Actuarial Society of South Africa

EXAMINERS REPORTS

October 2022

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

Many candidates can also increase their probability of passing this exam by practising good exam technique, which is often missing in its most basic form.

Question 1

$$a_{\overline{\infty}|}^{(p)} = \frac{1}{p}v^{\frac{1}{p}} + \frac{1}{p}v^{\frac{2}{p}} + \cdots . @ \text{ i}$$

Thus

$$a_{\overline{\infty}|\, i}^{(p)} = \frac{1}{p} \times v^{\frac{1}{p}} \left[1 + v^{\frac{1}{p}} + v^{\frac{2}{p}} + \ldots \right] \ @ i$$

This is a converging, infinite geometric sum, since $v^{\frac{1}{p}} < 1$.

$$a_{\overline{\infty}|\, i}^{(p)} = \frac{1}{p} \times v^{\frac{1}{p}} \left[\frac{1}{1 - v^{\frac{1}{p}}} \right]$$

$$= \frac{1}{p} \times \left[\frac{1}{(1+i)^{\frac{1}{p}} - 1} \right]$$

$$= \frac{1}{i^{(p)}}$$

Question was moderately well done.

Common errors were

- *Starting with an incorrect series*
- *Applying the geometric series incorrectly.*

Question 2

$50{,}000 = Xv^{6.25} + Xv^{7.25} + Xv^{8.25} + Xv^{9.25} + Xv^{10.75} + Xv^{11.75} + Xv^{12.75} + Xv^{13.75}$

@i = 8.25%

$= Xv^{9.75}a_{\overline{4}|} + Xv^{5.25}a_{\overline{4}|}$ $@i = 8.25\%$

$a_{\overline{4}|} = 3.293779093$

$v^{5.25} = 0.659558769$

$v^{9.75} = 0.461666037$

$$X = \frac{50{,}000}{(v^{9.75} + v^{5.25}) \times a_{\overline{4}|}} = R13{,}538.88$$

Question was done well, although calculation errors were common if calculations were done without converting to annuities.

Question 3

i. a. A scenario-based model takes into consideration a particular scenario, and the series of corresponding input parameters that describe that scenario.

b. One can model the financial performance of a company under different future scenarios,
such as recession or economic boom.
The values of input parameters (e.g. inflation and interest rate)
would be selected to be appropriate to each specific scenario.

ii. No, model is not valid. The 50% withdrawal rate may no longer be valid as the higher retirement benefit may change the behavior of the employees. This would invalidate the model.

Performance in this question was mixed with parts (i)(b) and (ii) being answered very poorly.

Questions, on this part of the work, that requires application continues to be a problem.

Question 4

$PV_L = R80{,}500v^8 + 160{,}000v^{17} = 97{,}503.635391743$ ………*equation 1*

$PV_A = 97{,}503.635391743 = 800 \times 100v^5 + Xv^n$

$= 57{,}038.894358694 + Xv^n$……*equation 2*

From *equation 1 and 2*

$Xv^n = 97{,}503.635391743 - 57{,}038.894358694 = 40{,}464.74103305$

$-PV_L{}' = R80{,}500 \times 8v^9 + 160{,}000 \times 17v^{18} = 1{,}155{,}043.18261894$…*equation 3*

$-PV_A{}' = 1{,}155{,}043.18261894 = 800 \times 100 \times 5v^6 + Xnv^{n+1}$

$= 266{,}536.889526605 + Xnv^n$ …*equation 4*

From equation 3 and 4:

$Xnv^{n+1} = 888{,}506.293092334$

$$n = \frac{888{,}506.293092334}{40{,}464.74103305} \times (1.07) = 23.49$$

$X = \frac{40{,}464.75}{v^{23.49}} = R198{,}351.74$

[Total 7]

Question was answered well.

Question 5

For Scenario A, the index-linked bond would provide a higher real rate of return than fixed interest bond. This is because the inflation rate is positive, as indicated by an increasing inflation index , and so cashflows (coupons and redemptions) would be adjusted upward to compensate for the loss of purchasing power, and thus preserve the real rate of return.

For Scenario B, the fixed-interest security would proide a higher real rate of return. This is because the inflation rate is negative, as indicated by a decreasing inflation index and so cashflows (coupons and redemptions) for the fixed interest security would be higher than those paid under the index-linked bond, and thus produce a higher real rate of return.

For Scenario C, the index-linked bond would provide a higher real rate of return. The inflation rate (or inflation index) appears volatile but is increasing on average (up-steps greater than down-steps). Thus, the index-linked bond is likely to provide on average a higher real rate of return than the fixed interest security.

This was the worst answered question in the paper.

Some candidates still confuse retail price index (RPI) with inflation. The RPI is the instrument used to measure inflation. Increasing RPI implies positive inflation. While decreasing RPI implies negative inflation (i.e., deflation).

Candidates also failed to connect the appropriate bond with the appropriate scenario.

Question 6

$$PV = \int_4^9 40\exp(-0.03t + 0.002t^2)\exp\left(-\int_2^t (0.06 + 0.004s)ds\right)dt$$
$$+ 300\exp\left(-\int_2^5 (0.06 + 0.004s)ds\right)$$

$$= \int_4^9 40\exp(-0.03t + 0.002t^2) \times \exp[-0.06t - 0.002s^2]_2^t dt$$
$$+ 300\exp[-0.06t - 0.002s^2]_2^5$$

$$= \int_4^9 40\exp(-0.03t + 0.002t^2)\exp(-0.06t + 0.128 - 0.002t^2)\,dt$$
$$+300\exp\,(-0.222)$$

$$= \int_4^9 40\exp\,(0.128 - 0.09t)dt + 240.2746093$$

$$= 40\exp(0.128)\int_4^9 \exp(-0.09t)\,dt + 240.2746093$$

$$= 40\exp(0.128)\left(-\frac{1}{0.09}\exp(-0.09t)\right)_4^9 + 240.2746093$$

$$= 40\exp(0.128)\,(2.809091776) + 240.2746093$$

$$= 127.70726277 + 240.2746093 = 367.98$$

> *This was the best answered question in the paper.*
>
> *Candidates should however remember that the examiners need to see the integration. A correct answer without showing all your working will NOT gain full credit.*

Question 7

i. a. TRUE: Major investments institutions have the alternative of borrowing from abroad. A government may therefor want to keep interest rates in a country low relative to other countries to encourage local borrowing

b. FALSE: Investors expect nominal interest rates to outstrip inflation.

ii.

Sub values into formulae and rearrange:

$100 = 7((1+y_1)^{-1} + (1+y_2)^{-2}) + 100(1+y_2)^{-2}$……*equation 1*

$98 = 5.5((1+y_1)^{-1} + (1+y_2)^{-2}) + 101(1+y_2)^{-2}$……. *equation 2*

From equation 1

$(1+y_2)^{-2} = (100 - 7(1+y_1)^{-1})\left(\frac{1}{107}\right)$ ……*equation 3*

From equation 2

$(1+y_2)^{-2} = (98 - 5.5(1+y_1)^{-1})\left(\frac{1}{106.5}\right)$ ……*equation 4*

equation 3 = equation 4

$$(98 - 5.5(1+y_1)^{-1})\left(\frac{1}{106.5}\right) = (100 - 7(1+y_1)^{-1})\left(\frac{1}{107}\right)$$

Solve for y_1

$$-0.014391645 = -0.013777368(1+y_1)^{-1}$$
$$(1+y_1)^{-1} = 1.044585987$$

$$y_1 = -0.042682926$$

Solve for $f_{1,1}$

$$(1+y_2)^{-2} = (1+y_1)^{-1}\left(1+f_{1,1}\right)^{-1}$$
$$= (100 - 7(0.957317073)^{-1})\left(\frac{1}{107}\right)$$

$$\left(1+f_{1,1}\right)^{-1} = 0.866242038(0.957317073) = 0.829268292$$

$$f_{1,1} = 0.205882353$$

For clarity part (i)(b), in the exam paper, referred to nominal interest rates.

Part (ii) was done moderately well.

The common errors in part (ii) were

- *incorrect definition of a par yield*
- *using effective rates of interest (which were not available) instead of spot rates and forward rates.*

Question 8

i.

$$i^{(4)} = 0.0682341 > \frac{(1-t_1)D}{R} = 0.75 \times \frac{9}{104} = 0.064903846$$

Capital Gain for investor, worst case scenario is choose late redemption.

$$P_A = 0.75 \times 9a_{\overline{20|}}^{(4)} + 104v^{20} - 0.35(104 - P_A)v^{20}$$

$$a_{\overline{20|}}^{(4)} = 10.868187565$$

$$P_A - 0.35P_A v^{20} = 0.75 \times 9a_{\overline{20|}}^{(4)} + 0.65 \times 104v^{20}$$

$$P_A = \frac{0.75 \times 9a_{\overline{20}|}^{(4)} + 0.65 \times 104v^{20}}{1 - 0.35v^{20}}$$

$$P_A = R99.86\ per\ R100\ nominal$$

ii.

$$i^{(4)} = 0.053900698 < 0.65 \times \frac{9}{104} = 0.05625$$

Capital loss for investor, though still a gain for borrower too (funds were raised at issue). However, choose earlier date for pricing to lock in the minimum (borrower might redeem later for some reason).

$$P_B = 0.65 \times 9a_{\overline{11}|}^{(4)} + 104v^{11}$$

$$a_{\overline{11}|}^{(4)} = 8.257583235$$

$$P_B = 106.018\ per\ R100\ nominal$$

iii.

Capital gains tax paid by investor A, because P_B, the price that investor A receives, is now larger than P_A, the price that investor A paid for the bond

Question was done well.

Candidates should note that in part (ii) the details of new investor (investor B) should be used to determine whether a capital gain/loss will be made.

Question 9

i.

Prospective:

$L_t = X a_{\overline{n-t|}} @ i\%$

Retrospective

$L_t = L_0(1+i)^t - X s_{\overline{t|}} @ i\%$

ii.

We know that $L_0 = Xv + Xv^2 + .. + Xv^t + \cdots + Xv^n$

Multiply this by $(1+i)^t$

$L_0(1+i)^t = X(1+i)^{t-1} + \cdots + X + X(1+i)^{t-(t+1)} \ldots + X(1+i)^{t-n}$

Rearranging we get: $L_0(1+i)^t - X(1+i)^{t-1} - X(1+i)^{t-2} - X = Xv + \cdots + Xv^{n-t}$

$L_0(1+i)^t - X s_{\overline{t|}} = X a_{\overline{n-t|}}$

iii.

$850{,}000 = X_1 a_{\overline{20|}} @ i = 8\%$

$X_1 = \dfrac{850{,}000}{9.818147407} \ldots = R86{,}574.38$

iv.

$L_7 = X_1 a_{\overline{13|}8\%}$

$= R684{,}264.48$

Then calculate the new repayment:

$L_7 = X_2 a_{\overline{13|}6\%}$

$\rightarrow X_2 = \dfrac{684{,}264.482040710}{8.852682963} = R77{,}294.59$

Prospective

$L_{13} = X_2 a_{\overline{7|}6\%} = 431{,}487.87$

v.

Retrospective

$850,000(1.08)^7 \times (1.06)^6 - 86,574.377499678 s_{\overline{7}|8\%} \times (1.06)^6 - 77,294.587971753 s_{\overline{6}|6\%}$

$= 2,066,428.608265130 - 772,486.146421873 \times (1.06)^6 - 539,154.372335525$

$= 431,487.87$

Question was answered well in general.

Part (ii) of the question was answered very poorly, which is surprising as it is straight forward theory.

Question 10

i.

$NPV = -5m + 0.1 \times (36,500 \times 150\bar{a}_{\overline{1}|}$

$+43,800 \times 165\bar{a}_{\overline{1}|}v + 43,800 \times 165\bar{a}_{\overline{1}|}1.03^2 v^2 + 43,800 \times 165\bar{a}_{\overline{1}|}1.03^4 v^3$

$+ \cdots + 43,800 \times 165\bar{a}_{\overline{1}|}1.03^{26} v^{14}) + 0.1 \times 15,000,000 v^{15}$

$NPV = -5m + 0.1 \times (36,500 \times 150\bar{a}_{\overline{1}|}$

$+43,800 \times 165\bar{a}_{\overline{1}|}v(1 + 1.03^2 v^1 + 1.03^4 v^2 + 1.03^6 v^3 + \cdots + 1.03^{26} v^{13})$

$+0.1 \times 15,000,000 v^{15}$

$$j = \left(\left(\frac{1.03^2}{1.08}\right)^{-1} - 1\right) = 0.02083137$$

$= -5m + 0.1 \times \left(5,475,000\bar{a}_{\overline{1}|} + 7,227,000\bar{a}_{\overline{1}|}v\left(\ddot{a}_{\overline{14}|@j}\right)\right) + 0.1 \times 15mv^{15}$

$\bar{a}_{\overline{1}|8.3\%} = 0.961171337$

$\ddot{a}_{\overline{14}|@j} = 12.286434893$

$NPV = -5m + 0.1(5,262,413.07140078 + 78,805457.2201779)$

$+453,590.979978783 = R3,860,378$

ii. The NPV is greater than zero, when cashflows are discounted at 8.3%

Hence, under the conditions stated, this is a profitable investment.

iii. The loan is repaid later during the project (in arrears vs continuous)

Additional interest will accrue on the loan between repayments which are now further apart.

The profitability of the project will be reduced.

Part (i) of this question was done well.

Many marks were available for writing down the equation of value correctly.

Common errors were

- *missing that only 10% of the revenue would flow to the fund*
- *dealing incorrectly with the growth, both the number of people and ticket price had to grow each year*
- *applying the geometric series formula incorrectly.*

