EXAMINATION

20 September 2019

Subject A211 — Financial Mathematics

Time allowed: Two hours and 15 minutes reading time

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have 2 hours to complete the paper.*

4. *The use of calculators is not permitted during the reading time.*

5. *Mark allocations are shown in brackets.*

6. *Attempt all questions, beginning your answer to each question on a new page.*

7. *You should show calculations where this is appropriate.*

8. *Assume that months are all of equal length, unless otherwise stated.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

> *In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.*

QUESTION 1

i. Define algebraically $(\overline{Ia})_{\overline{n}|}$.

[1]

ii. Hence, prove that $(\overline{Ia})_{\overline{n}|} = \dfrac{\overline{a}_{\overline{n}|} - nv^n}{\delta}$.

[4]
[Total 5]

QUESTION 2

A man deposits R1,000 into a savings account which earns interest as follows:

- Simple interest at a rate of 5% per annum over the first five years.
- Effective compound interest of 7% per annum over the next three years.
- Effective compound interest of 0.5% per month over the next four years.
- Nominal rate of discount of 8% per annum compounded monthly over the last four years.

Calculate the accumulated value in the account after 16 years.

[Total 4]

QUESTION 3

i. Show that $d^{(p)}$, the nominal rate of discount payable p times a year, can be written as $d^{(p)} = p\left(1 - e^{-\delta/p}\right)$, where δ is the force of interest.

[2]

ii. Starting from the equation derived in i., show that $\lim\limits_{p \to \infty} d^{(p)} = \delta$.

[3]
[Total 5]

QUESTION 4

i. List four issues or events that should be considered when setting parameter values for use in a cashflow model of a life office.

[2]

ii. Explain the differences between a stochastic and deterministic model.

[3]

iii. a. Briefly outline a proxy model.
      b. Explain why a proxy model may be used.

[3]
[Total 8]

PLEASE TURN OVER

QUESTION 5

An investor deposits the following amounts into a fund:
- R100 at time 0 and
- RX at time 3

The fund earns interest at a force of interest, $\delta(t) = \dfrac{t^2}{100}$ where t $\geq$ 0.

The amount of interest earned on the fund from time 3 to time 6 is also equal to RX.

Determine the value of *X*.

[Total 8]

QUESTION 6

On 25 October 2008 a certain government issued a five-year index-linked bond. The bond paid coupons at a nominal rate of 3% per annum payable half-yearly in arrear and the bond was redeemed at par. The coupons and redemption payments were index-linked to a retail price index at the month of payment.

An investor, who is not subject to tax, bought R10,000 nominal of the bond on 26 October 2012. The investor held the bond until redemption.

	Retail price index	
	April	*October*
2008	…	149.2
	…	…
2012	…	169.4
2013	171.4	173.8

i.   Calculate all the payments received by the investor on 25 April 2013 and 25 October 2013.

[3]

ii.   Calculate the purchase price that the investor paid on 26 October 2012 if the investor achieved an effective real yield of 3.5% per annum on this investment.

[5]

[Total 8]

QUESTION 7

A student wishes to investigate whether the pass rate of first year actuarial science students at universities is correlated to a mark obtained in a mathematical entrance exam.

Describe the key steps to follow in the data analysis process, for this investigation, and give an example, applicable in this scenario, for each of the steps.

[Total 10]

PLEASE TURN OVER

QUESTION 8

A local company is considering investing in a gas project. The project requires an initial investment of three payments, each of R120,000. The first is due at the start of the project, the second three months later, and the third payment is due one year after the start of the project.

At the beginning of the 15th year, it is assumed that a major refurbishment of the infrastructure will be required, costing R300,000. The project is expected to provide a continuous income stream as follows:

- R30,000 in the second year
- R33,000 in the third year
- R36,000 in the fourth year
- R39,000 in the fifth year

Thereafter the continuous income stream is expected to increase by 5% per annum (compound) at the start of each year. The income stream is expected to cease at the end of the 35th year from the start of the project.

Calculate the discounted payback period for the project, assuming an effective rate of interest of 10% per annum. Note: The discounted payback period is after the major refurbishment.

[Total 10]

QUESTION 9

Bonds pay annual coupons at a rate of 6% per annum, in arrear, and are redeemable at par. The bonds, redeemable in exactly one, two, three, four and five years respectively, are all priced at R96 per R100 nominal.

i.   Determine the one-year, two-year and three-year spot rates.

[3]

ii.   Calculate $f_{0,1}$, $f_{1,2}$ and $f_{2,2}$ where $f_{t,r}$ is the annual forward interest rate agreed at time 0 for an investment made at time t (where $t > 0$) for a period of r years.

[5]

iii. Comment on the term structure of the spot rates in i. with reference to expectations theory.

[3]
[Total 11]

QUESTION 10

A pension fund has a liability of R400,000 due in ten years' time.

The pension fund has exactly enough funds to cover the liability based on an effective rate of interest of 8% per annum. This is also the interest rate at which current market prices are calculated and the rate earned on cash.

The pension fund wishes to hold 10% of its funds in cash, and to invest the balance in the following securities:
- A zero-coupon bond redeemable at par in 12 years' time
- A fixed-coupon bond paying half-yearly coupons of 8% per annum, in arrear, redeemable at 110% in 16 years' time

i.   Determine the amount to invest in the zero-coupon bond and the fixed-coupon bond, respectively, for the pension fund's portfolio of assets and liabilities to satisfy Redington's first two conditions for immunisation.

[9]

ii.   Explain, without further calculation, whether the pension fund would be immunised against small changes in the interest rate if the quantities of stock in i. are purchased.

[3]
[Total 12]

QUESTION 11

An investor takes out a 15-year loan. The loan is repaid by monthly payments in arrear with the first repayment being R1,000 and the repayments increasing by R200 each month until the repayments reach R10,000. The repayments then remain at R10,000 for the rest of the term. The annual effective rate of interest is 8.5% per annum.

i.   Show that the amount of the loan is R848,200.

[7]

ii.   Calculate, using the retrospective method, the amount of capital outstanding at the end of the third year, just after the repayment made at that time.

[5]

iii.   Comment on the numerical answer obtained in ii. and what this implies about repayments made.

[4]

iv.   Calculate the capital component and the interest component of the 37th payment.

[3]
[Total 19]

[GRAND TOTAL 100]

END OF EXAMINATION