Actuarial Society of South Africa

EXAMINERS REPORT

September 2021

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediate figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

QUESTION 1

i. When the claims distribution has sparse data in the tails, a model which appears to accurately fit the mean may end up underestimating claims received in the tails of the distribution. This may be a particular problem for large claims because the impact of underestimating large claims may be much more significant than the benefits of accurately modelling the average (or small) claims.

   Thus, in this instance, a model that may be less accurate for the mean claim, but which gives more weight to the tails, may be preferable. This would particularly be true if the purpose of the modelling exercise is to set solvency reserves.

[4]

ii. a. Can be quicker and is more precise than Monte Carlo simulation.
   One can also analyse the effect of changes to assumptions more readily.

   b. Provides a check on any simulation methods used by providing an alternative method for calculating some value e.g. the mean or median.

[3]

iii. False, scenario testing would consider different series of input parameters (e.g., a scenario where inflation is low, unemployment is low (initially) etc.,) and not test the entire range of a single input parameter.

[3]
[Total 10]

Parts (i) and (iii) were the worst answered questions in the paper.

These questions focused on very specific sections of the chapter and a large majority of candidates did not know the theory.

Application of the theory was also lacking as very few candidates could correctly identify scenario testing.

Candidates continue to struggle with questions based on the theory of Chapter 1.

QUESTION 2

i. The index is decreasing, which means that annual inflation rates are negative.
The index is decreasing by an increasing amount, which means the inflation rates are becoming more negative (decreasing).

[3]

ii. Accumulated value in money terms: $15,000\times(1.03)^4 = 16,882.63215$

Accumulated value in real terms: $16,882.63215\times\frac{110}{90} = 20,634.32818$

Real rate of return: $20,634.32818 = 15,000\times(1+r)^4 \Rightarrow r = 0.082990803$

[3]

iii. The real rate of return is higher than the nominal savings rate.

This is due to negative inflation rates, which have increased the purchasing power of money.

[3]

[Total 9]

Parts (i) and (iii) were poorly answered and candidates would do well to follow the instructions in the question.

Part (i) referred to inflation as summarized by the indices and part (iii) referred to the purchasing power of money. Many candidates confused the two concepts.

Common errors in part (i) were:

- *to equate reducing indices with reducing inflation instead of negative inflation*
- *not mentioning anything about the indices*
- *making comments about the purchasing power of money, which would have gained no credit.*

The common error in part (iii) was not taking the results of part (i) and (ii) into account.

Part (ii) was well answered.

QUESTION 3

$$PV = PV_1 + PV_2 = \frac{5{,}000}{A(0{,}4)A(4{,}8)A(8{,}11)} + \int_3^4 \frac{1{,}000\exp(0.005t^2)}{A(0,t)}dt$$

$$A(0{,}4) = \exp\left(\int_0^4 0.075 + 0.01t dt\right)$$
$$= \exp\left(0.075 \times 4 + \tfrac{0.01}{2} \times 4^2\right) = \exp(0.38) = 1.462285$$

$$A(4{,}8) = \exp(0.09 \times 4) = \exp(0.36) = 1.433329$$

$$A(8{,}11) = \exp\left(\int_8^{11} 0.01 + 0.001t^2 dt\right) = \exp\left(0.01 \times 3 + \frac{0.001}{3}(11^3 - 8^3)\right) = \exp(0.303)$$
$$= 1.353914$$

$$PV_1 = \frac{5{,}000}{A(0{,}4)A(4{,}8)A(8{,}11)} = 1{,}761.9797534$$

$$PV_2 = \int_3^4 \frac{1{,}000\exp(0.005t^2)}{A(0,t)}dt = \int_3^4 \frac{1{,}000\exp(0.005t^2)}{\exp\left(\int_0^t 0.075 + 0.01s\,ds\right)}dt$$
$$= \int_3^4 \frac{1{,}000\exp(0.005t^2)}{\exp(0.075t + 0.005t^2)}dt = \int_3^4 1{,}000\exp(-0.075t)\,dt$$
$$= \frac{1{,}000}{-0.075}(\exp(-0.075 \times 4) - \exp(-0.075 \times 3))$$
$$= 769.306641$$

[Total 14]

The best answered question in the paper.

QUESTION 4

i. $f_{t,r}$ = the discrete time forward rate agreed at time *0* for an investment made at time *t*, $t > 0$, for a period of *r* years

y_t = a measure of the average (per annum) interest rate over the period from **now** until *t* years' time

P_t = price at issue of a unit zero coupon bond maturing in *t* years

$$(1+f_{t,r})^r = \frac{(1+y_{t+r})^{t+r}}{(1+y_t)^t} = \frac{P_t}{P_{t+r}}$$

[4]

ii. $90 = 100(1+y_2)^{-2} \Rightarrow y_2 = 0.054092553$

$(1+y_4)^4 = (1+y_1)\exp(F_{1,3} \times 3) \Rightarrow y_4 = 0.056334798$

$(1+y_5)^5 = (1+y_2)^2(1+f_{2,3})^3 \Rightarrow y_5 = 0.060623542$

$P_{6\%} = 5a_{\overline{5|}} + 104v^5 = R98.77666891$

$$98.7767 = \frac{5}{1.04} + \frac{5}{(1.05409)^2} + \frac{5}{(1+y_3)^3} + \frac{5}{(1.05633)^4} + \frac{109}{(1.06062)^5}$$

$\Rightarrow y_3 = 0.056391958$

[7]

[Total 11]

Part (ii) was poorly answered, with many candidates writing down incorrect equations from information provided.

Common errors were:

- *taking $F_{1,3}$ as an annual effective rate of interest instead of a force of interest*
- *assuming y_2 is only applicable for the 2nd year, y_3 is only applicable for the 3rd year etc.*

QUESTION 5

$PV_L = 10{,}000v^3 + 10{,}000v^5 + 80{,}000v^9 = R54{,}764.07175$

$PV_A = 3{,}000a_{\overline{7|}} + Xv^7 + Yv^{11} = 15{,}619.11 + Xv^7 + Yv^{11}$

$39{,}144.96 = Xv^7 + Yv^{11}\ (PV_A = PV_L)\ (Eq\ 1)$

$DMT_L(numerator) = 30{,}000v^3 + 50{,}000v^5 + 720{,}000v^9 = 418{,}023.3834$

$DMT_A(numerator) = 3{,}000(Ia)_{\overline{7|}} + 7Xv^7 + 11Yv^{11}$

$= 57{,}691.76 + 7Xv^7 + 11Yv^{11}$

$360{,}331.62 = 7Xv^7 + 11Yv^{11}\ (DMT_A = DMT_L)(Eq\ 2)$

$(Eq\ 2) - (Eq\ 1) \times 7 \ \Rightarrow\ 86{,}316.9 = 4Yv^{11} \Rightarrow Y = 50{,}314.96$

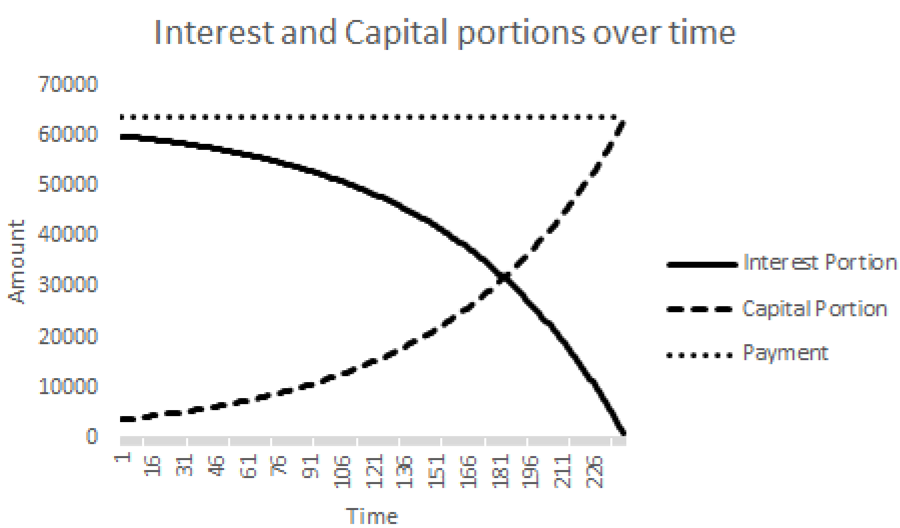
$Sub\ Y\ into\ (Eq\ 1) \Rightarrow 39{,}144.96 = Xv^7 + Yv^{11} \Rightarrow X = 30{,}104.58$

[Total 12]

Question was answered well.

QUESTION 6

i.



[5]

ii. The quarterly repayment amount would be smaller in monetary terms.
Although the interest rate stays the same, the repayment frequency increases.
Capital is therefore repaid earlier, and so interest charged on the outstanding loan is less, in monetary terms.
As a result, the size of the payment would be less than a quarter of the original payment.

[3]

iii. The total interest payable increases.
This is due to the repayments being made over a longer time period implying smaller repayments, which reduces capital by a smaller amount with each repayment.
This leads to higher accrual of interest on the outstanding loan.

[2]

[Total 12]

Common errors in part (i) were:

- *Not to have the convex/concave shape*
- *Not to have a crossover point between P_I and P_c.*

It is also important to note that neither P_I or P_c will ever be equal to 0 or P.

QUESTION 7

i. $P = 6{,}000 \times a_{\overline{8|}}^{(2)} + 100{,}000 \times v^8 - v^{\frac{5}{12}} \times (6{,}000 \times 0.32) \times a_{\overline{8|}}$
$- 0.2 \times (100{,}000 - P) \times v^{8+\frac{5}{12}}$

$a_{\overline{8|}} = 5.971298506$
$a_{\overline{8|}}^{(2)} = 6.074028855$

$$P = \frac{6{,}000 \times a_{\overline{8|}}^{(2)} - v^{\frac{5}{12}} \times (1{,}920) \times a_{\overline{8|}} + 100{,}000 \times v^8 - 20{,}000 v \times^{8+\frac{5}{12}}}{1 - 0.2 v^{8+\frac{5}{12}}}$$

$$= \frac{36{,}444.173129581 - 11{,}146.1984266 + 46{,}884.295655177}{0.886834}$$

$= R81{,}393.2285020$

[7]

ii. The investor will want to pay slightly less for the bond to obtain the same yield.
The tax cashflow –which is an outgo – is now received earlier on the timeline.
Outgo is discounted less than when tax was deferred and would decrease the PV of the cashflow. [3]

[Total 10]

Part (i) was answered reasonably well.

Common errors in part (i) were

- *incorrect timing of income and capital gains tax*
- *assuming that income tax will be payable twice a year.*

Part (ii) was answered poorly as candidates were expected to use general compound interest theory to answer the question.

QUESTION 8

$$(1-nD) = (1+i)^{-n}$$

$$1-(1+i)^{-n} = nD$$

$$D = \frac{1}{n}\left[1-(1+i)^{-n}\right]$$

[Total 3]

Question was answered poorly.

The instruction stated "derive from first principles". The only acceptable way of deriving the equation was to start with the first line in the examiner's report.

QUESTION 9

i. **Project A**

$PV_{out}(6.615\%) = 100{,}000 + 50{,}000 \times \bar{a}_{\overline{3|}}$
$= 100{,}000 + 136{,}468.302591290$
$= R236{,}468.302591290$

$PV_{in}(6.615\%)$
$= 30{,}000 \times v^2 \times (1 + (1.05) \times v + \cdots + (1.05)^6 \times v^6) + 100{,}000v \times^8$
$= 30{,}000 \times (1.06615)^{-2} \times \left[\frac{\left(1-\left(\frac{1.05}{1.06615}\right)^7\right)}{1-\left(\frac{1.05}{1.06615}\right)}\right] + 100{,}000v^8$
=176,562.3029 +59,903.681000795 $= R236{,}465.983927803$

$NPV = -2.318663487 \cong 0$

$\therefore IRR\ is\ 6.615\%$

[9]

ii. **Project B**

$PV_{out}(6.615\%) = 80{,}000\ddot{a}_{\overline{5|}} = 353{,}348.833$

$PV_{in}(6.615\%) = 50{,}000a_{\overline{8|}} + 10{,}000(Ia)_{\overline{8|}}$
$= 303{,}071.1942494 + 252{,}470.53589 = 555{,}541.73040327$
$(Ia)_{\overline{8|}} = 25.247053589$

$NPV_B = PV_{in} - PV_{out} = R202{,}192.896342735 > 0$ and since NPV decreases with increasing valuation interest
$IRR_B > 6.615\% \Rightarrow IRR_B > IRR_A$

[9]

iii. Borrowing rates should be less than 6.615% for both projects to be profitable.

NPV (IRR) = 0

$NPV\ (i<IRR) \Rightarrow NPV > 0$ where i=valuation interest rate

[3]
[Total 21]

Part (i) and (ii) were answered reasonably well but part (iii) was answered poorly.

In part (i) the command verb is "show". Although linear interpolation is a valid alternative, the more efficient way of performing this calculation was to substitute the internal rate of return into the NPV of project A and determine whether the NPV of project A is equal to zero.

Common errors in part (i) were:

- *to assume the PV(income) = PV(outgo) and*
- *when linear interpolation was used, to choose interest rates too far away from the target internal rate of return.*

Linear interpolation is a numerical method which yields an approximate result. As the target internal rate of return was given, the interest rates for linear interpolation could be chosen close to the target internal rate of return, to calculate a more accurate answer.

Part (ii) did not ask for the determination of the numerical value of the internal rate of return on Project B but only asked to determine whether the internal rate of return on Project B was higher or lower than the internal rate of return on Project A. There are several ways of doing this. After doing the necessary calculations, candidates should explain their result and how their result ultimately answers the question.

In Part (ii) linear interpolation is also an alternative, although it requires many more calculations than the solution presented here.

