

# Actuarial Society of South Africa

## WRITTEN EXAMINATION

17 MAY 2022

### Subject A213 — Contingencies

*Time allowed:*

*Two hours and fifteen minutes – examination time  
20 minutes (at the end of the examination) – scan and upload time*

#### **INSTRUCTIONS TO THE CANDIDATE**

1. *Once you have entered the ASSA Exam Platform, ensure that you have accessed the **Video Room** Invigilation link with both your camera and microphone on, **before you attempt the examination.***
2. *Your PC must be placed, and camera angled, so that your writing area on your desk is visible to the invigilator.*
3. *Ensure that you have your candidate number handy to input **as part of the 2 hours 15 minutes examination.** Write your candidate number at the top of each page. (DO NOT WRITE YOUR NAME OR MEMBER NUMBER.)*
4. *Your cell phone that will be used to scan your final answer script must be switched **OFF** during the 2 hours and 15 minutes examination time. Place your cell phone at the top of your examination pad / writing pages in view of the invigilator.*
5. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may commence answering the paper whenever you are ready. You then have two hours to complete the paper.*
6. *Questions are only available in the ASSA Exam Platform and may not be printed or copied outside of the ASSA Exam Platform.*
7. *You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.*
8. *Attempt all questions, beginning your answer to each question on a new page and numbering your answers clearly.*
9. *Write in black or dark blue pen.*
10. *You should show calculations where this is appropriate.*
11. *You may not use any other computer program (e.g. Email, MS Word or Excel) or files, nor open any other browser during the examination.*

12. You **MAY NOT** make use of a *Formulae and Tables* book during the examination. Any such information that may be required will be provided to you within the examination.
13. Mark allocations are shown in brackets.
14. You may use additional scrap paper to make notes where this is appropriate. This paper **MUST NOT BE SCANNED** as part of your answer script.
15. Assume that months are all of equal length, unless otherwise stated.
16. At the end of the 2 hours and 15 minutes examination time, you must stop writing and may start scanning and uploading your script. **Do not continue writing into upload time.**
17. Access to your PC will be opened-up after the examination time so you can access your scanned file. You may now also switch on your cell phone to scan.
18. Scan **ALL** your answer pages to .pdf so that your candidate number at the top of the page is clear.
19. **Save your .pdf scanned file using your candidate number as file name. (DO NOT USE YOUR NAME OR MEMBER NUMBER AS FILE NAME.)**
20. Transfer your .pdf script to your PC and click on the **UPLOAD ANSWERS** link below the examination paper link.
21. Upload your answer file into the ASSA Exam Platform and ensure you click on **FINISH** below the upload box and again on **FINISH all and SUBMIT**, **before** the 20 minute upload time is up. (If the status on the summary page indicates “Answer saved” your file was uploaded. You can click on Review attempt to see the file you have uploaded.)
22. An option to opt out of the examination will become available one hour after the official examination start time. If you select the Opt-Out option, you agree and understand that your entire script/answers will be deleted and cannot be retrieved at a later stage and that your script or part thereof will not be put forward for marking.

**Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.**

**END OF INSTRUCTIONS**

## QUESTION 1

A joint life insurance policy is sold by a South African insurer. The policy covers two lives, a male and female.

The sum assured under the policy is R2 000 000 and is payable when the male dies within a 15-year period. The sum assured will be payable immediately on the male's death if the female dies before the male. However, if the female is still alive at the time of the male's death, the payment of the sum assured will be deferred until the end of the 15-year period. Both lives are aged 45 exactly.

- (i) Let the random variable  $T_{xy}$  represent the joint future lifetime of lives  $(x)$  and  $(y)$ . By first defining  $T_{xy}$  as a function of the complete future lifetime random variables of the two lives, prove that the probability density function of  $T_{xy}$  is  ${}_t p_{xt} p_y (\mu_{x+t} + \mu_{y+t})$ .

[8]

- (ii) Calculate the single premium for this policy.

Basis:

Mortality: Constant force of mortality for both lives of 0.004 at all ages.

Interest: Constant annual force of interest of 0.05 throughout.

Expenses: Initial underwriting expenses of R2 500 per policy.

Profit: Profit equal to 40% of single premium.

[9]

[Total 17]

## QUESTION 2

A life insurance company sells a 30-year with-profits endowment assurance policy to a life aged 35 exactly. The policy provides a basic sum assured of R1 100 000 plus declared bonuses.

Death benefits are paid immediately on death. A premium of R6 800 is payable quarterly in advance throughout the term of the policy or until earlier death.

By the end of the 25<sup>th</sup> policy year, the actual past bonuses that were added to the policy amounted to R1 595 000.

- i) Write down a generic expression for the profit that can be earned for the year between policy durations  $t$  and  $t+1$  clearly defining any notation used. [5]
- ii) Calculate the gross premium prospective reserve at the start of the 26<sup>th</sup> policy year immediately before the premium due. [9]

Basis:

Mortality: AM92 Ultimate

Interest: 4% per annum

Bonus loading: 4% of the sum assured and attaching bonuses, compounded and vesting at the end of each policy year.

Renewal commission: 2.75% of each quarterly premium

Renewal expenses: R990 at the start of each policy year

Claim expenses: R1 100 on death and R550 on maturity.

[Total 14]

**PLEASE TURN OVER**

### QUESTION 3

A company issues a 35-year non-profit endowment assurance policy. Level premiums are payable monthly in advance throughout the term of the policy. The sum assured is R1 000 000.

(i)

Calculate the monthly premium for a male aged 30 exactly using the equivalence principle.

Basis:

Mortality:	AM92 Select
Interest:	6% per annum
Initial expenses:	R2 500 plus 50% of the gross annual premium
Renewal expenses:	R750 quoted per annum at the start of the policy. Renewal expenses start at the start of the second policy year and inflate at 1.92308% per annum. An additional 2.5% of the second and subsequent monthly premiums is also incurred as an expense.
Claims expense:	R500 at the start of the policy inflating at 1.92308% per annum. The expense is incurred either on death or maturity.

[9]

(ii)

Explain the impact of the following adjustments on the premium calculated above. Support your answer with information from the actuarial tables where relevant (although no additional calculations are required):

- a) Renewal expenses are increased [1]
- b) Initial expenses are reduced [1]
- c) Mortality table is adjusted to AM92 Ultimate [4]

[Total 15]

### QUESTION 4

An insurer sells single premium deferred annuity policies to female lives aged 45 exactly.

Each policy provides an annuity income of R240 000 per annum payable annually in advance, commencing at age 60. The policy also provides for a death benefit of R1 200 000, payable immediately on death after age 60. All expenses relating to this policy are incurred at the beginning of a year.

Basis:

Mortality	AM92 Ultimate
Interest	6% per annum
Initial expenses	R5 000
Renewal expenses	R200 per annum payable from the 2 <sup>nd</sup> policy year onwards

- i. Calculate the prospective reserve at the end of the 20<sup>th</sup> policy year. [4]
- ii. There are 1 000 policies in force at the end of the 19<sup>th</sup> policy year. It is also known that 25 lives died during the 20<sup>th</sup> policy year. Calculate the mortality profit during the 20<sup>th</sup> policy year. [5]
- iii. Explain whether your answer above in (ii) is reasonable. [3]

[Total 12]

**PLEASE TURN OVER**

### QUESTION 5

An insurer has recently completed a mortality investigation. The following select and ultimate mortality values have been provided by the actuarial team.

Age	Duration 0 $q_{[x]}$	Duration 1 $q_{[x-1]+1}$	Duration 2+ $q_x$
35	0.000638		
36	0.000724	0.001100	
37	0.000918	0.001084	0.001323
38		0.001330	0.001454
39			0.001528

Calculate the values of  $l_{[35]}$ ,  $l_{[36]}$ ,  $l_{[35]+1}$  and  $l_{[36]+1}$  assuming that  $l_{37} = 3000$ .

[Total 5]

### QUESTION 6

Let  $K$  denote the curtate future lifetime random variable for a life aged  $x$ .

- i) Write down an expression for the present value random variable,  $Z$ , representing an annuity that pays R200 000 annually in advance for a maximum of ten years and ceasing on earlier death. [3]
- ii) Derive the standard deviation of the above policy assuming a constant force of mortality of 2% per year and a constant force of interest of 4% per year for all ages. [11]
- iii) Describe what your result in ii) means for the insurer. [3]

[Total 17]

### QUESTION 7

An insurer has recently launched a 20-year with-profits endowment assurance policy. The policy pays a sum assured of R1 000 000 to a life aged 45 exactly. Level premiums are payable monthly in advance.

The sum assured plus declared bonuses are payable at the end of year of death or on maturity of the policy, if earlier.

A simple bonus vests at the beginning of each year including the first. Calculate the level simple bonus rate that can be supported each year if the monthly premium is R5 500.

Basis:

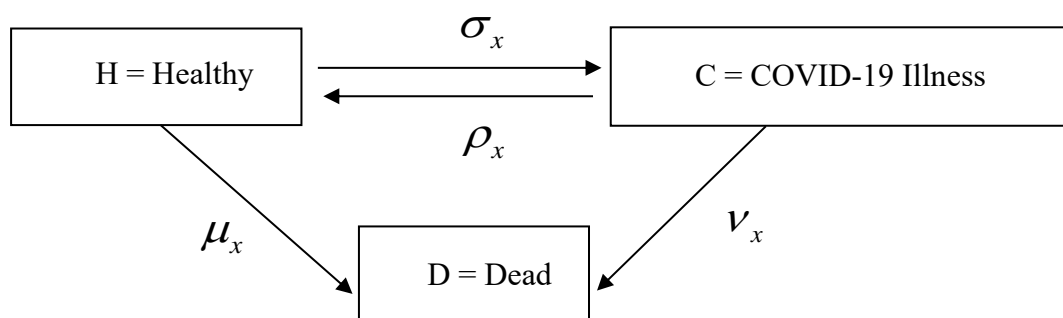
Mortality	AM92 Ultimate
Rate of interest	4% per annum
Initial expenses	25% of the first year's premiums
Renewal expenses	3.5% of each premium payable
Claim expenses	R2 000 at termination of the contract

[Total 13]

**PLEASE TURN OVER**

## QUESTION 8

Consider the following multi-state model:



A policyholder is aged 30 exactly and takes out a whole life insurance policy that pays the following benefits:

- R20 000 per year is paid continuously while the policyholder is sick with COVID-19
- R200 000 is paid on the death of policyholder following death whilst in the COVID-19 illness state
- R400 000 is paid on the death of policyholder following death whilst in the healthy state
- A COVID-19 sickness income benefit of R200 000 per annum, whilst the life remains in the COVID-19 state, but paying for a maximum period of 24 months.

Premiums are paid continuously up until age 65 while the policyholder is healthy.

Write down an expression in terms of integrals that can be used to calculate:

- The expected present value of the premiums only. [2]
- The expected present value of the death benefit. [2]
- The expected present value of the sickness benefits. [3]

Assume interest at a continuous force of interest of  $\delta$ .

[Total 7]

[GRAND TOTAL 100]

**END OF EXAMINATION**