Actuarial Society of South Africa

EXAMINERS' REPORT

September 2019

Subject A211

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General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

Question 1

(i)

$$(\overline{Ia})_{\overline{n}|} = \int_0^n tv^t dt$$

OR

$$(\overline{Ia})_{\overline{n}|} = \int_0^n te^{-\delta t} dt$$

(ii)

$$(\overline{Ia})_{\overline{n}|} = \int_0^n tv^t dt = \int_0^n te^{-\delta t} dt$$

Integrating by parts:

$$u = t$$

$$\frac{dv}{dt} = e^{-\delta t}$$

$$(\overline{Ia})_{\overline{n}|} = \left[t\left(\frac{1}{-\delta} e^{-\delta t} \right) \right]_0^n - \int_0^n \frac{1}{-\delta} e^{-\delta t} dt = -\frac{nv^n}{\delta} + \frac{1}{\delta} \int_0^n v^t dt = -\frac{nv^n}{\delta} + \frac{\overline{a}_{\overline{n}|}}{\delta}$$

$$= \frac{\overline{a}_{\overline{n}|} - nv^n}{\delta}$$

Part (i) was done well by most candidates. *In part (ii) the common error was mistakes made in the integration by parts.*

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Question 2

$$1,000\times(1+0.05(5))\times(1.07)^3\times(1.005)^{48}\times\left(1-\frac{0.08}{12}\right)^{-48}=2,682.08$$

This was the best answered question in the paper. However, many candidates lost time by converting the interest rates given into annual effective interest rates before calculating the accumulated value, this is an unnecessary step.

Question 3

(i)

$$\left(1-\frac{d^{(p)}}{p}\right)^p=v=e^{-\delta}\Rightarrow\left(1-\frac{d^{(p)}}{p}\right)=e^{-\delta/p}$$

$$\frac{d^{(p)}}{p}=1-e^{-\delta/p}\Rightarrow d^{(p)}=p\left(1-e^{-\delta/p}\right)$$

(ii)

$$\lim_{p\to\infty}d^{(p)}=\lim_{p\to\infty}\left[p\left(1-e^{-\delta/p}\right)\right]=\lim_{p\to\infty}\left[p\left(1-\left(1-\frac{\delta}{p}+\frac{\delta^2}{2!\,p^2}-\dots\right)\right)\right]$$

$$=\lim_{p\to\infty}\left[p\left(\frac{\delta}{p}+\frac{\delta^2}{2!\,p^2}-\dots\right)\right]=\lim_{p\to\infty}\left[\delta-\frac{\delta^2}{2!\,p}+\frac{\delta^3}{3!\,p^2}\dots\right]=\delta$$

Part (i) was answered well.

Part (ii) was one of the worst answered questions in the paper. Candidates need to avail themselves of all the information available in the Table book. See page 2 to 5 in the Table book for Mathematical methods.

An alternative method for part (ii) is L'Hopital rule.

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Question 4

(i)

Any four of the following

- Regulations
- Taxation
- Cancellation terms
- Future investment returns
- Future inflation
- New business
- Lapses
- Mortality
- Expenses

(ii)

Deterministic model

- contains no random components
- is deterministic in nature.
- gives one result for a single scenario.

Stochastic model

- recognizes the random nature of input components
- gives distributions of relevant results for a distribution of scenarios,
- output also random in nature

(iii)

Proxy Model

- a simplified formula of a Monte Carlo simulation
- developed to predict results with reasonable accuracy
- as a substitute for running the full model.
- It is used to replace full Monte Carlo simulation,
- to provide faster but less accurate results.

Part (i) was the worst answered question in the paper with candidates focusing on the modelling process rather that the external factors driving the parameters. Very few candidates scored any marks in this part of the question. *Parts (ii) and (iii) were answered more successfully by the better prepared candidates but no candidates scored full marks for either part. This is indicative of the scoring opportunities that exist in the theory questions which candidates are missing out on.*

Question 5

Accumulation @ t=3

$$100\exp\left(\int_0^3 \frac{s^2}{100}ds\right)+X=100\exp\left(\left.\frac{s^3}{100(3)}\right|_0^3\right)+X=100e^{0.09}+X=109.4174284+X\ldots\boxed{1}$$

Accumulation @ *t=6*

$$(109.4174284+X)\exp\left(\int_3^6 \frac{s^2}{100}ds\right)=(109.4174284+X)\exp\left(\left.\frac{s^3}{100(3)}\right|_3^6\right)$$

$$=(109.4174284+X)\times\exp(0.72-0.09))=(109.4174284+X)\times(1.877610579)\ldots\boxed{2}$$

Interest earned from t=3 to t=6 from (2) - (1) :

$$(109.4174284+X)\times(1.877610579)-109.4174284-X=(109.4174284+X)\times(0.877610579)$$

and this equals X

$$(109.4174284+X)\times(0.877610579)=X$$

$$96.02589269=0.122389421X$$

$$X=784.59$$

This question called for some logical thinking combined with some basic integration.

It was pleasing to see well prepared candidates applying their minds to the question and solving the problem very efficiently.

For marginal candidates this question was challenging with many leaving it out all together or not realizing that the interest earned is the difference between the accumulated value at time $t=3$ and $t=6$.

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Question 6

(i)

25 April 2013: $10,000*0.015*\frac{171.4}{149.2}=172.3190349$

25 October 2013: $10,000*0.015*\frac{173.8}{149.2}=174.7319035$

$$10,000*\frac{173.8}{149.2}=1,1648.79357$$

(ii)

Q(0) = RPI on 25 October 2008 (issue date)

$Q(t_0)$ = RPI on 25 October 2012 (purchase date)

$$\text{Price}=1.5\times\left[\frac{Q(t_1)}{Q(0)}\right]\times\left[\frac{Q(t_0)}{Q(t_1)}\right]\times v_{3.5\%}^{0.5}+1.5\times\left[\frac{Q(t_2)}{Q(0)}\right]\times\left[\frac{Q(t_0)}{Q(t_2)}\right]\times v_{3.5\%}^{1}+100\times\left[\frac{Q(t_2)}{Q(0)}\right]\times\left[\frac{Q(t_0)}{Q(t_2)}\right]\times v_{3.5\%}^{1}$$

$$=1.5\times\left[\frac{Q(t_0)}{Q(0)}\right]\times v_{3.5\%}^{0.5}+1.5\times\left[\frac{Q(t_0)}{Q(0)}\right]\times v_{3.5\%}^{1}+100\times\left[\frac{Q(t_0)}{Q(0)}\right]\times v_{3.5\%}^{1}$$

$$=1.5\times\left[\frac{169.4}{149.2}\right]\times v_{3.5\%}^{0.5}+1.5\times\left[\frac{169.4}{149.2}\right]\times v_{3.5\%}^{1}+100\times\left[\frac{169.4}{149.2}\right]\times v_{3.5\%}^{1}$$

$$=1.674039368+1.645490927+109.6993952=113.02$$

This was an easy question on a topic many candidates struggle with. Well prepared candidates easily obtained full credit for this question.

In part (i) the common errors were using the incorrect indexes, not multiplying the coupon rate with the nominal value and not calculating the redemption value.

In part (ii) the common errors were using the incorrect indexes in removing the inflation or not removing the inflation at all.

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Question 7

- Develop a well-defined set of objectives that need to be met by the results of the data analysis
    - Here the objective is to determine if mathematical entrance exam results is correlated to the Actuarial Science 1st year students' performance
- Identify the data items required for the analysis
    - The data items needed would be the mathematical entrance mark for the 1st year students doing Actuarial Science in the different South African Universities and the 1st year final year results for such students over a period of time
- Collection of the data from appropriate sources
    - The data can be obtained from the Universities offering Actuarial Science degrees.
- Processing and formatting the data for analysis, e.g. inputting into a spreadsheet, database or other model.
    - The data will need to be extracted from the administrative system of the Universities and loaded into whichever statistical package is being used for the analysis.
- Cleaning data, e.g. addressing unusual , missing or inconsistent values
    - For example, a student may be recorded as registered at University X but the marks may be missing or marks which are unrealistic e.g. negative numbers or marks more than 100% per subject X
- Exploratory data analysis,
    - Here takes the form of inferential analysis as we are here testing the hypothesis that mathematical entrance mark is correlated with 1st year Actuarial Science performance at universities.
- Modelling the data.
    - In this case we need to choose the correct statistical method e.g. a Chi-squared test for the analysis
- Communicating the results, which include: describing the data sources used, the analysis performed and the conclusion of the analysis.
- Monitoring the process. Updating the data and repeating the process if required.
    - May mean choosing another statistical package to use, or adjusting the level of significance chosen.

The common error was not giving an appropriate example at each key step.

Question 8

$$-120,000\left(1+v^{0.25}+v\right)-300,000v^{14}+\bar{a}_{\overline{1}|}\left(30,000v+33,000v^2+36,000v^3+39,000v^4\right)$$

$$+39,000\bar{a}_{\overline{1}|}\left((1.05)v^5+(1.05)^2v^6+\ldots+(1.05)^{t-5}v^t\right)=0 \quad @i=10\%$$

$$-322,032.1592+39,000\bar{a}_{\overline{1}|}v^4\left((1.05)v^1+(1.05)^2v^2+\ldots+(1.05)^{t-5}v^{t-5}\right)=0 \quad @i=10\%$$

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With $\frac{1.05}{1.1}=\frac{1}{1+j} \Rightarrow j=4.7619048\%$

$$-322,032.1592+39,000\bar{a}_{\overline{1}|10\%}v_{10\%}^{4}\left(v_j^1+v_j^2+\ldots+v_j^{t-5}\right)=0$$

$$-322,032.1592+39,000\bar{a}_{\overline{1}|10\%}v_{10\%}^{4}a_{\overline{t-5}|j\%}=0$$

$$a_{\overline{t-5}|j\%}=12.67469016$$

t-5= 19.888690318

DPP = 19.888690318 +5 =24.888690318

This question was answered well in general with most candidates scoring partial credit but very few candidates were able to calculate the final DPP.

The common errors were discounting the refurbishment cost of R300,000, for 15 years instead of the correct 14 years and assuming the cashflows before time t = 14 could be ignored.

Question 9

(i)

$$106(1+y_1)^{-1}=96$$

$$y_1=0.1041666$$

$$106(1+y_2)^{-2}+6(1+y_1)^{-1}=96 \Rightarrow 106(1+y_2)^{-2}+6(1+0.104167)^{-1}=96$$

$$106(1+y_2)^{-2}=90.56604$$

$$y_2=0.081858$$

$$106(1+y_3)^{-3}+6(1+y_2)^{-2}+6(1+y_1)^{-1}=96$$

$$106(1+y_3)^{-3}=85.43965824$$

$$y_3=0.074522$$

(ii)

$$f_{0,1} = y_1 = 10.41666\%$$

$$(1+f_{1,2})^2 = \frac{(1+y_3)^3}{(1+y_1)} = \frac{(1+0.074522)^3}{(1+0.1041666)} = 1.1236$$

$$f_{1,2} = 0.06$$

$$106(1+y_4)^{-4} + 6(1+y_3)^{-3} + 6(1+y_2)^{-2} + 6(1+y_1)^{-1} = 96$$

$$(1+y_4)^{-4} = \frac{80.60345}{106} = 0.760409917$$

$$(1+f_{2,2})^2 = \frac{(1+y_4)^4}{(1+y_2)^2} = \frac{(0.760409917)^{-1}}{(1+0.08185797)^2} = 1.1236$$

$$f_{2,2} = 0.06$$

(iii)

- Spot rates are a decreasing function of the term.
- The expectations theory posits that investors are expecting a decrease in short term spot rates.
- This makes short-term investments less attractive and longer-term investments more attractive.
- In these circumstances, yields on short-term investments will rise and yields on long-term investments will fall.

Part (i) and (ii) were answered well. *The common mistakes were the use of a zero – coupon bond instead of a fixed interest bond in part (i) and not realizing that forward rates $f_{1,2}$ and $f_{2,2}$ apply for a two years period each.* *Part (iii) was very poorly answered with many candidates confusing expectation theory with other yield curve theories*

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Question 10

(i)

Let X = nominal amount of zero-coupon bond
Y = nominal amount of fixed-coupon bond

PV(L) = $400,000v_{8\%}^{10} = 185,277.3952$

10% is in cash thus 18,527.73952 and remainder 166,749.6557 will be invested in 2 bonds

PV(A)= $18,527.73952 + Xv_{8\%}^{12} + 0.08Ya_{\overline{16}|8\%}^{(2)} + 1.1Yv_{8\%}^{16}$

$$18,527.73952 + Xv_{8\%}^{12} + 0.08Ya_{\overline{16}|8\%}^{(2)} + 1.1Yv_{8\%}^{16} = 185,277.3952$$

$$Xv_{8\%}^{12} + Y0.08a_{\overline{16}|8\%}^{(2)} + 1.1Yv_{8\%}^{16} = 166,749.6557\ldots\ldots\boxed{1}$$

Numerator of DMT(L): $(10)400,000v_{8\%}^{10} = 1,852,773.952$

$$i = 8\% \Rightarrow i^{(2)} = 7.864096908\% \Rightarrow \frac{i^{(2)}}{2} = 3.923048454\%$$

Numerator of DMT(A):

$$12Xv_{8\%}^{12} + Y\left(0.04(\tfrac{1}{2})(Ia)_{\overline{32}|3.923048\%} + (16)(1.1)v_{8\%}^{16}\right) = 1,852,773.93\ldots\ldots\boxed{2}$$

From (1)

$$X = \frac{(166,749.6557 - 1.043078787Y)}{0.397113759}$$

sub into (2)

$Y = 57,483.53505$
sub into (1) and $X = 268,914.8824$

(ii)

- Spread of asset around DMT is greater than spread of liabilities around DMT
- Convexity of assets > Convexity of liabilities
- Immunized

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This question referenced Redington's first two conditions for immunisation. As the second condition is set out in terms of the volatility it would be most appropriate to work with the volatility of the assets and liabilities. As there is a direct relationship between volatility and DMT the examiners accepted the marking schedule as stated above as an alternative. Both methods would lead to the same numerical answers.

This question was answered poorly. Although many candidates had a broad idea as to what needed to be done the details in the question were not handled well.

Common errors were dealing incorrectly with the cash or ignoring the cash and not assigning unknown values to the nominal values of the bonds as a first step.

Question 11

(i)

$10000 = 1000 + 200(t-1) \Rightarrow t = 46$ months

$$800 \times a_{\overline{46}|} + 200 \times (Ia)_{\overline{46}|} + 10,000 \times v^{46} \times a_{\overline{134}|} \qquad @ \frac{i^{(12)}}{12} = 0.0068214933$$

$$= 31,494.3014483263 + 175,610.2038747990 + 641,080.1491508610$$

$$= 848,184.654 \approx 848200$$

(ii)

$$848,200 \times (1.085)^3 - 800 \times s_{\overline{36}|\frac{i^{(12)}}{12}} - 200 \times (Ia)_{\overline{36}|\frac{i^{(12)}}{12}} \times (1.085)^3 = 906,433.21$$

(iii)

- Loan outstanding is larger than original loan
- Payments at start of loan term was not large enough to cover the interest
- Interest on loan was R57,859.90673 and repayment was R1,000
- Interest not covered was added to outstanding loan

(iv)

Payment: 8,200
Interest component: $906,433.210189080 * 0.0068214933 = 6,183.23$
Capital component: 8,200 - 6,183.22813 = $2,016.77$

Part (i) was answered very well.

In part (ii) many candidates ignored the instruction of the question and used the prospective method for calculating the outstanding loan. This gained no credit.

Parts (iii) and (iv) were answered poorly. Candidates that gave sensible comments on their numerical answers (even if incorrect) were given credit.

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