Actuarial Society of South Africa

EXAMINATION SAMPLE SOLUTIONS AND EXAMINER COMMENTS

17 MAY 2022

Subject A213 — Contingencies

WRITTEN EXAM

QUESTION 1

A joint life insurance policy is sold by a South African insurer. The policy covers two lives, a male and female.

The sum assured under the policy is R2 000 000 and is payable when the male dies within a 15-year period. The sum assured will be payable immediately on the male's death if the female dies before the male. However, if the female is still alive at the time of the male's death, the payment of the sum assured will be deferred until the end of the 15-year period. Both lives are aged 45 exactly.

(i) Let the random variable T_{xy} represent the joint future lifetime of lives (x) and (y). By first defining T_{xy} as a function of the complete future lifetime random variables of the two lives, prove that the probability density function of T_{xy} is ${}_t p_{xt} p_y \left(\mu_{x+t} + \mu_{y+t}\right)$.

[8]

(ii) Calculate the single premium for this policy.

Basis:
Mortality: Constant force of mortality for both lives of 0.004 at all ages.
Interest: Constant annual force of interest of 0.05 throughout.
Expenses: Initial underwriting expenses of R2 500 per policy.
Profit: Profit equal to 40% of single premium.

[9]

[Total 17]

Examiner comments:

Generally, students performed below expectations on part i) which was a standard bookwork questions. In many cases, this question was the result of student not achieving a pass grade and should be a focus area for those attempting the exam in future. Part ii) was generally well done by well-prepared students.

Question average performance: 35% to 40%

Sample Solution:

i)

$T_{xy} = \min\left\{T_x; T_y\right\}$

CDF

$F_{T_{xy}}(t) = P\left[T_{xy} \leq t\right]$

$or\ F_{T_{xy}}(t) = 1 - {}_t\, p_{xy}$

$= P\left[\min\left\{T_x; T_y\right\} \leq t\right]$

$= 1 - P\left[T_x > t\ and\ T_y > t\right]$

$= 1 - P\left[T_x > t\right] P\left[T_y > t\right]$

From independence

Hence $F_{T_{xy}}(t) = 1 - {}_t\, p_{xt} p_y$

Now, the density function of T_{xy} can be obtained by differentiating the CDF

$$f_{T_{xy}}(t) = \frac{d}{dt}\left[1 - {}_t p_x {}_t p_y\right]$$

$$= -{}_t p_x \frac{d}{dt} {}_t p_y - {}_t p_y \frac{d}{dt} {}_t p_x$$

Using product rule

$$= -{}_t p_x \left(-{}_t p_y \mu_{y+t}\right) - {}_t p_y \left(-{}_t p_x \mu_{x+t}\right)$$

Hence $f_{T_{xy}}(t) = {}_t p_x {}_t p_y \left(\mu_{x+t} + \mu_{y+t}\right)$

ii)

$Let\ x = male\ and\ y = female$

We value two components: a) and b). The assurance is payable immediately on x's death if x dies after y and within 15 years.

The first part a)

$$EPV = 2\,000\,000 \bar{A}^{2}_{45:45:\overline{15}|}$$

$$= 2\,000\,000\left(\bar{A}^{1}_{45:\overline{15}|} - \bar{A}^{1}_{45:45:\overline{15}|}\right)$$

and then b) the assurance is payable when x dies while y is alive, in which case the payment is only made at the end of 15 years.

$$EPV = 2\,000\,000\, e^{-15\delta}\, {}_{15}q^{1}_{45:45}$$

Now, the function values:

$$\bar{A}^{1}_{45:\overline{15}|} = \int_0^{15} e^{-\delta t}\, {}_t p_{45} \mu_{45+t} dt = \int_0^{15} e^{-0.05t} e^{-0.004t}\, 0.004 dt = 0.0411216$$

$$\bar{A}^{1}_{45:45:\overline{15}|} = \int_0^{15} e^{-\delta t}\, {}_t p_{45:45} \mu_{45+t} dt = \int_0^{15} e^{-0.05t} e^{-0.008t}\, 0.004 dt = 0.040072$$

$${}_{15}q^{1}_{45:45} = \int_0^{15} {}_t p_{45:45} \mu_{45+t} dt = \int_0^{15} e^{-0.008t}\, 0.004 dt = 0.05654$$

Hence, let P be the premium solving the equation:

$$P = 0.4P + 2500 + 2000000\left(0.0411216 - 0.04007\right) + 2000000 e^{-15(0.05)}\, 0.05654$$

$$\therefore P = R96\,697.35$$

PLEASE TURN OVER

QUESTION 2

A life insurance company sells a 30-year with-profits endowment assurance policy to a life aged 35 exactly. The policy provides a basic sum assured of R1 100 000 plus declared bonuses.

Death benefits are paid immediately on death. A premium of R6 800 is payable quarterly in advance throughout the term of the policy or until earlier death.

By the end of the 25th policy year, the actual past bonusses that were added to the policy amounted to R1 595 000.

i) Write down a generic expression for the profit that can be earned for the year between policy durations *t* and *t+1* clearly defining any notation used. [5]

ii) Calculate the gross premium prospective reserve at the start of the 26th policy year immediately before the premium due. [9]

Basis:

Mortality:	AM92 Ultimate
Interest:	4% per annum
Bonus loading:	4% of the sum assured and attaching bonuses, compounded and vesting at the end of each policy year.
Renewal commission:	2.75% of each quarterly premium
Renewal expenses:	R990 at the start of each policy year
Claim expenses:	R1 100 on death and R550 on maturity.

[Total 15]

Examiner comments:

Part i) was done well by most students. In part ii) many students did not use the correct sum assured in calculating the prospective reserve.

Question average performance: 45% to 50%

Sample Solution:

i) $PRO_t = \left({}_tV' + G - e\right)(1+i) - q_{x+t}\left(S+f\right) - p_{x+t} \cdot {}_{t+1}V'$ with PRO being the profit earned between t and t+1; tV being the gross premium reserve; G being the Gross premiums, e being renewal expenses, i being interest, qx being mortality rate, S being sum assured, f being claim expenses and p being survival probability

ii)

SA plus bonusses declared: R2 695 000

EPV of death benefits

$$= 2\,695\,000\left[q_{60}v^{\frac{1}{2}} + 1.04\,{}_{1|}q_{60}v^{1\frac{1}{2}} + 1.04^2\,{}_{2|}q_{60}v^{2\frac{1}{2}} + 1.04^3\,{}_{3|}q_{60}v^{3\frac{1}{2}} + 1.04^4\,{}_{4|}q_{60}v^{4\frac{1}{2}}\right]$$

$$2\,695\,000\,v^{\frac{1}{2}}\left[q_{60} + {}_{1|}q_{60} + {}_{2|}q_{60} + {}_{3|}q_{60} + {}_{4|}q_{60}\right]$$

Simplified to = $= 2695000v^{\frac{1}{2}}\,{}_5q_{60} = 2695000\dfrac{1}{1.04^{\frac{1}{2}}}\left(1 - \dfrac{8821.2612}{9287.2164}\right)$

$$= R132\,586.92$$

Similarly, the EPV of survival benefits:

$$2695000 \times {}_5p_{60} \times v^5 \times 1.04^5 = 2695000\frac{8821.2612}{9287.2164} = 2559787.337$$

EPV of claim expenses on maturity:
$$550 \times {}_5p_{60} \times v_{4\%}^{5} = 429.3795$$

EPV of claim expenses on death:
$$1100\bar{A}^{1\;4\%}_{60:\overline{5}|} = 1100 \times 1.05^{\frac{1}{2}}\left[A_{60:\overline{5}|} - \frac{D_{65}}{D_{60}}\right] = 1100 \times 1.05^{\frac{1}{2}}\left[0.82499 - \frac{689.23}{882.85}\right] = 49.70$$

So the gross prospective reserve at time 25 =

$$ {}_{25}V^{pro} = 132586.92 + 2559787.337 + 990\ddot{a}^{4\%}_{60:\overline{5}|} + 48.7322 + 429.3795$$
$$+0.0275 \times 4 \times 6800\ddot{a}^{(4)\,4\%}_{60:\overline{5}|} - 4 \times 6800\ddot{a}^{(4)\,@4\%}_{60:\overline{5}|}$$

With the following values:

$$\ddot{a}^{4\%}_{60:\overline{5}|} = 4.550$$
$$\ddot{a}^{(4)}_{60:\overline{5}|} \approx 4.550 - \frac{3}{8}\left(1 - \frac{D_{65}}{D_{60}}\right) = 4.468$$
$$ {}_{25}V^{pro} = 132586.92 + 2559787.337 + 990(4.550) + 49.70 + 429.3795$$
$$+0.0275 \times 4 \times 6800(4.468) - 4 \times 6800(4.468)$$
$$= 2\,579\,170.30$$

QUESTION 3

A company issues a 35-year non-profit endowment assurance policy. Level premiums are payable monthly in advance throughout the term of the policy. The sum assured is R1 000 000.

(i)

Calculate the monthly premium for a male aged 30 exactly using the equivalence principle.

Basis:

Mortality:	AM92 Select
Interest:	6% per annum
Initial expenses:	R2 500 plus 50% of the gross annual premium
Renewal expenses:	R750 quoted per annum at the start of the policy. Renewal expenses start at the start of the second policy year and inflate at 1.92308% per annum. An additional 2.5% of the second and subsequent monthly premiums is also incurred as an expense.
Claims expense:	R500 at the start of the policy inflating at 1.92308% per annum. The expense is incurred either on death or maturity.

[9]

(ii)

Explain the impact of the following adjustments on the premium calculated above. Support your answer with information from the actuarial tables where relevant (although no additional calculations are required):

a) Renewal expenses are increased [1]
b) Initial expenses are reduced [1]
c) Mortality table is adjusted to AM92 Ultimate [4]

[Total 15]

Examiner comments:

Part i) was done well by most students. In part ii) many students did not reference how they made use of the actuarial tables in their answer.

Question average performance: 60% to 65%

Sample Solution

Assuming benefits paid at the end of year:

Let P be the monthly premiums in EPV Premiums = EPV Benefits + EPV Expenses

EPV of premiums:

$$12P\ddot{a}^{(12)}_{[30]:\overline{35}|} = 12P\left[\ddot{a}_{[30]:\overline{35}|} - \frac{11}{24}\left(1 - v^{35}\frac{l_{65}}{l_{[30]}}\right)\right] = 176.96P$$

EPV of benefits:

$$1000000A_{[30]:\overline{35}|} = 1000000(0.14234) = 142340$$

EPV of inflationary expenses @ j = 4%

$$750\left(\ddot{a}_{[30]:\overline{35}|} - 1\right) + 500A_{[30]:\overline{35}|} = 13687.235$$

EPV of non-inflationary expenses:

$$2500 + 0.5\times12P + 0.025\times12\times P\ddot{a}^{(12)}_{[30]:\overline{35}|} - 0.025P = 2500 + 10.399P$$

Noting that @ 4%:

$$\ddot{a}_{[30]:\overline{35}|} = 19.072$$

$$A_{[30]:\overline{35}|} = 0.26647$$

$$\ddot{a}_{[30]:\overline{35}|} @6\% = 15.152$$

$$A_{[30]:\overline{35}|} @6\% = 0.14234$$

Hence solving for P = R951.77

Alternatively, assuming benefits paid immediately:

EPV of benefits:

$$1000000\bar{A}_{[30]:\overline{35}|} = 1000000\times1.06^{\frac{1}{2}}(0.14234) = 146547.9994$$

Solving for P = R977.03

ii)

a) The premium will be higher because the expense allowance is higher.

b) The premium will be lower premium because the expense allowance is lower.

c) The benefit of the assurance benefit increases when considering the assurance factors and the EPV of premiums also decreases when considering the annuity factors as per the tables, therefore the premium should increase.

$\ddot{a}_{[30]:\overline{35|}} = 19.072 > \ddot{a}_{30:\overline{35|}} = 19.069$

$A_{[30]:\overline{35|}} = 0.26647 < A_{30:\overline{35|}} = 0.26657$

QUESTION 4

An insurer sells single premium deferred annuity policies to female lives aged 45 exactly.

Each policy provides an annuity income of R240 000 per annum payable annually in advance, commencing at age 60. The policy also provides for a death benefit of R1 200 000, payable immediately on death after age 60. All expenses relating to this policy are incurred at the beginning of a year.

Basis:	
Mortality	AM92 Ultimate
Interest	6% per annum
Initial expenses	R5 000
Renewal expenses	R200 per annum payable from the 2nd policy year onwards

i. Calculate the prospective reserve at the end of the 20th policy year. [4]

ii. There are 1 000 policies in force at the end of the 19th policy year. It is also known that 25 lives died during the 20th policy year. Calculate the mortality profit during the 20th policy year. [5]

iii. Explain whether your answer above in ii) is reasonable. [3]

[Total 12]

Examiner comments:

This question was well done by most. The comment mistakes included not allowing for the half year interest adjustment in the DSAR calculation and including the annuity amount in the DSAR calculation which is incorrect due to the timing of the annuity payments.

Question average performance: 65% to 70%

Sample Solution

(i) $_{20}V = (240000 + 200)\ddot{a}_{65} + SA\bar{A}_{65}$

$= (240\,200 \times 10.569) + 1200\,000(1.06)^{0.5} * (0.40177)$

= 3 035 050.84

(ii) DSAR = $1200000(1+i)^{0.5} - {_{20}V}$

= 1200000(1.06)$^{0.5}$- 3 035 050.84

= -1 799 575.223

ADS = 25×-1 799 575.223

=- 44 989 380.58

EDS = 1 000× q_{64} ×DSAR

= 1 000× 0.012716×(-1 799 575.223)

= -22 883 398.54

Mortality Profit = EDS-ADS

=-22 883 398.54 – (-44 989 380.58)

= <u>22 105 982.04</u>

(iii) The insurer has incurred a mortality profit because

- The death strain at risk is negative in the 20th year
- Actual number of deaths (25) is greater than the expected number of deaths (12.7)
- With negative death strain at risk, more reserves are released on each death, hence the prof

QUESTION 5

An insurer has recently completed a mortality investigation. The following select and ultimate mortality values have been provided by the actuarial team.

Age	Duration 0 $q_{[x]}$	Duration 1 $q_{[x-1]+1}$	Duration 2+ q_x
35	0.000638		
36	0.000724	0.001100	
37	0.000918	0.001084	0.001323
38		0.001330	0.001454
39			0.001528

Calculate the values of $l_{[35]}$, $l_{[36]}$, $l_{[35]+1}$ and $l_{[36]+1}$ assuming that $l_{37} = 3000$.

[Total 5]

Examiner comments:

Overall performance on this question was disappointing given the fact that the question assesses the basic construction of a life table which is fundamental to the subject.

Question average performance: 45% to 50%

Sample Solution

$$l_{37} = 3000$$

$$l_{[35]+1} = \frac{l_{37}}{1-q_{[35]+1}} = 3003.30$$

$$l_{[35]} = \frac{l_{[35]+1}}{1-q_{[35]}} = 3005.22$$

$$l_{[36]+1} = \frac{l_{38}}{1-q_{[36]+1}} = 2999.28$$

$$l_{[36]} = \frac{l_{[36]+1}}{1-q_{[36]}} = 3001.46$$

QUESTION 6

Let K denote the curtate future lifetime random variable for a life aged x.

i) Write down an expression for the present value random variable, Z, representing an annuity that pays R200 000 annually in advance for a maximum of ten years and ceasing on earlier death. [3]

ii) Derive the standard deviation of the above policy assuming a constant force of mortality of 2% per year and a constant force of interest of 4% per year for all ages. [11]

iii) Describe what your result in ii) means for the insurer. [3]

[Total 17]

Examiner comments:

Students generally scored well below expectations on this question which was testing basic application of relatively standard bookwork. Part iii) was most concerning with many students not being able to interpret what the standard deviation means.

Question average performance: 35% to 40%

Sample Solution

i)

$$Z = 200\ 000 \begin{cases} \ddot{a}_{\overline{K_x+1}|} & K_x < 10 \\ \ddot{a}_{\overline{10}|} & K_x \geq 10 \end{cases}$$

or

$$Z = 200\ 000\, \ddot{a}_{\overline{\min(K_x+1,10)}|}$$

ii)

EPV=

$$E[Z] = 200000\, \ddot{a}_{x:\overline{10}|}$$

$$= 200000 \sum_{k=0}^{9} v^k\, {}_k p_x$$

$$= 200000 \sum_{k=0}^{9} e^{-0.04k} e^{-0.02k}$$

$$= 200000 \left(\frac{1-e^{-0.06(10)}}{1-e^{-0.06}} \right)$$

$$= 1549531.2$$

$$\text{var}\left[Z\right]=E\left[Z^2\right]-E^2\left[Z\right]$$

$$\text{var}\left[Z\right]=200000^2\ \text{var}\left[\ddot{a}_{\overline{\min(K_x+1,10)}|}\right]$$

$$=\frac{200000^2}{d^2}\text{var}\left[v^{\min[K+1,10]}\right]$$

and

$$\text{var}\left[v^{\min[K+1,10]}\right]={}^2A_{x:\overline{10}|}-\left(A_{x:\overline{10}|}\right)^2$$

hence

$$A_{x:\overline{10}|}=1-da_{x:\overline{10}|}=1-\left(1-e^{-0.04}\right)\times 7.747650=0.696210$$

$$\ddot{a}_{x:\overline{10}|}^{\delta=8\%}=\sum_{k=0}^{9}e^{-0.08k}e^{-0.02k}=\frac{1-e^{-0.1(10)}}{1-e^{-0.1}}=6.642533$$

hence

$${}^2A_{x:\overline{10}|}=1-\left(1-e^{-0.08}\right)\times 6.642533=0.489298$$

$$\therefore\ \text{var}\left[v^{\min[K+1,10]}\right]=0.004589$$

so

$$stdev=\sqrt{\text{var}(z)}=\sqrt{\frac{200000^2}{d^2}0.004589}=345\ 530.17$$

iii) The standard deviation of the present value of the policy represents the degree of variability or risk associated with the policy in terms of its value to the insurer.

A higher stdev means the policy present value is more uncertain and would therefore be riskier for the insurer to sell than a lower stdev.

Assuming a normal distribution, 95% of the actual values that is likely to be observed in real life would be expected to fall within the range approximately R1.5 +- 1.96*(0.3m) [max 3]

QUESTION 7

An insurer has recently launched a 20-year with-profits endowment assurance policy. The policy pays a sum assured of R1 000 000 to a life aged 45 exactly. Level premiums are payable monthly in advance.

The sum assured plus declared bonuses are payable at the end of year of death or on maturity of the policy, if earlier.

A simple bonus vests at the beginning of each year including the first. Calculate the level simple bonus rate that can be supported each year if the monthly premium is R5 500.

Basis:

Mortality	AM92 Ultimate
Rate of interest	4% per annum
Initial expenses	25% of the first year's premiums
Renewal expenses	3.5% of each premium payable
Claim expenses	R2 000 at termination of the contract

[Total 13]

Examiner comments:

Many students performed well on this question. Answers that were clearly laid out with the values of intermediate steps showed scored better.

Question average performance: 55% to 60%

Sample Solution

$$0.965(12)P\left(\ddot{a}^{(12)}_{45:\overline{20}|}\right) = 1000000A^{@4\%}_{45:\overline{20}|} + 0.25\times 12P\times \ddot{a}^{(12)}_{45:\overline{1}|} + 2000A_{45:\overline{20}|} + B\left(IA\right)_{45:\overline{20}|}$$

With

$$A_{45:\overline{20}|} = 0.46998$$

$$\ddot{a}^{(12)}_{45:\overline{20}|} = \ddot{a}_{45:\overline{20}|} - \frac{11}{24}\left(1-\frac{D_{65}}{D_{45}}\right) = 13.78 - \frac{11}{24}\left(1-\frac{689.23}{1677.97}\right)$$

$$= 13.78 - 0.27007187$$

$$= 13.509928$$

$$\left(IA\right)_{45:\overline{20}|} = \left(IA\right)_{45} - \frac{D_{65}}{D_{45}}\left(20A_{65} + \left(IA\right)_{65} - 20\right)$$

$$= 8.33628 - \frac{689.23}{1677.97}\left(20\times 0.52786 + 7.89442 - 20\right) = 8.9722806$$

$$\therefore 0.965(12)5500\left(13.509928\right) = 1000000(0.46998) + 0.25\times 12(5500)\times 0.981726431$$

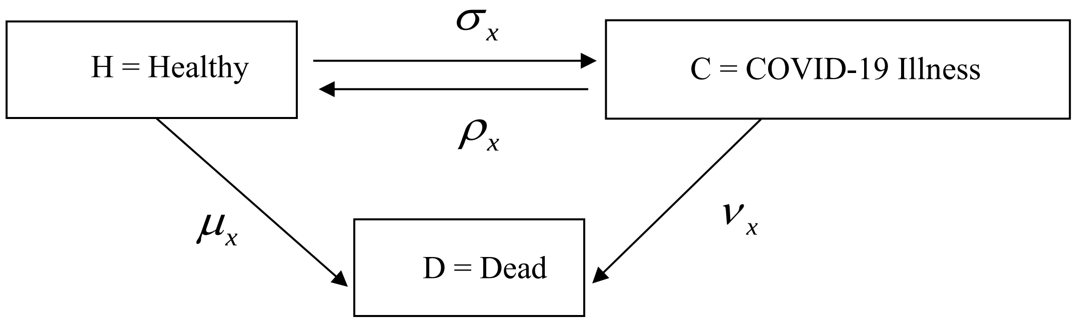
$$+2000(0.46998) + B(8.972280612)$$

$$\therefore B = 41609.14$$

or as percentage 4.16%

QUESTION 8

Consider the following multi-state model:



A policyholder is aged 30 exactly and takes out a whole life insurance policy that pays the following benefits:

- R20 000 per year is paid continuously while the policyholder is sick with COVID-19
- R200 000 is paid on the death of policyholder following death whilst in the COVID-19 illness state
- R400 000 is paid on the death of policyholder following death whilst in the healthy state
- A COVID-19 sickness income benefit of R200 000 per annum, whilst the life remains in the COVID-19 state, but paying for a maximum period of 24 months,

Premiums are paid continuously up until age 65 while the policyholder is healthy.

Write down an expression in terms of integrals that can be used to calculate:

i) The expected present value of the premiums only. [2]

ii) The expected present value of the death benefit. [2]

iii) The expected present value of the sickness benefits. [3]

Assume interest at a continuous force of interest of δ.

[Total 7]

Examiner comments:

Time management appeared to become an issue for many students which resulted in an underperformance in this question.

Question average performance: 45% to 50%

Sample Solution:

Present value of Premiums $= P\int_0^{35} e^{-\delta t}\,{}_tp_{30}^{HH}\,dt$

Death benefits $= 200\,000\int_0^{\infty} e^{-\delta t}\left(2\,{}_tp_{30}^{HH}\,\mu_{30+t} + {}_tp_{30}^{HC}\,\nu_{30+t}\right)dt$

Sickness benefits $= (200000)\int_0^{\infty} e^{-\delta t}\,{}_tp_{30}^{HH}\,\sigma_{30+t}\left(\int_0^{2} e^{-\delta s}\,{}_sp_{30+t}^{\overline{CC}}\,ds\right)dt + 20000\int_0^{\infty} e^{-\delta t}\,{}_tp_{30}^{HC}\,dt$

[GRAND TOTAL 100]

