.25(0.01965)} = 0.98519$

${}_{0.25}\,p_{65.25} = 0.98519 \times 0.97801 \times 0.99388 = 0.95763$

$EPV = 100000v^{2}\,{}_{2}p_{63.25}$

$= 100000v^{2}\left(0.95763\right) = R88538.28$

ii)

Under CFM:

$EPV = 100000v^{2}\,{}_{2}p_{63.25}$

${}_{2}p_{63.25} = {}_{0.75}\,p_{63.25} \times p_{64} \times {}_{0.25}\,p_{65}$

$p_{64} = 1 - q_{64} = 1 - 0.02199 = 0.97801$

${}_{0.25}\,p_{65} = \left(p_{65}\right)^{0.25} = \left(1 - q_{65}\right)^{0.25} = 0.97553^{0.25} = 0.99383$

${}_{0.75}\,p_{63.25} = \left(p_{63}\right)^{0.75} = \left(1 - q_{63}\right)^{0.75} = \left(0.98035\right)^{0.75} = 0.98523$

hence

${}_{2}p_{63.25} = 0.98523 \times 0.97801 \times 0.99383 = 0.95761$

$$EPV = 100000v^2 {}_2p_{63.25}$$
$$= 100000v^2 (0.95761) = R88536.43$$

Alternative using AM92:

i)

$$EPV = 100000v^2 {}_2p_{63.25}$$
$${}_2p_{63.25} = {}_{0.75}\ p_{63.25} \times p_{64} \times {}_{0.25}\ p_{65}$$
$$p_{64} = 1 - q_{64} = 1 - 0.012716 = 0.987284$$

Under UDD:

$${}_{0.25}p_{65} = 1 - 0.25q_{65} = 1 - 0.25(0.014243) = 0.99644$$
and
$${}_{0.75}p_{63.25} = \frac{p_{63}}{{}_{0.25}p_{63}} = \frac{p_{63}}{1 - 0.25q_{63}} = \frac{0.988656}{1 - 0.25(0.011344)} = 0.991468$$
$${}_{0.25}p_{65.25} = 0.987284 \times 0.99644 \times 0.991468 = 0.9753757$$
$$EPV = 100000v^2 {}_2p_{63.25}$$
$$= 100000v^2 (0.9753757) = R90178.97$$

ii)

Under CFM:

$$EPV = 100000v^2 {}_2p_{63.25}$$
$${}_2p_{63.25} = {}_{0.75}\ p_{63.25} \times p_{64} \times {}_{0.25}\ p_{65}$$
$$p_{64} = 1 - q_{64} = 1 - 0.012716 = 0.987284$$
$${}_{0.25}p_{65} = (p_{65})^{0.25} = (1 - q_{65})^{0.25} = 0.985757^{0.25} = 0.99642$$
$${}_{0.75}p_{63.25} = (p_{63})^{0.75} = (1 - q_{63})^{0.75} = (0.988656)^{0.75} = 0.9914799$$
hence
$${}_2p_{63.25} = 0.9872584 \times 0.99642 \times 0.9914799 = 0.975343$$
$$EPV = 100000v^2 {}_2p_{63.25}$$
$$= 100000v^2 (0.975343) = R90175.90$$

EXAMINER COMMENTS:

Question performance: 60-65%

Students performed well in this question irrespective of the fact that the ELT15 table was not available at the start of the exam session. The results from both tables were there accepted and overall performance was pleasing.

QUESTION 9:

$$EPV = 240000v^{15}\left(\frac{l_{65}}{l_{50}}\right)\times\int_0^{\infty}1.02^t v^t\,{}_tp_{65}dt$$

$$1.02^t v^t = \left[\frac{1.02}{1.03}\right]^t = 0.9902913^t = e^{(\ln 0.9902913)t} = e^{-0.0097562t}$$

hence

$$1.02^t v^t\,{}_tp_{65} = e^{-0.0097562t}e^{-0.0502438t} = e^{-0.06t}$$

$$EPV = 240000v^{15}\left(\frac{9647.797}{9941.923}\right)\times\int_0^{\infty}e^{-0.06t}dt$$

with

$$\int_0^{\infty}e^{-0.06t}dt = \left[\frac{e^{-0.06t}}{-0.06}\right]_0^{\infty} = -\frac{0-e^0}{0.06} = \frac{1}{0.06}$$

so

$$EPV = (240000)(1.03^{-15})\left(\frac{9647.797}{9941.923}\right)\left(\frac{1}{0.06}\right) = R2\,491491.342$$

EXAMINER COMMENTS:

Question performance: 45-50%

Students' performance has been varied with students doing either well or not attempting the question at all. Time pressure is starting to show towards the end of the paper.

QUESTION 10:

The principle of an insurance policy being "self-funding" means that the premiums collected from policyholders should be sufficient to cover the claims, expenses, and other costs associated with providing the insurance cover. In other words, the insurance company should be able to pay for its liabilities (future claims), operational costs, and maintain its financial stability using the premiums it receives and initial reserves set up from its customers without relying on external funding or other sources of income.

The self-funding requirement mean that insurance company would need to set up reserves. Reserves are typically invested more conservatively meaning that overall profitability would likely be reduced.

EXAMINER COMMENTS:

Question performance: 50-55%

[GRAND TOTAL 100]

END OF EXAMINATION

