Actuarial Society of South Africa

WRITTEN EXAMINATION

APRIL 2020 SAMPLE SOLUTIONS

Subject A213 — Contingencies

Examiner's Comments:

Generally, students would have scored better marks where the following exam technique has been applied:

- Label each question clearly
- Start each question on a new page
- Use more space than less and make it clear how you break up key parts of a question. This makes it easier to see your workings and to understand what was done in order to give principle marks for incorrect solutions.
- Some students lost marks because the electronic scans were not clear.

QUESTION 1

A standard book work question in which students did surprisingly poorly. Many students got the main idea but were not able to provide a complete answer and hence lost out on some easy marks.

Average mark: 30-40%

Features of accumulating with-profits

The benefits take the form of an accumulating fund of premiums

The fund accumulates with interest

- Where the interest rate may be partly guaranteed
- With the remainder being discretionary 'bonus' interest which can vary over time

Interest rates cannot be negative

They will reflect the underlying profits made by the insurer (including investment profits)

- But will be smoothed over time so as to produce a more stable progression compared to the underlying asset returns

On death or maturity a terminal bonus can be paid out in addition to the accumulated fund value

At the discretion of the insurance company

QUESTION 2

A standard calculation question in which many students did well. Some maybe took too long on this question so time management on this question was key.

Average mark: 70-80%

i)

$$P\ddot{a}_{\overline{50:50}} = 500000\bar{A}_{\overline{50:50}} + 500 + 0.05P\left(\ddot{a}_{\overline{50:50}} - 1\right)$$

$$\ddot{a}_{\overline{50:50}} = \ddot{a}_{50}{}^{m} + \ddot{a}_{50}{}^{f} - \ddot{a}_{50:50} = 18.843 + 19.539 - 17.688 = 20.694$$

$$\bar{A}_{\overline{50:50}} \approx\sim 1.04^{0.5} A_{\overline{50:50}} = 1.04^{0.5}\left(1 - d\ddot{a}_{\overline{50:50}}\right) = 1.04^{0.5}\left(1 - \frac{0.04}{1.04} \times 20.694\right) = 0.208118$$

Hence $P = \dfrac{104559}{19.7093} = 5305.06$

ii)

Just before the 11th premium payment is time 10 and both parents are aged 60.

$${}_{10}V = 500000\bar{A}_{\overline{60:60}} + 0.05P\left(\ddot{a}_{\overline{60:60}}\right) - P\ddot{a}_{\overline{60:60}}$$

$$P = 5305.06$$

$$\ddot{a}_{\overline{60:60}} = \ddot{a}_{60}{}^{m} + \ddot{a}_{60}{}^{f} - \ddot{a}_{60:60} = 15.632 + 16.652 - 14.090 = 18.194$$

$$\bar{A}_{\overline{60:60}} \approx\sim 1.04^{0.5} A_{\overline{60:60}} = 1.04^{0.5}\left(1 - d\ddot{a}_{\overline{60:60}}\right) = 1.04^{0.5}\left(1 - \frac{0.04}{1.04}\times 18.194\right) = 0.30617651$$

$${}_{10}V = 500000\left(0.30617651\right) - 0.95P\left(18.194\right) = R61394$$

QUESTION 3

Students who noted the major aspect was to split the whole life terms in two components scored well on this question; as follows:

$$A_x \quad = \quad A_{\substack{1\\x:\overline{t}|}} + v^t \cdot \frac{l_{x+t}}{l_x} \cdot A_{x+t}$$

$$\ddot{a}_x \quad = \quad \ddot{a}_{x:\overline{t}|} + v^t \cdot \frac{l_{x+t}}{l_x} \cdot \ddot{a}_{x+t}$$

Average mark: 60-70%

(i) Gross premium retrospective and prospective reserves will be equal if:

(1) the retrospective and prospective reserves are calculated on the same basis, and

(2) this basis is the same as the basis used to calculate the premiums used in the reserve calculation.

(ii)

The original gross premium is given by:

$$G\ddot{a}_x - SA_x - e\ddot{a}_x - fA_x - I \quad = \quad 0$$

$$\left(G - e\right)\cdot\ddot{a}_x - \left(S + f\right)\cdot A_x - I \quad = \quad 0 \quad (1)$$

where:

- S is the sum assured;
- I is the initial expense;
- e is the annual renewal expenses;
- f is the claim expense; and
- G is the annual gross premium.

The prospective reserve at time t $(t=1,2,\dots)$ is given by

$$\begin{aligned} {}_tV^{pro} &= SA_{x+t} + e\ddot{a}_{x+t} + fA_{x+t} - G\ddot{a}_{x+t} \\ &= (S+f)\cdot A_{x+t} - (G-e)\cdot \ddot{a}_{x+t} \quad (2) \end{aligned}$$

Considering equation (1), we consider the whole life assurance and annuity functions:

$$A_x = A_{\overset{1}{x}:\overline{t}|} + v^t \cdot \frac{l_{x+t}}{l_x} \cdot A_{x+t} \quad (3)$$

$$\ddot{a}_x = \ddot{a}_{x:\overline{t}|} + v^t \cdot \frac{l_{x+t}}{l_x} \cdot \ddot{a}_{x+t} \quad (4)$$

Substituting (3) and (4) in equation (1) yields:

$$(G-e)\cdot\left(\ddot{a}_{x:\overline{t}|} + v^t \cdot \frac{l_{x+t}}{l_x}\cdot \ddot{a}_{x+t}\right) - (S+f)\cdot\left(A_{\overset{1}{x}:\overline{t}|} + v^t \cdot \frac{l_{x+t}}{l_x}\cdot A_{x+t}\right) - I = 0 \quad (4)$$

Grouping like-terms (namely those that relate to the first t years, and those thereafter) in equation (4) results in:

$$(G-e)\cdot\ddot{a}_{x:\overline{t}|} - (S+f)\cdot A_{\overset{1}{x}:\overline{t}|} - I = (S+f)\cdot\left(v^t \cdot \frac{l_{x+t}}{l_x}\cdot A_{x+t}\right) - (G-e)\cdot\left(v^t \cdot \frac{l_{x+t}}{l_x}\cdot \ddot{a}_{x+t}\right) \quad (5)$$

Multiplying both sides of equation (5) by $(1+i)^t \times \frac{l_x}{l_{x+t}}$ gives:

$$\begin{aligned} (1+i)^t \times \frac{l_x}{l_{x+t}} \times \left[(G-e)\cdot\ddot{a}_{x:\overline{t}|} - (S+f)\cdot A_{\overset{1}{x}:\overline{t}|} - I\right] &= (S+f)\cdot A_{x+t} - (G-e)\cdot\ddot{a}_{x+t} \quad (6) \\ {}_tV^{retro} &= {}_tV^{pro} \end{aligned}$$

Clearly in equation (6), the left-hand side is the gross premium retrospective reserve, and the right-hand side is the gross premium prospective reserve.

QUESTION 4

A non-standard question which students generally struggled with as it required of student to think through the question from first principles rather than to regurgitate a standard method which they might be used to. It was encouraging to see some students that scored well.

Average mark: 20-30%

The value of 1 per annum payable monthly for 1 year is: $\ddot{a}^{(12)}_{x:\overline{1}|}$

$$\ddot{a}^{(12)}_{x:\overline{1}|} = \ddot{a}_{x:\overline{1}|} - \frac{11}{24}(1 - v \cdot p_x)$$

where:

$\ddot{a}_{x:\overline{1}|} = 1$; and

$p_x = (1 - 0.005) = 0.995$

Therefore:

$$a^{(12)}_{x:\overline{1}|} = 1 - \frac{11}{24}\left[1 - \frac{0.995}{1.075}\right] = 0.965891473$$

The respective probabilities of reaching the beginning of each year are:

Year 1 = 1

Year 2 = $0.995 * 0.95 = 0.94525$

Year 3 = $0.94525 * 0.995 * 0.9 = 0.846471375$

Expected Present Value of 150 contracts is:

$$= 150 * (100 - 5) * 12 * 0.965891473 * \left[1 + \frac{0.94525}{1.075} + \frac{0.846471375}{(1.075)^2}\right] - (15 * 150)$$

$= R429,131.29$

QUESTION 5

Students performed reasonably well on this question, but many wasted unnecessary time by first calculating premiums which was not necessary.

Average mark: 60-70%

i) The expected death strain is the amount the insurer expects to pay on death over and above the reserves held for policies at the end of the year.

ii) Let us first consider the Death Strain at Risk (DSAR).

$$\text{DSAR} = 500,000 \cdot \left(1 - {}_1V_{45:\overline{15}|}\right)$$

with:

$${}_1V_{45:\overline{15}|} = 1 - \frac{\ddot{a}_{46:\overline{14}|}}{\ddot{a}_{45:\overline{15}|}}$$

$$1 - {}_1V_{45:\overline{15}|} = \frac{\ddot{a}_{46:\overline{14}|}}{\ddot{a}_{45:\overline{15}|}}$$

$$= \frac{9.712}{10.149}$$

$$= 0.95694157$$

$$\text{DSAR} = 500,000 \cdot \left(1 - 0.95694157\right) = 478,470.785$$

Finally, the Expected Death Strain (EDS) is:

$$\text{EDS} = 100 * q_{45} \ * \text{DSAR}$$

$$\text{EDS} = 100 * 0.001465 * 478,470.785 = R70,095.97$$

iii) First, let us first consider the Death Strain at Risk of the "Healthy" lives (DSAR-H).

$$\text{DSAR-H} = 500,000 \cdot \left(1 - {}_1V_{[45]:\overline{15}|}\right)$$

$${}_1V_{[45]:\overline{15}|} = 1 - \frac{\ddot{a}_{[45]+1:\overline{14}|}}{\ddot{a}_{[45]:\overline{15}|}}$$

$$1 - {}_1V_{[45]:\overline{15}|} = \frac{\ddot{a}_{[45]+1:\overline{14}|}}{\ddot{a}_{[45]:\overline{15}|}}$$

$$= \frac{9.712^{*1}}{10.151}$$

$$= 0.956753029$$

$$\ddot{a}_{[x]+1:\overline{14}|} = (\ddot{a}_{[x]:\overline{15}|} - 1) * \frac{l_{[x]}}{l_{[x]+1}} * (1+i); \quad \text{or}$$

$$\ddot{a}_{[x]+1:\overline{14}|} = 1 + v * \frac{l_{x+2}}{l_{[x]+1}} * \ddot{a}_{x+2:\overline{13}|}$$

$$\ddot{a}_{[45]+1:\overline{14}|} = (10.151-1)*(1.06)*\frac{9,798.0837}{9,786.3162} = 9.711723782 = 9.712 \ \text{(3 DP)}$$

So that: $\text{DSAR-H} = 500,000 \cdot \left(1 - {}_1V_{[45]:\overline{15}|}\right) = 478,376.5146$

Secondly, the Death Strain at Risk of the "Unhealthy" lives (DSAR-U) is:

$$\text{DSAR-U} = 500,000 \cdot \left(1 - {}_1V_{[50]:\overline{15}|}\right)$$

$${}_1V_{[50]:\overline{15}|} = 1 - \frac{\ddot{a}_{[50]+1:\overline{14}|}}{\ddot{a}_{[45]:\overline{15}|}}$$

$$1 - {}_1V_{[50]:\overline{15}|} = \frac{\ddot{a}_{[50]+1:\overline{14}|}}{\ddot{a}_{[50]:\overline{15}|}}$$

$$= \frac{9.564^{*2}}{10.044}$$

$$= 0.952210275$$

$$\ddot{a}_{[50]+1:\overline{14}|} = (10.044-1)*(1.06)*\frac{9,706.0977}{9,686.9669} = 9.563513766 = 9.564 \ \text{(3 DP)}$$

So that: $\text{DSAR-U} = 500,000 \cdot \left(1 - {}_1V_{[50]:\overline{15}|}\right) = 476,105.1374$

Finally, the Expected Death Strain (EDS) is a weighted sum:

$$\text{EDS} = 95 * q_{[45]} * (\text{DSAR-H}) + 5 * q_{[50]} * (\text{DSAR-U})$$

$$= 95*(0.001201)*(478,376.5146) + 5*(0.001971)*(476,105.1374)$$
$$= 54,580.36843 + 4,692.016 = R59,272.38$$

iv) The expected death strain is much lower under the second basis. This is expected as the weighted average of the select rates at ages 45 and 50 are lighter than the ultimate mortality at age 45.

QUESTION 6

A typical joint life question. Well prepared students scored well. Students that broke up their solution into distinct parts also tend to score better.

Average mark: 40-50%

Split the benefit into three parts

Guaranteed annuity for first five years (*B*)

$$B = 50,000 * 12 * a_{\overline{5}|}^{(12)}$$

where $a_{\overline{5}|}^{(12)} = \dfrac{i}{1^{(12)}} * a_{\overline{5}|} = 1.018204 * 4.4518 = 4.53283 = 4.5328$ (4 DP)

$$B = R2,719,680$$

Benefit if both alive at the end of five years (*C*)

$$C = 120,000 * {}_5p_{65}^{(m)} * {}_5p_{60}^{(f)} * v^5 \quad * \left[5 * a_{\overline{70(m):65(f)}}^{(12)} \quad + 1 * a_{70(m):65(f)}^{(12)} \quad \right]$$

where:

$${}_5p_{65}^{(m)} = \frac{l_{70}}{l_{65}} = \frac{9,238.134}{9,647.797} = 0.957538182$$

$${}_5p_{60}^{(f)} = \frac{l_{65}}{l_{60}} = \frac{9,703.708}{9,848.431} = 0.985304969$$

$$a_{70(m):65(f)}^{(12)} = a_{70(m):65(f)} - \frac{13}{24} = 10.494 - 0.542 = 9.952 \quad \text{(3 DP)}$$

$$\begin{aligned} a_{\overline{70(m):65(f)}}^{(12)} &= a_{70(m)}^{(12)} + a_{65(f)}^{(12)} - a_{70(m):65(f)}^{(12)} \\ &= \left[\ddot{a}_{70(m)} - \frac{13}{24} \right] + \left[\ddot{a}_{65(f)} - \frac{13}{24} \right] - \left[\ddot{a}_{70(m):65(f)} - \frac{13}{24} \right] \\ &= \ddot{a}_{70(m)} + \ddot{a}_{65(f)} - \ddot{a}_{70(m):65(f)} - \frac{13}{24} \\ &= 11.562 + 14.871 - 10.494 - 0.542 \\ &= 15.397 \quad \text{(3 DP)} \end{aligned}$$

$$\begin{aligned} C &= 120,000 * (0.957538) * (0.985305) * (0.82193) * [5 * (15.379) + (9.952)] \\ &= R8,081,604.70 \end{aligned}$$

Benefit if only one is alive at the end of 5th year (E)

$$E = \left[{}_5p_{65}^{(m)} * {}_5q_{60}^{(f)} * v^5 * 2{,}500{,}000 \right] + \left[{}_5q_{65}^{(m)} * {}_5p_{60}^{(f)} * v^5 * 600{,}000 * a_{65(f)}^{(12)} \right]$$

where the only factor we have not yet calculated is:

$$a_{65(f)}^{(12)} = a_{65(f)} - \frac{13}{24} = 14.871 - 0.542 = 14.392 \quad \text{(3 DP)}$$

$$\begin{aligned} E &= (0.957538)*(1-0.985304)*(0.82193)*(2{,}500{,}000) \\ &\quad +(1-0.957538)*(0.985304)*(0.82193)*(600{,}000)*(14.329) \\ &= 28{,}915.453 + 295{,}646.442 \\ &= R324{,}561.90 \end{aligned}$$

Finally, the single premium is the sum of B, C and E.

$$\begin{aligned} P &= B + C + E \\ &= 2{,}719{,}680 + 8{,}081{,}604.70 + 324{,}561.90 \\ &= R11{,}125{,}846.60 \end{aligned}$$

QUESTION 7

A long question which required some careful planning from students. Well laid out answers where the different actuarial terms were calculated separately and clearly scored well. Maybe students did not give this question a decent attempt and many made simple errors in the calculations of the increasing functions.

Average mark: 40-50%

Let P be the level annual premium and S be the sum assured.

First, let us consider the Expected Present Value (EPV) of the Benefits.

$$\text{EPV Benefits} = 0.96 * S * A_{[45]:\overline{20}|} + 0.04 * S * (IA)^{1}_{45:\overline{20}|} + 21 * 0.04 * S * A_{[45]:\overline{20}|}^{\ \ \ 1}$$

Secondly, the EPV of the Expenses is:

$$\text{EPV Expenses} = (250 + 0.5P) + 300 * A_{[45]:\overline{20}|} - 150 * A_{[45]:\overline{20}|}^{\ \ \ 1}$$

From the *Tables* and the formula for an increasing term assurance, we get the following values:

$$\begin{aligned}
A_{[45]:\overline{20}|} &= 0.46982 \\
A_{[45]:\overline{20}|}^{\ \ \ 1} &= \frac{D_{65}}{D_{[45]}} = \frac{689.23}{1{,}677.42} \\
&= 0.41089 \quad (5\text{ DP}) \\
(IA)^{1}_{[45]:\overline{20}|} &= (IA)_{[45]} - A_{[45]:\overline{20}|}^{\ \ \ 1} * \left[(IA)_{65} + 20 * A_{65}\right] \\
&= (8.33865) - (0.41089) * \left[(7.89442) + 20 * (0.52786)\right] \\
&= (8.33865) - (7.58159) \\
&= 0.75706
\end{aligned}$$

Then the EPV of Benefits and Expenses are respectively:

$$\begin{aligned}
\text{EPV Benefits} &= 0.96 * S * (0.46982) + 0.04 * S * (0.75706) + 0.84 * S * (0.41089) \\
&= S * (0.451027 + 0.030282 + 0.345148) \\
&= 0.82646 * S
\end{aligned}$$

$$\begin{aligned}
\text{EPV Expenses} &= (250 + 0.5P) + 300 * (0.46982) - 150 * (0.41089) \\
&= 329.3125 + 0.5P
\end{aligned}$$

So that the EPV of Benefits and Expenses is:

$$\text{EPV Benefits and Expenses} = 0.82646 * S + 329.3125 + 0.5P$$

(i) Level annual premiums of R12,000 payable in advance which increase every year by R500 for the second policy year onwards.

The EPV of Premiums is:

$$\text{EPV Premiums} = 11{,}500 * \ddot{a}_{[45]:\overline{20}|} \quad + 500 * (I\ddot{a})_{[45]:\overline{20}|}$$

From the *Tables* and the formula for an increasing temporary annuity, we get the following values:

$$\ddot{a}_{[45]:\overline{20}|} = 13.785$$

$$(I\ddot{a})_{[45]:\overline{20}|} = (I\ddot{a})_{[45]} - A_{[45]:\overline{20}|}^{\;\;\;\;1} * \left[(I\ddot{a})_{65} + 20 * \ddot{a}_{65} \right]$$

$$= (272.737) - (0.41089) * \left[(113.911) + 20 * (12.276) \right]$$

$$= (272.737) - (147.687)$$

$$= 125.050$$

Then the EPV of Premiums is:

$$\text{EPV Premiums} = 11{,}500 * (13.785) + 500 * (125.050)$$

$$= 158{,}527.50 + 62{,}525$$

$$= 221{,}052.50$$

Setting the EPV of Premiums equal to the sum of the EPV of the Benefits and Expenses gives:

$$221{,}052.50 = 0.82646 * S + 329.3125 + 0.5 * 12{,}000$$

$$0.82646 * S = 214{,}723.1875$$

$$S = R259{,}810.74$$

ii) Level annual premiums of R18,000 payable in advance for the first 15 years and then reducing to R7,000 from the 16th policy year onwards.

The EPV of Premiums is:

$$\begin{aligned}\text{EPV Premiums} &= 18,000 * \ddot{a}_{[45]:\overline{20}|} - 11,000 * {}_{15|}\ddot{a}_{[45]:\overline{5}|} \\ &= 18,000 * \ddot{a}_{[45]:\overline{20}|} - 11,000 * A_{\substack{1 \\ [45]:\overline{15}|}} * \ddot{a}_{60:\overline{5}|}\end{aligned}$$

From the *Tables*, we get the following values:

$$\begin{aligned}\ddot{a}_{60:\overline{5}|} &= 4.550 \\ A_{\substack{1 \\ [45]:\overline{15}|}} &= \frac{D_{60}}{D_{[45]}} = \frac{882.85}{1,677.42} = 0.52631 \quad (5\ \text{DP})\end{aligned}$$

Then the EPV of Premiums is:

$$\begin{aligned}\text{EPV Premiums} &= 18,000 * (13.785) - 11,000 * (0.52631) * (4.550) \\ &= 248,130 - 26,341.816 \\ &= 221,788.18\end{aligned}$$

Setting the EPV of Premiums equal to the sum of the EPV of the Benefits and Expenses gives:

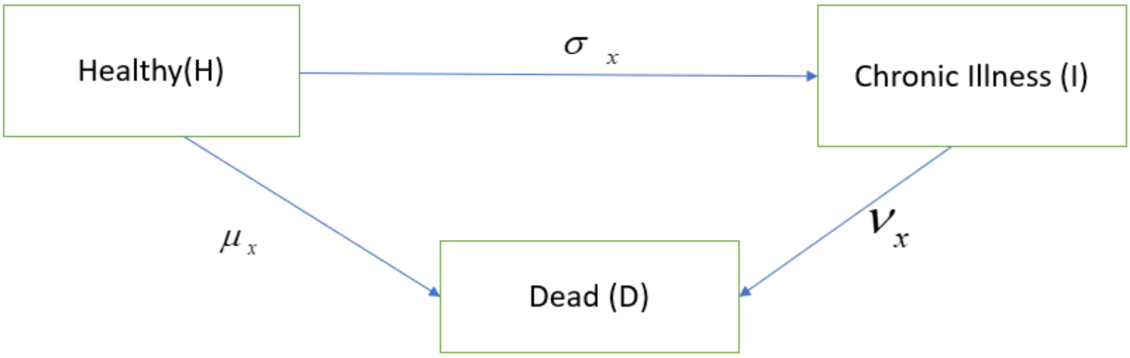
$$\begin{aligned}221,788.18 &= 0.82646 * S + 329.3125 + 0.5 * 18,000 \\ 0.82646 * S &= 212,458.8675 \\ S &= R257,070.96\end{aligned}$$

QUESTION 8

As expected, by this question the time pressure on students started showing. The marks on this question varied materially with many students scoring full marks but in contrast many scoring no marks. This would have been an ideal question to do earlier on as little effort was rewarded handsomely with marks. Students that planned their approach well, scored well here.

Average mark: 50-60%

(i)



(ii)

$$\text{EPV} = 1{,}000{,}000 * \int_0^{20} v^t \; * {}_tp_{45}^{HH} \; *(\mu_{45+t} + \sigma_{45+t}) \; dt$$

where:

$$v^t = e^{-\ln(1.055)t} = e^{-0.053540767t}$$

$${}_tp_{45}^{HH} = {}_tp_{45}^{\overline{HH}} = \exp\left(-\int_{45}^{45+t}(\mu_s + \sigma_s)ds\right) = \exp\left(-\int_{45}^{45+t}(0.03 + 0.01)ds\right) = e^{-0.04t}$$

Then the EPV of the benefits under the policy is:

$$=1,000,000*0.04*\int_0^{20} e^{-0.053540767t}*e^{-0.04t}dt$$

$$=40,000*\int_0^{20} e^{-0.093540767t}dt$$

$$=40,000*\left[\frac{e^{-0.093540767t}}{-0.093540767}\right]_0^{20}$$

$$=40,000*(10.69052599-1.646320153)$$

=R361,768.23

(iii)

EPV of no-claim bonus is:

$$=15,000*v^{20}*{}_{20}p_{45}^{HH}$$
$$=15,000*e^{-(20*0.053540767)}*e^{-(20*0.04)}$$
$$=15,000*e^{-(20*0.093540767)}$$
$$=15,000*e^{-1.870815339}$$

=R2,309.97

