

EXAMINATION

3 October 2023 (am)

Subject A211 — Financial Mathematics Intermediate Technical

Time allowed: Two hours and fifteen minutes (which includes 15 minutes of reading time)

Scan and upload time: Twenty minutes (at the end of the examination)

Total marks: 100

INSTRUCTIONS TO THE CANDIDATE

- 1. Once you have entered the ASSA Exam Platform, ensure that you have accessed the **Examity** Invigilation link with both your camera and microphone switched on. Your examination will be proctored by Live Proctor Examity agents.*
- 2. Your computer must be placed, and camera angled, so that your head and shoulders as well as your writing area on your desk is visible to the invigilator. Readjust your camera if you bump or move your computer by accident.*
- 3. Ensure that you have your exam permit handy. It reflects your candidate number to input **as part of the two hours 15 minutes examination and not before the start of the examination**. Write your candidate number at the top of each page during the examination time only. Do not use your name or member number anywhere on your answer script.*
- 4. The cell phone to be used to scan your final answer script must be switched **OFF** during the two hours and 15 minutes examination time. Place the cell phone at the top of your examination pad / writing pages in view of the invigilator.*
- 5. You are strongly encouraged to use the first 15 minutes as reading time only, however, you may start answering the paper whenever you are ready.*
- 6. The question paper is only available on the ASSA Exam Platform as a PDF download and may not be printed.*

7. *You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.*
8. *Attempt all questions, beginning your answer to each question on a new page.*
9. *Write in black or dark blue pen.*
10. *Show calculations where appropriate. You may use blank paper to carry out rough work calculations. This rough work paper must not be scanned as part of your answer script. You may use a calculator from the approved list only.*
11. *You may not access any file from your computer, use any other computer program (e.g. Email, MS Word or Excel), or open any other browser during the examination. The use of Grammarly and Grammarly Premium or similar is also not permitted in examinations.*
12. *You may not use any other material (e.g. a Formulae and Tables book) during the examination. Any such information that may be required will be provided to you within the examination.*
13. *Mark allocations are shown in brackets.*
14. *Assume that months are all equal length, unless otherwise stated.*
15. *At the end of the two hours and 15 minutes examination time, you must stop writing and start scanning and uploading your script. **You may NOT continue to write or review your script during this time.***
16. *Scan ALL your answer pages to PDF so that your candidate number is clear at the top of each page.*
17. *Save your PDF scanned file using your candidate number as file name. Do not use your name or member number anywhere in your answer script nor as file name.*
18. *Transfer your scanned script file to your computer and upload into the ASSA Exam Platform.*
19. *Click on the **Upload Answers** link below the examination paper link. Ensure you click on **Finish** below the upload box and again on **Finish All and Submit**, before the 20-minute upload time is up. (After submission the number of files successfully submitted will be reflected.)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

A group of demography researchers want to build a model that will accurately predict the growth of the South African population.

Discuss, in the context of this specific model, the steps of the modelling process. Give concrete examples where possible.

[Total 9]

QUESTION 2

An investor is considering buying a 16-seater taxi. To finance the project the investor will borrow the capital from a local commercial bank.

Describe, the certainty/uncertainty of the cashflows (from the investor's perspective) regarding size and timing of all cashflows in operating the taxi.

[Total 4]

QUESTION 3

The force of interest $\delta(t)$, at time t (measured in years), is given by

$$\delta(t) = \begin{cases} 0.05 & 0 \leq t < 5 \\ 0.05 - 0.006t & 5 \leq t \leq 10 \end{cases}$$

The following payments are made,

- R200 paid a year from now
- R100 paid at time $t = 3$
- A continuous payment stream paid at the rate of $(50 - 6t)$ per unit time between time $t = 5$ and time $t = 10$.

Calculate, showing all steps, the combined present value of the payments at time $t = 0$.

[Total 11]

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QUESTION 4

The price, P_t , of a zero-coupon bond (at time zero), redeemable at par in t years' time, is denoted by

$$P_t = \frac{\beta \times (\beta + 1)}{(\beta + t) \times (\beta + t + 1)} \quad t \geq 0 \text{ and where } \beta \text{ is a given positive constant.}$$

- i. Derive a formula for Y_t , continuous time t - year spot rate, in terms of β and t . [2]
 - ii. Derive a formula for F_5 , the instantaneous forward rate, in terms of β . [5]
- [Total 7]

QUESTION 5

A local company has two liabilities

- a lump-sum amount of R300,000 payable at the end of 20 years and
- an annuity of R150,000 per annum payable half-yearly in arrears for 30 years, starting in 15 years' time.

The company currently holds an amount of cash which equals the present value of the two liabilities valued at an effective rate of interest of 6.5% per annum. The company wishes to invest the cash in two zero coupon bonds to immunize its portfolio of assets and liabilities against small movements in the interest rate.

The company has decided to invest an amount in Bond A sufficient to provide a lump sum of R450,000 when Bond A is redeemed in ten years' time. The remainder of the cash is invested in Bond B.

- i. Calculate the amount of money invested in Bond B. [7]

The numerator of the discounted mean term for the liabilities has been calculated at the prevailing interest rate, and is equal to R21,704,293.03.

- ii. Calculate the term for Bond B by using Redington's theorem. [6]
- [Total 13]

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QUESTION 6

Consider a loan of R100,000 repayable annually in arrears over 30 years.

The loan is repaid by a special compound increasing annuity in arrears where the first payment is RY. The repayments then increase as follows

- at a rate of 6% per annum with the first increase at $t=1$ and last increase at $t=15$ and
- at a rate of 10% per annum over the second 15 years (with the first increase of 10% at $t=16$).

The repayment is calculated using a rate of interest of 9% per annum convertible quarterly.

- Calculate Y. [10]
 - Calculate the total amount of interest paid on this loan. [6]
 - Calculate the flat rate on the loan. [2]
- [Total 18]

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QUESTION 7

An index-linked bond was issued on 1 April 2020. An investor purchased the bond three months later for R110 per R100 nominal just after the coupon, due at that time, has been paid.

The coupons on the bond are of a nominal amount of 8% per annum payable half-yearly in arrears and are payable on 1 January and 1 July every year. The bond is redeemed at par on 1 July 2022.

Coupon and redemption payments are indexed by reference to the value of an inflation index with a time lag of three months.

You are given the following values for the inflation index:

<i>Date</i>	<i>Index</i>
1 <i>January</i> 2020	131.2
1 <i>April</i> 2020	132.8
1 <i>July</i> 2020	133.1
1 <i>October</i> 2020	133.6
1 <i>January</i> 2021	133.9
1 <i>April</i> 2021	134.2
1 <i>July</i> 2021	134.4
1 <i>October</i> 2021	134.9
1 <i>January</i> 2022	135.2
1 <i>April</i> 2022	136.0
1 <i>July</i> 2022	136.1

Calculate the real effective yield per annum earned by the investor.

[Total 14]

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QUESTION 8

A property investor bought a complex of flats for R11,000,000 to let out to university students. The investor will receive annual rental income of R500,000 per annum payable half-yearly in advance, in the first year. Thereafter, the annual rent will increase by R12,500 per annum and the investor will sell the property for R21,000,000 in 20 years' time.

The city council demands that every owner refurbishes their building after every 15 years of ownership. The total cost of refurbishments will be R1,500,000.

The investor pays utility bills. These bills are paid continuously at a rate of R15,000 per annum in the first year and thereafter the rate of payment increases by R2,000 per annum until the property is sold.

- i. Calculate the net present value of this investment at an effective interest rate of 5% per annum.

[20]

The property investor will borrow R11,000,000 from a bank at a rate of 5% per annum effective for the investment. Excess income from the project will immediately be used as instalments to repay the loan. Once the loan has been repaid money can be invested at 6% per annum effective.

- ii. Show that the discounted payback period is 20 years.

[4]

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Compound Interest

n	$(1+i)^n$	v^n	$s_{\overline{n} }$	$a_{\overline{n} }$	$(Ia)_{\overline{n} }$	$(Da)_{\overline{n} }$	n	5%
1	1.050 00	0.952 38	1.000 0	0.952 4	0.952 4	0.952 4	1	i 0.050 000
2	1.102 50	0.907 03	2.050 0	1.859 4	2.766 4	2.811 8	2	$i^{(2)}$ 0.049 390
3	1.157 63	0.863 84	3.152 5	2.723 2	5.358 0	5.535 0	3	$i^{(4)}$ 0.049 089
4	1.215 51	0.822 70	4.310 1	3.546 0	8.648 8	9.081 0	4	$i^{(12)}$ 0.048 889
5	1.276 28	0.783 53	5.525 6	4.329 5	12.566 4	13.410 5	5	
6	1.340 10	0.746 22	6.801 9	5.075 7	17.043 7	18.486 2	6	δ 0.048 790
7	1.407 10	0.710 68	8.142 0	5.786 4	22.018 5	24.272 5	7	
8	1.477 46	0.676 84	9.549 1	6.463 2	27.433 2	30.735 7	8	$(1+i)^{1/2}$ 1.024 695
9	1.551 33	0.644 61	11.026 6	7.107 8	33.234 7	37.843 6	9	$(1+i)^{1/4}$ 1.012 272
10	1.628 89	0.613 91	12.577 9	7.721 7	39.373 8	45.565 3	10	$(1+i)^{1/12}$ 1.004 074
11	1.710 34	0.584 68	14.206 8	8.306 4	45.805 3	53.871 7	11	v 0.952 381
12	1.795 86	0.556 84	15.917 1	8.863 3	52.487 3	62.735 0	12	$v^{1/2}$ 0.975 900
13	1.885 65	0.530 32	17.713 0	9.393 6	59.381 5	72.128 5	13	$v^{1/4}$ 0.987 877
14	1.979 93	0.505 07	19.598 6	9.898 6	66.452 4	82.027 2	14	$v^{1/12}$ 0.995 942
15	2.078 93	0.481 02	21.578 6	10.379 7	73.667 7	92.406 8	15	d 0.047 619
16	2.182 87	0.458 11	23.657 5	10.837 8	80.997 5	103.244 6	16	$d^{(2)}$ 0.048 200
17	2.292 02	0.436 30	25.840 4	11.274 1	88.414 5	114.518 7	17	$d^{(4)}$ 0.048 494
18	2.406 62	0.415 52	28.132 4	11.689 6	95.893 9	126.208 3	18	$d^{(12)}$ 0.048 691
19	2.526 95	0.395 73	30.539 0	12.085 3	103.412 8	138.293 6	19	
20	2.653 30	0.376 89	33.066 0	12.462 2	110.950 6	150.755 8	20	$i/i^{(2)}$ 1.012 348
21	2.785 96	0.358 94	35.719 3	12.821 2	118.488 4	163.576 9	21	$i/i^{(4)}$ 1.018 559
22	2.925 26	0.341 85	38.505 2	13.163 0	126.009 1	176.739 9	22	$i/i^{(12)}$ 1.022 715
23	3.071 52	0.325 57	41.430 5	13.488 6	133.497 3	190.228 5	23	i/δ 1.024 797
24	3.225 10	0.310 07	44.502 0	13.798 6	140.938 9	204.027 2	24	$i/d^{(2)}$ 1.037 348
25	3.386 35	0.295 30	47.727 1	14.093 9	148.321 5	218.121 1	25	$i/d^{(4)}$ 1.031 059
26	3.555 67	0.281 24	51.113 5	14.375 2	155.633 7	232.496 3	26	$i/d^{(12)}$ 1.026 881
27	3.733 46	0.267 85	54.669 1	14.643 0	162.865 6	247.139 3	27	
28	3.920 13	0.255 09	58.402 6	14.898 1	170.008 2	262.037 5	28	
29	4.116 14	0.242 95	62.322 7	15.141 1	177.053 7	277.178 5	29	
30	4.321 94	0.231 38	66.438 8	15.372 5	183.995 0	292.551 0	30	
31	4.538 04	0.220 36	70.760 8	15.592 8	190.826 1	308.143 8	31	
32	4.764 94	0.209 87	75.298 8	15.802 7	197.541 9	323.946 5	32	
33	5.003 19	0.199 87	80.063 8	16.002 5	204.137 7	339.949 0	33	
34	5.253 35	0.190 35	85.067 0	16.192 9	210.609 7	356.141 9	34	
35	5.516 02	0.181 29	90.320 3	16.374 2	216.954 9	372.516 1	35	
36	5.791 82	0.172 66	95.836 3	16.546 9	223.170 5	389.063 0	36	
37	6.081 41	0.164 44	101.628 1	16.711 3	229.254 7	405.774 3	37	
38	6.385 48	0.156 61	107.709 5	16.867 9	235.205 7	422.642 1	38	
39	6.704 75	0.149 15	114.095 0	17.017 0	241.022 4	439.659 2	39	
40	7.039 99	0.142 05	120.799 8	17.159 1	246.704 3	456.818 3	40	
41	7.391 99	0.135 28	127.839 8	17.294 4	252.250 8	474.112 6	41	
42	7.761 59	0.128 84	135.231 8	17.423 2	257.662 1	491.535 8	42	
43	8.149 67	0.122 70	142.993 3	17.545 9	262.938 4	509.081 8	43	
44	8.557 15	0.116 86	151.143 0	17.662 8	268.080 3	526.744 5	44	
45	8.985 01	0.111 30	159.700 2	17.774 1	273.088 6	544.518 6	45	
46	9.434 26	0.106 00	168.685 2	17.880 1	277.964 5	562.398 7	46	
47	9.905 97	0.100 95	178.119 4	17.981 0	282.709 1	580.379 7	47	
48	10.401 27	0.096 14	188.025 4	18.077 2	287.323 9	598.456 8	48	
49	10.921 33	0.091 56	198.426 7	18.168 7	291.810 5	616.625 6	49	
50	11.467 40	0.087 20	209.348 0	18.255 9	296.170 7	634.881 5	50	
60	18.679 19	0.053 54	353.583 7	18.929 3	333.272 5	821.414 2	60	
70	30.426 43	0.032 87	588.528 5	19.342 7	360.183 6	1 013.146 5	70	
80	49.561 44	0.020 18	971.228 8	19.596 5	379.242 5	1 208.070 8	80	
90	80.730 37	0.012 39	1 594.607 3	19.752 3	392.501 1	1 404.954 8	90	
100	131.501 26	0.007 60	2 610.025 2	19.847 9	401.597 1	1 603.041 8	100	