EXAMINATION

7 October 2022

Subject A211 — Financial Mathematics

Time allowed:

Examination time - Two hours and fifteen minutes

Scan and upload time - Twenty minutes (at the end of the examination)

INSTRUCTIONS TO THE CANDIDATE

1. *Once you have entered the ASSA Exam Platform, ensure that you have accessed the* ***Video Room*** *Invigilation link with both your camera and microphone switched on* ***before you attempt the examination.***
2. *Your computer must be placed, and camera angled, so that your writing area on your desk is visible to the invigilator. Readjust your camera if you bump or move your computer by accident.*
3. *Ensure that you have your exam permit handy. It reflects your candidate number to input* ***as part of the two hours 15 minutes examination and <u>not before</u> the start of the examination****. Write your candidate number at the top of each page. <u>Do not use your name or member number anywhere on your answer script.</u>*
4. *Your cell phone that will be used to scan your final answer script must be switched* ***OFF*** *during the two hours and 15 minutes examination time. <u>Place your cell phone at the top of your examination pad / writing pages in view of the invigilator.</u>*
5. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may start answering the paper whenever you are ready.*
6. *The question paper is only available on the ASSA Exam Platform as a PDF download and may not be printed.*
7. *You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.*
8. *Attempt all questions, beginning your answer to each question on a new page.*

9. *Write in black or dark blue pen.*
10. *Show calculations where appropriate. You may use blank paper to make notes. This paper must not be scanned as part of your answer script.*
11. *You may not access any file from your computer, use any other computer program (e.g. Email, MS Word or Excel), or open any browser during the examination.*
12. *You may not use any other material (e.g. a Formulae and Tables book) during the examination. Any such information that may be required will be provided to you in the examination.*
13. *Mark allocations are shown in brackets.*
14. *Assume that months are all equal length, unless otherwise stated.*
15. *At the end of the two hours and 15 minutes examination time, you must stop writing and start scanning and uploading your script.* ***You may NOT continue to write or review your script during this time.***
16. *Scan ALL your answer pages to PDF so that your candidate number is clear at the top of each page.*
17. *Save your PDF scanned file using your candidate number as file name. Do not use your name or member number anywhere in your answer document nor as file name.*
18. *Transfer your scanned script file to your computer and upload it to the ASSA Exam Platform.*
19. *Click on the* ***Upload Answers*** *link below the examination paper link. Ensure you click on* ***Finish*** *below the upload box and again on* ***Finish All and Submit****, before the 20-minute upload time is up. (After submission the number of files successfully submitted will be reflected.)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

Derive the expression $a_{\overline{\infty}|i}^{(p)} = \frac{1}{i^{(p)}}$, where $i^{(p)}$ is the nominal annual interest rate compounded p^{th}-*ly* per annum. Start by writing $a_{\overline{\infty}|i}^{(p)}$ as a series.

Assume that interest rates are positive.

[Total 4]

QUESTION 2

A man has two daughters, aged seven years and three months, and eleven years and nine months, respectively.

He invests a lump sum of R50,000 today at an interest rate of 8.25% per annum effective. This investment must provide amounts of R*X* on each of the two daughters' 18^{th}, 19^{th}, 20^{th}, and 21^{st} birthdays.

Calculate R*X*.

[Total 4]

QUESTION 3

i. a. Define scenario-based models.

[2]

b. Give an example of where scenario-based models can be used in decision analysis to evaluate the expected impact of a course of action.

[3]

ii. An employer provides a fixed lump sum benefit to all employees that retire at age 65. It is using a model to calculate how much the employer must contribute every year to fund this benefit. An assumption in this model is that 50% of employees will leave the company before retirement and therefore not receive the fixed lump sum.

The employer wants to assess by how much the amount that it contributes every year increases, if the fixed lump sum is doubled.

Is the current model valid for the purpose of answering this question? Explain your answer.

[2]

[Total 7]

PLEASE TURN OVER

QUESTION 4

An insurance company has liabilities of R80,500 due in eight years' time and R160,000 due in 17 years' time. Its assets consist of 800 zero-coupon bonds each with a R100 nominal amount, payable after five years, and a zero coupon bond paying RX in n years' time.

The current interest rate is 7% per annum effective and the portfolio is immunized against small movements in interest rates using Redington's theory of immunization.

Calculate X and n.

[Total 7]

QUESTION 5

On 1 June 2019, the Government issued a two-year index-linked bond with nominal coupons of 2% per annum payable half-yearly in arrears and a nominal redemption at par. The actual coupon and redemption payments were indexed to an inflation index with a six-month time lag.

On the same date, the Government also issued a two-year fixed interest bond, also with nominal coupons of 2% per annum payable half-yearly in arrears and a nominal redemption price of 100%.

These are the only two bonds available for investment in this market and they have the same price. Once purchased, bonds will be held to redemption.

Values of the different retail price indices (RPI) for different economic scenarios (Scenario A, B and C) are given below:

Date	***RPI***		
	Scenario A	***Scenario B***	***Scenario C***
1 December 2018	112	112	112
1 June 2019	113	111	118
1 December 2019	116	108	116
1 June 2020	117	106	126
1 December 2020	121	103	124
1 June 2021	122	101	129

An investor who is a resident of this country and only invests in local assets wishes to maximize his real effective rate of return.

Explain, for each economic scenario, which of the two investment options, fixed interest or index-linked bond, would have resulted in the highest real effective rate of return for the investor. **No calculations are required.**

[Total 7]

PLEASE TURN OVER

QUESTION 6

You are given that the annual force of interest is

$$\delta(t) = 0.06 + 0.004t \qquad \text{where } t \geq 0$$

Calculate the combined present value at *t=2* of

- an annuity payable continuously at an annual rate of $40\exp\left(-0.03t + 0.002t^2\right)$ from time *t=4* to *t=9* and
- *R300* invested at time *t=5*.

[Total 10]

QUESTION 7

i. State, with a reason, whether the following statements are TRUE or FALSE:

   a. The interest rate in a particular country will be influenced by the cost of borrowing in other countries.

   [1]

   b. Periods of high inflation tend to be associated with low interest rates.

   [1]

In the current bond market, the two-year par yield at time *t=0* is 7%. The issue price of a two-year fixed interest bond is 98% of par. This fixed interest bond pays coupons of 5.5% annually in arrears and is redeemable at 101% of par.

ii. Calculate the one-year spot rate, y_1, and the one-year forward rate applicable from time 1 to time 2, $f_{1,1}$.

[9]

[Total 11]

PLEASE TURN OVER

QUESTION 8

Investor A purchased R1,000,000 nominal of a bond at issue on 1 January 2015.

The bond is redeemable at 104% of par and redemption is at the option of the borrower on any coupon date between 15 and 20 years from the date of issue. It pays quarterly coupons of 9% per annum in arrears.

At the time of purchase, Investor A was subject to 25% income tax and 35% capital gains tax and planned to hold the bond until redemption.

i. Calculate the price Investor A should have paid for the bond to secure a minimum net redemption yield of 7% per annum effective.

[7]

After four years, directly after the coupon then due, Investor A sold the bond to Investor B.

Investor B is subject to 35% income tax and 27% capital gains tax.

ii. Calculate the price Investor B should pay for the bond to secure a minimum net redemption yield of 5.5% per annum effective, assuming Investor B holds the bond until redemption.

[6]

iii. Explain, with a reason, whether investor A will pay capital gains tax at the sale of the bond.

[2]

[Total 15]

PLEASE TURN OVER

Compound Interest

7%

n	$(1+i)^n$	v^n	$s_{\overline{n}\mid}$	$a_{\overline{n}\mid}$	$(Ia)_{\overline{n}\mid}$	$(Da)_{\overline{n}\mid}$	n
1	1.070 00	0.934 58	1.000 0	0.934 6	0.934 6	0.934 6	1
2	1.144 90	0.873 44	2.070 0	1.808 0	2.681 5	2.742 6	2
3	1.225 04	0.816 30	3.214 9	2.624 3	5.130 4	5.366 9	3
4	1.310 80	0.762 90	4.439 9	3.387 2	8.181 9	8.754 1	4
5	1.402 55	0.712 99	5.750 7	4.100 2	11.746 9	12.854 3	5
6	1.500 73	0.666 34	7.153 3	4.766 5	15.744 9	17.620 9	6
7	1.605 78	0.622 75	8.654 0	5.389 3	20.104 2	23.010 2	7
8	1.718 19	0.582 01	10.259 8	5.971 3	24.760 2	28.981 4	8
9	1.838 46	0.543 93	11.978 0	6.515 2	29.655 6	35.496 7	9
10	1.967 15	0.508 35	13.816 4	7.023 6	34.739 1	42.520 3	10
11	2.104 85	0.475 09	15.783 6	7.498 7	39.965 2	50.018 9	11
12	2.252 19	0.444 01	17.888 5	7.942 7	45.293 3	57.961 6	12
13	2.409 85	0.414 96	20.140 6	8.357 7	50.687 8	66.319 3	13
14	2.578 53	0.387 82	22.550 5	8.745 5	56.117 3	75.064 7	14
15	2.759 03	0.362 45	25.129 0	9.107 9	61.554 0	84.172 7	15
16	2.952 16	0.338 73	27.888 1	9.446 6	66.973 7	93.619 3	16
17	3.158 82	0.316 57	30.840 2	9.763 2	72.355 5	103.382 5	17
18	3.379 93	0.295 86	33.999 0	10.059 1	77.681 0	113.441 6	18
19	3.616 53	0.276 51	37.379 0	10.335 6	82.934 7	123.777 2	19
20	3.869 68	0.258 42	40.995 5	10.594 0	88.103 1	134.371 2	20
21	4.140 56	0.241 51	44.865 2	10.835 5	93.174 8	145.206 8	21
22	4.430 40	0.225 71	49.005 7	11.061 2	98.140 5	156.268 0	22
23	4.740 53	0.210 95	53.436 1	11.272 2	102.992 3	167.540 2	23
24	5.072 37	0.197 15	58.176 7	11.469 3	107.723 8	179.009 5	24
25	5.427 43	0.184 25	63.249 0	11.653 6	112.330 1	190.663 1	25
26	5.807 35	0.172 20	68.676 5	11.825 8	116.807 1	202.488 9	26
27	6.213 87	0.160 93	74.483 8	11.986 7	121.152 3	214.475 6	27
28	6.648 84	0.150 40	80.697 7	12.137 1	125.363 5	226.612 7	28
29	7.114 26	0.140 56	87.346 5	12.277 7	129.439 9	238.890 4	29
30	7.612 26	0.131 37	94.460 8	12.409 0	133.380 9	251.299 4	30
31	8.145 11	0.122 77	102.073 0	12.531 8	137.186 8	263.831 2	31
32	8.715 27	0.114 74	110.218 2	12.646 6	140.858 5	276.477 8	32
33	9.325 34	0.107 23	118.933 4	12.753 8	144.397 3	289.231 6	33
34	9.978 11	0.100 22	128.258 8	12.854 0	147.804 7	302.085 6	34
35	10.676 58	0.093 66	138.236 9	12.947 7	151.082 9	315.033 3	35
36	11.423 94	0.087 54	148.913 5	13.035 2	154.234 2	328.068 5	36
37	12.223 62	0.081 81	160.337 4	13.117 0	157.261 2	341.185 5	37
38	13.079 27	0.076 46	172.561 0	13.193 5	160.166 5	354.379 0	38
39	13.994 82	0.071 46	185.640 3	13.264 9	162.953 3	367.643 9	39
40	14.974 46	0.066 78	199.635 1	13.331 7	165.624 5	380.975 6	40
41	16.022 67	0.062 41	214.609 6	13.394 1	168.183 3	394.369 7	41
42	17.144 26	0.058 33	230.632 2	13.452 4	170.633 1	407.822 2	42
43	18.344 35	0.054 51	247.776 5	13.507 0	172.977 2	421.329 1	43
44	19.628 46	0.050 95	266.120 9	13.557 9	175.218 8	434.887 0	44
45	21.002 45	0.047 61	285.749 3	13.605 5	177.361 4	448.492 5	45
46	22.472 62	0.044 50	306.751 8	13.650 0	179.408 4	462.142 6	46
47	24.045 71	0.041 59	329.224 4	13.691 6	181.363 0	475.834 2	47
48	25.728 91	0.038 87	353.270 1	13.730 5	183.228 6	489.564 7	48
49	27.529 93	0.036 32	378.999 0	13.766 8	185.008 5	503.331 4	49
50	29.457 03	0.033 95	406.528 9	13.800 7	186.705 9	517.132 2	50
60	57.946 43	0.017 26	813.520 4	14.039 2	199.806 9	656.583 1	60
70	113.989 39	0.008 77	1 614.134 2	14.160 4	207.678 9	797.708 7	70
80	224.234 39	0.004 46	3 189.062 7	14.222 0	212.296 8	939.685 6	80
90	441.102 98	0.002 27	6 287.185 4	14.253 3	214.957 5	1 082.095 3	90
100	867.716 33	0.001 15	12 381.661 8	14.269 3	216.469 3	1 224.725 0	100

i	0.070 000
$i^{(2)}$	0.068 816
$i^{(4)}$	0.068 234
$i^{(12)}$	0.067 850
δ	0.067 659
$(1+i)^{1/2}$	1.034 408
$(1+i)^{1/4}$	1.017 059
$(1+i)^{1/12}$	1.005 654
v	0.934 579
$v^{1/2}$	0.966 736
$v^{1/4}$	0.983 228
$v^{1/12}$	0.994 378
d	0.065 421
$d^{(2)}$	0.066 527
$d^{(4)}$	0.067 090
$d^{(12)}$	0.067 468
$i/i^{(2)}$	1.017 204
$i/i^{(4)}$	1.025 880
$i/i^{(12)}$	1.031 691
i/δ	1.034 605
$i/d^{(2)}$	1.052 204
$i/d^{(4)}$	1.043 380
$i/d^{(12)}$	1.037 525

PLEASE TURN OVER

QUESTION 9

A loan of amount L_0 is taken out at time 0. It is repayable by equal annual instalments of size X over a period of n years. The annual effective interest rate charged on the loan is $i\%$.

Let L_t denote the loan outstanding at time t (where $t < n$), directly after the instalment at time t has been paid.

i. State expressions for L_t using
   - the prospective method and
   - the retrospective method.

[3]

ii. Show algebraically that the two equations in (i) are equivalent.

[3]

Consider a loan of R850,000 repayable by level annual instalments in arrears over 20 years. The interest rate agreed on is *8%* effective per annum.

iii. Calculate the annual instalments.

[2]

Directly after the seventh repayment, the bank adjusts the interest rate downward to *6%* per annum effective and adjusts the repayments accordingly.

iv. Calculate the loan outstanding directly after the 13^{th} payment using the prospective method.

[4]

v. Show that the value in (iv) is equal to the loan outstanding calculated using the retrospective method.

[4]

[Total 16]

PLEASE TURN OVER

Compound Interest

8%

i	0.080 000
$i^{(2)}$	0.078 461
$i^{(4)}$	0.077 706
$i^{(12)}$	0.077 208
δ	0.076 961
$(1+i)^{1/2}$	1.039 230
$(1+i)^{1/4}$	1.019 427
$(1+i)^{1/12}$	1.006 434
v	0.925 926
$v^{1/2}$	0.962 250
$v^{1/4}$	0.980 944
$v^{1/12}$	0.993 607
d	0.074 074
$d^{(2)}$	0.075 499
$d^{(4)}$	0.076 225
$d^{(12)}$	0.076 715
$i/i^{(2)}$	1.019 615
$i/i^{(4)}$	1.029 519
$i/i^{(12)}$	1.036 157
i/δ	1.039 487
$i/d^{(2)}$	1.059 615
$i/d^{(4)}$	1.049 519
$i/d^{(12)}$	1.042 824

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.080 00	0.925 93	1.000 0	0.925 9	0.925 9	0.925 9	1
2	1.166 40	0.857 34	2.080 0	1.783 3	2.640 6	2.709 2	2
3	1.259 71	0.793 83	3.246 4	2.577 1	5.022 1	5.286 3	3
4	1.360 49	0.735 03	4.506 1	3.312 1	7.962 2	8.598 4	4
5	1.469 33	0.680 58	5.866 6	3.992 7	11.365 1	12.591 1	5
6	1.586 87	0.630 17	7.335 9	4.622 9	15.146 2	17.214 0	6
7	1.713 82	0.583 49	8.922 8	5.206 4	19.230 6	22.420 4	7
8	1.850 93	0.540 27	10.636 6	5.746 6	23.552 7	28.167 0	8
9	1.999 00	0.500 25	12.487 6	6.246 9	28.055 0	34.413 9	9
10	2.158 92	0.463 19	14.486 6	6.710 1	32.686 9	41.124 0	10
11	2.331 64	0.428 88	16.645 5	7.139 0	37.404 6	48.262 9	11
12	2.518 17	0.397 11	18.977 1	7.536 1	42.170 0	55.799 0	12
13	2.719 62	0.367 70	21.495 3	7.903 8	46.950 1	63.702 8	13
14	2.937 19	0.340 46	24.214 9	8.244 2	51.716 5	71.947 0	14
15	3.172 17	0.315 24	27.152 1	8.559 5	56.445 1	80.506 5	15
16	3.425 94	0.291 89	30.324 3	8.851 4	61.115 4	89.357 9	16
17	3.700 02	0.270 27	33.750 2	9.121 6	65.710 0	98.479 5	17
18	3.996 02	0.250 25	37.450 2	9.371 9	70.214 4	107.851 4	18
19	4.315 70	0.231 71	41.446 3	9.603 6	74.617 0	117.455 0	19
20	4.660 96	0.214 55	45.762 0	9.818 1	78.907 9	127.273 2	20
21	5.033 83	0.198 66	50.422 9	10.016 8	83.079 7	137.290 0	21
22	5.436 54	0.183 94	55.456 8	10.200 7	87.126 4	147.490 7	22
23	5.871 46	0.170 32	60.893 3	10.371 1	91.043 7	157.861 8	23
24	6.341 18	0.157 70	66.764 8	10.528 8	94.828 4	168.390 5	24
25	6.848 48	0.146 02	73.105 9	10.674 8	98.478 9	179.065 3	25
26	7.396 35	0.135 20	79.954 4	10.810 0	101.994 1	189.875 3	26
27	7.988 06	0.125 19	87.350 8	10.935 2	105.374 2	200.810 4	27
28	8.627 11	0.115 91	95.338 8	11.051 1	108.619 8	211.861 5	28
29	9.317 27	0.107 33	103.965 9	11.158 4	111.732 3	223.019 9	29
30	10.062 66	0.099 38	113.283 2	11.257 8	114.713 6	234.277 7	30
31	10.867 67	0.092 02	123.345 9	11.349 8	117.566 1	245.627 5	31
32	11.737 08	0.085 20	134.213 5	11.435 0	120.292 5	257.062 5	32
33	12.676 05	0.078 89	145.950 6	11.513 9	122.895 8	268.576 4	33
34	13.690 13	0.073 05	158.626 7	11.586 9	125.379 3	280.163 3	34
35	14.785 34	0.067 63	172.316 8	11.654 6	127.746 6	291.817 9	35
36	15.968 17	0.062 62	187.102 1	11.717 2	130.001 0	303.535 1	36
37	17.245 63	0.057 99	203.070 3	11.775 2	132.146 5	315.310 3	37
38	18.625 28	0.053 69	220.315 9	11.828 9	134.186 8	327.139 1	38
39	20.115 30	0.049 71	238.941 2	11.878 6	136.125 6	339.017 7	39
40	21.724 52	0.046 03	259.056 5	11.924 6	137.966 8	350.942 3	40
41	23.462 48	0.042 62	280.781 0	11.967 2	139.714 3	362.909 6	41
42	25.339 48	0.039 46	304.243 5	12.006 7	141.371 8	374.916 3	42
43	27.366 64	0.036 54	329.583 0	12.043 2	142.943 0	386.959 5	43
44	29.555 97	0.033 83	356.949 6	12.077 1	144.431 7	399.036 6	44
45	31.920 45	0.031 33	386.505 6	12.108 4	145.841 5	411.145 0	45
46	34.474 09	0.029 01	418.426 1	12.137 4	147.175 8	423.282 4	46
47	37.232 01	0.026 86	452.900 2	12.164 3	148.438 2	435.446 7	47
48	40.210 57	0.024 87	490.132 2	12.189 1	149.631 9	447.635 8	48
49	43.427 42	0.023 03	530.342 7	12.212 2	150.760 2	459.848 0	49
50	46.901 61	0.021 32	573.770 2	12.233 5	151.826 3	472.081 4	50
60	101.257 06	0.009 88	1 253.213 3	12.376 6	159.676 6	595.293 1	60
70	218.606 41	0.004 57	2 720.080 1	12.442 8	163.975 4	719.464 8	70
80	471.954 83	0.002 12	5 886.935 4	12.473 5	166.273 6	844.081 1	80
90	1 018.915 09	0.000 98	12 723.938 6	12.487 7	167.480 3	968.903 3	90
100	2 199.761 26	0.000 45	27 484.515 7	12.494 3	168.105 0	1 093.821 0	100

60

PLEASE TURN OVER

Compound Interest

6%

i	0.060 000
$i^{(2)}$	0.059 126
$i^{(4)}$	0.058 695
$i^{(12)}$	0.058 411
δ	0.058 269
$(1+i)^{1/2}$	1.029 563
$(1+i)^{1/4}$	1.014 674
$(1+i)^{1/12}$	1.004 868
v	0.943 396
$v^{1/2}$	0.971 286
$v^{1/4}$	0.985 538
$v^{1/12}$	0.995 156
d	0.056 604
$d^{(2)}$	0.057 428
$d^{(4)}$	0.057 847
$d^{(12)}$	0.058 128
$i/i^{(2)}$	1.014 782
$i/i^{(4)}$	1.022 227
$i/i^{(12)}$	1.027 211
i/δ	1.029 709
$i/d^{(2)}$	1.044 782
$i/d^{(4)}$	1.037 227
$i/d^{(12)}$	1.032 211

n	$(1+i)^n$	v^n	$s_{\overline{n}\rceil}$	$a_{\overline{n}\rceil}$	$(Ia)_{\overline{n}\rceil}$	$(Da)_{\overline{n}\rceil}$	n
1	1.060 00	0.943 40	1.000 0	0.943 4	0.943 4	0.943 4	1
2	1.123 60	0.890 00	2.060 0	1.833 4	2.723 4	2.776 8	2
3	1.191 02	0.839 62	3.183 6	2.673 0	5.242 2	5.449 8	3
4	1.262 48	0.792 09	4.374 6	3.465 1	8.410 6	8.914 9	4
5	1.338 23	0.747 26	5.637 1	4.212 4	12.146 9	13.127 3	5
6	1.418 52	0.704 96	6.975 3	4.917 3	16.376 7	18.044 6	6
7	1.503 63	0.665 06	8.393 8	5.582 4	21.032 1	23.627 0	7
8	1.593 85	0.627 41	9.897 5	6.209 8	26.051 4	29.836 8	8
9	1.689 48	0.591 90	11.491 3	6.801 7	31.378 5	36.638 5	9
10	1.790 85	0.558 39	13.180 8	7.360 1	36.962 4	43.998 5	10
11	1.898 30	0.526 79	14.971 6	7.886 9	42.757 1	51.885 4	11
12	2.012 20	0.496 97	16.869 9	8.383 8	48.720 7	60.269 3	12
13	2.132 93	0.468 84	18.882 1	8.852 7	54.815 6	69.122 0	13
14	2.260 90	0.442 30	21.015 1	9.295 0	61.007 8	78.416 9	14
15	2.396 56	0.417 27	23.276 0	9.712 2	67.266 8	88.129 2	15
16	2.540 35	0.393 65	25.672 5	10.105 9	73.565 1	98.235 1	16
17	2.692 77	0.371 36	28.212 9	10.477 3	79.878 3	108.712 3	17
18	2.854 34	0.350 34	30.905 7	10.827 6	86.184 5	119.539 9	18
19	3.025 60	0.330 51	33.760 0	11.158 1	92.464 3	130.698 1	19
20	3.207 14	0.311 80	36.785 6	11.469 9	98.700 4	142.168 0	20
21	3.399 56	0.294 16	39.992 7	11.764 1	104.877 6	153.932 1	21
22	3.603 54	0.277 51	43.392 3	12.041 6	110.982 7	165.973 6	22
23	3.819 75	0.261 80	46.995 8	12.303 4	117.004 1	178.277 0	23
24	4.048 93	0.246 98	50.815 6	12.550 4	122.931 6	190.827 4	24
25	4.291 87	0.233 00	54.864 5	12.783 4	128.756 5	203.610 7	25
26	4.549 38	0.219 81	59.156 4	13.003 2	134.471 6	216.613 9	26
27	4.822 35	0.207 37	63.705 8	13.210 5	140.070 5	229.824 4	27
28	5.111 69	0.195 63	68.528 1	13.406 2	145.548 2	243.230 6	28
29	5.418 39	0.184 56	73.639 8	13.590 7	150.900 3	256.821 3	29
30	5.743 49	0.174 11	79.058 2	13.764 8	156.123 6	270.586 1	30
31	6.088 10	0.164 25	84.801 7	13.929 1	161.215 5	284.515 2	31
32	6.453 39	0.154 96	90.889 8	14.084 0	166.174 2	298.599 3	32
33	6.840 59	0.146 19	97.343 2	14.230 2	170.998 3	312.829 5	33
34	7.251 03	0.137 91	104.183 8	14.368 1	175.687 3	327.197 6	34
35	7.686 09	0.130 11	111.434 8	14.498 2	180.241 0	341.695 9	35
36	8.147 25	0.122 74	119.120 9	14.621 0	184.659 6	356.316 9	36
37	8.636 09	0.115 79	127.268 1	14.736 8	188.944 0	371.053 7	37
38	9.154 25	0.109 24	135.904 2	14.846 0	193.095 1	385.899 7	38
39	9.703 51	0.103 06	145.058 5	14.949 1	197.114 2	400.848 8	39
40	10.285 72	0.097 22	154.762 0	15.046 3	201.003 1	415.895 1	40
41	10.902 86	0.091 72	165.047 7	15.138 0	204.763 6	431.033 1	41
42	11.557 03	0.086 53	175.950 5	15.224 5	208.397 8	446.257 6	42
43	12.250 45	0.081 63	187.507 6	15.306 2	211.907 8	461.563 8	43
44	12.985 48	0.077 01	199.758 0	15.383 2	215.296 2	476.947 0	44
45	13.764 61	0.072 65	212.743 5	15.455 8	218.565 5	492.402 8	45
46	14.590 49	0.068 54	226.508 1	15.524 4	221.718 2	507.927 2	46
47	15.465 92	0.064 66	241.098 6	15.589 0	224.757 2	523.516 2	47
48	16.393 87	0.061 00	256.564 5	15.650 0	227.685 1	539.166 2	48
49	17.377 50	0.057 55	272.958 4	15.707 6	230.504 8	554.873 8	49
50	18.420 15	0.054 29	290.335 9	15.761 9	233.219 2	570.635 7	50
60	32.987 69	0.030 31	533.128 2	16.161 4	255.204 2	730.642 9	60
70	59.075 93	0.016 93	967.932 2	16.384 5	269.711 7	893.590 9	70
80	105.795 99	0.009 45	1 746.599 9	16.509 1	279.058 4	1 058.181 2	80
90	189.464 51	0.005 28	3 141.075 2	16.578 7	284.973 3	1 223.688 3	90
100	339.302 08	0.002 95	5 638.368 1	16.617 5	288.664 6	1 389.707 6	100

PLEASE TURN OVER

QUESTION 10

A private equity fund is considering investing in a major theme park. The fund has been asked to make an investment of R5 million for a 10% share in the revenues from the theme park. No other costs will be incurred by the private equity fund.

The following revenues are expected to arise from the project:

- In the first year, 36,500 people will purchase tickets at a price of R150 each.
- In the second year, 43,800 people will purchase tickets at a price of R165 each.
- At the start of the third year, both the number of people and the ticket price will increase by 3% from their level in the previous year. Thereafter, both will continue to rise by 3% per annum compound, with increases taking place at the start of every year.
- At the end of the 15^{th} year, the theme park will be sold for R15 million, and the private equity fund will also receive 10% of the sale revenue.
- Assume that all ticket revenue is received continuously throughout the year.

i. Calculate the net present value of the investment in the theme park at a rate of interest of 8.3% per annum effective.

[15]

The private equity fund has to borrow the initial R5 million to invest in the theme park at a rate of 8.3% per annum effective. The loan is to be repaid continuously with proceeds of the theme park and once the loan is fully paid off, proceeds will be invested at a rate of 4% per annum effective.

ii. Explain whether the theme park is a profitable investment for the private equity fund.

[1]

The terms of the loan have been changed and the loan can now only be repaid annually in arrears.

iii. Explain how this change will affect the profitability of the project.

[3]

[Total 19]

[GRAND TOTAL 100]

END OF EXAMINATION

