Actuarial Society of South Africa

EXAMINERS REPORTS

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Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

The examiners mark with the candidates answers where a question has several parts which follow on each other. It is therefore essential that candidates work consistently with their answers.

Many candidates can also increase their probability of passing this exam by practising good exam technique, which is often missing in its most basic form.

Question 1

The objectives of the modelling exercise.

The validity of the model for the purpose to which it is to be put.

The validity of the data to be used.

The validity of the assumptions.

The possible errors associated with the model or that the parameters used is not a perfect representation of the real-word situation being modeled.

The impact of correlations between the random variables that 'drive' the model.

The extent of correlations between the various results produced form the model.

The current relevance of models written and used in the past.

The credibility of the data input.

The credibility of the results output.

The dangers of spurious accuracy.

The ease with which the model and its results can be communicated.

Regulatory requirements.

The question was well done in general.

Question 2

i.
The amount and timing of monthly interest payments and the lump sum payment at the end of the term is known in advance by Bank X, so amounts and timing are certain.

However, these cashflows are subject to default by the Person A, which adds uncertainty to whether cashflows will be received on time and in full.

ii.
This reduces the uncertainty of whether the principle amount of the loan will be repaid, as Bank X is sure to receive their original loan amount if the policyholder dies.

However, the timing of the repayment is still uncertain as it is unknown when, or if, the policyholder will die.

Default risk (credit risk) of the insurance company increases the uncertainty of the payment.

Term assurance do not pay out if cause of death not covered by insurance contract (e.g. suicide) – increase uncertainty of payment

Part (ii) was one of the worst answered questions in the paper.

Candidates showed poor understanding of a term assurance.

Although assurances are covered in A213, brief descriptions of assurances are part of the A211 syllabus.

Question 3

i.

$$(1+i)^2 = \exp\left(\int_2^4 0.002t^3\,dt\right)$$
$$= \exp\left(\frac{0.002}{4}t^4\right)\Big|_2^4$$
$$= \exp(0.12)$$
$$= 1.127496852$$

$$i = 0.06183655 = 6.18\%$$

ii.

$$(1-d)^{-1} = \exp\left(\int_1^2 0.002t^3\, dt\right)$$
$$= \exp\left(\frac{0.0002}{4}t^4\right)\Big|_1^2$$
$$= \exp(0.0075)$$
$$= 1.0075$$

$$d = 0.00747195 = 0.75\%$$

iii.

$$PV = 1{,}000 + 1{,}000 \times \exp\left(-\int_0^1 0.002t^3\, dt\right) + 1{,}000 \times \exp\left(-\int_0^2 0.002t^3\, dt\right) + 3{,}000 \times \exp\left(-\int_0^4 0.002t^3\, dt\right)$$

$$= R5{,}631.09$$

Part (i) and (ii) was well answered.

Common errors in part (iii) were

- *Errors in the timeline of the payments.*
- *Calculating a constant interest rate for the full three/four year period.*

When interest rates are a function of t, a separate corresponding constant interest rate must be calculated for the period of each individual cashflow.

Question 4

i.

$v = (1+i)^{-1}$ with i → effective interest rate per unit of time.

$i^{(p)}$ → nominal interest rate per unit of time compounded $p$thly.

$$a_{\overline{n}|}^{(p)} = \sum_{t=1}^{np} \frac{1}{p} v^{\frac{t}{p}}$$

$$= \frac{1}{p} \times \frac{v^{\frac{1}{p}}(1-v^n)}{1-v^{\frac{1}{p}}}$$

$$= \frac{1-v^n}{p \times \left[(1+i)^{\frac{1}{p}} - 1\right]}$$

$$= \frac{1-v^n}{i^{(p)}}$$

ii.

$$p = \frac{1}{4} = 0.25$$

$$n = 60$$

$$i^{(0.25)} = 0.25(1.075^4 - 1) = 0.08386$$

Payment =12,500

$$X = 12{,}500 \times a_{\overline{60}|}^{(0.25)}$$

$$= 12{,}500 \times \left(\frac{1-(1.075)^{-60}}{0.08386}\right)$$

$$= 12{,}500 \times (11.76803)$$

$$= R147{,}100.50$$

Alternative

$p = 1$

$n = 15$

$\frac{i^{(0.25)}}{0.25} = 0.33546914$

Payment=50,000

$X = 50{,}000 \times a_{\overline{15|}}$

Common errors in part (i) were

- *Not defining the notation as asked for in the question.*
- *Making errors in writing down the initial payment series.*
- *Appling the geometric series incorrectly.*

The common error in part (ii) was using the incorrect payment with the annuity factor.

Question 5

$FV = 50 \times \left(1 + f_{1,7}\right)^7 + 50 \times e^{F_{4,1}} \times \left(1 + f_{5,3}\right)^3$

$(1 + y_8)^8 = (1 + y_1) \times \left(1 + f_{1,7}\right)^7$

$\frac{100}{45} = \frac{100}{95} \times \left(1 + f_{1,7}\right)^7$

$\left(1 + f_{1,7}\right)^7 = \frac{95}{45}$

$(1 + y_5)^5 \times \left(1 + f_{5,3}\right)^3 = (1 + y_8)^8$

$\left(1 + f_{5,3}\right)^3 = \frac{65}{45}$

$$FV = 50 \times \left(\frac{95}{45}\right) + 50 \times (1.08545) \times \frac{65}{45}$$

$$= 105.555556 + 78.39403071$$

$$= R183.95$$

This was one of the worst answered questions in the paper.

The common error was the use of spot rates to calculate the accumulation factors.

The accumulation factors used forward rates as the investments were made in the future.

Spot rates were needed to calculate the appropriate forward rates.

Question 6

i.

It is the weighted mean term of the cashflows where the weights are the present values of the cashflows.

ii.

$$DMT = \frac{850 \times (Ia)_{\overline{22|}@\frac{i^{(2)}}{2}}}{850 \times a_{\overline{22|}@\frac{i^{(2)}}{2}}}$$

$$\frac{i^{(2)}}{2} = (1.035^{0.5} - 1) = 0.01734$$

$$(Ia)_{\overline{22|}@\frac{i^{(2)}}{2}} = 196.2894$$

$$DMT = \frac{196.2894}{18.1592} = 10.81\ half\ years \rightarrow 5.40\ years$$

iii.

The discounted mean term of the assets (bond) is more than the discounted mean term of the liabilities (annuity).
This means the present value of the assets will be more sensitive to a change in interest rates.
The insurance company will make a profit, as the present value of the assets will increase by more.

A common error in part (ii) was to allow incorrectly for the increasing time period of the half-yearly cashflows. The easiest way to do this, is to work with half-years as is done in the examiners report.

In part(iii) a common error was to make comments about immunization although no mention was made of immunization in the question. Candidates must use the information given which in the case was only information about the discounted mean term.

Question 7

i.

$$i^{(4)} = 4\left(\left(1+\frac{0.16}{12}\right)^{\frac{12}{4}} - 1\right) = 0.1621481$$

$$R150{,}000 = 4Xa_{\overline{5}|}^{(4)} \times (1 + 2v^5 + 3v^{10})$$

$$a_{\overline{5}|}^{(4)} = \frac{1-\left(1+\frac{0.16}{12}\right)^{-12\times 5}}{0.1621481} = 3.381521510$$

$v^5 = 0.451710584$
$v^{10} = 0.204042452$

$$X = \frac{150{,}000}{4 \times (3.38152) \times (1 + 2v^5 + 3v^{10})} = R4{,}408.455024307$$

42nd payment is in the third five-year period.

$3X = R13{,}225.38$

ii.
Loan outstanding after 41st payment (after the first payment in the third term):

$L_{41} = 4 \times (13{,}225.36507292) \times a^{(4)}_{\overline{4.75}|} = R172{,}913.388492408$

$a^{(4)}_{\overline{4.75}|} = 3.268593864$

Interest portion of 42nd payment:

$I_{42} = L_{41} \times \frac{i^{(4)}}{4} = R7{,}009.17$

$C_{42} = R13{,}225.38 - R7{,}009.17 = R6{,}216.21$

Part (i) was the best answered question in the paper.

The common error in part (i) was the conversion of the interest rate.

Part (ii) was answered less well with several candidates struggling to calculate the loan outstanding.

Question 8

i.
Capital Gains Test:

$i^{(2)} = 2 \times (1.0725^{0.5} - 1) = 0.07123518$

$\frac{D \times (1 - t_1)}{R} = \frac{0.095 \times (0.75)}{0.96} = 0.07421875$

$i^{(2)} < 0.07421875 \rightarrow no\ CGT$

$P = 100 \times \left(0.095 \times (0.75) \times a^{(2)}_{\overline{10}|} + 0.96 \times v^{10}\right)$

$a^{(2)}_{\overline{10}|} = 7.066771$

$P = R98.03\ per\ R100\ nominal$

ii.

$net\ running\ yield = \frac{D \times (1 - t_1)}{P} = \frac{0.095 \times (0.75)}{0.9803} = 0.072684377 = 7.3\%$

Part (i) was answered well but in Part (ii) candidates did not know the definition of a running yield.

Question 9

i.

$$PV_{income} = v^2 \times \left(99{,}000 \times \bar{a}_{\overline{4}|} + 99{,}000 \times (1.01) \times v^4 \times \bar{a}_{\overline{1}|} \begin{pmatrix} 1 + v^1 \times 1.01^1 + \cdots \\ +v^{13} \times 1.01^{13} \end{pmatrix}\right)$$
$$+ 3m \times v^{20}$$

$$PV_{income} = v^2 \times \left(99{,}000 \times \bar{a}_{\overline{4}|} + 99{,}000 \times (1.01) \times v^4 \times \bar{a}_{\overline{1}|} \times \left(\ddot{a}_{\overline{14}|@j}\right)\right)$$
$$j = \frac{\exp(0.034)}{1.01} - 1 = 0.02434119$$

$$\ddot{a}_{\overline{14}|@j} = 12.0303114$$

$$\bar{a}_{\overline{1}|} = 0.98319$$
$$\bar{a}_{\overline{4}|} = 3.739923$$

$$PV_{income} = R1{,}310{,}352.26 + R1{,}159{,}850.98 = R2{,}830{,}203.24$$

$$PV_{outflow} = 2{,}514{,}650 + (13{,}000 + 14{,}000v + 15{,}000v^2 + \cdots + 32{,}000v^{19})$$

$$PV_{running\ costs} = 12{,}000 \times \ddot{a}_{\overline{20}|} + 1{,}000 \times (I\ddot{a})_{\overline{20}|}$$

$$\ddot{a}_{\overline{20}|} = 14.759354326$$

$$(I\ddot{a})_{\overline{20}|} = 138.415277994$$

$$PV_{outflow} = R2{,}514{,}650 + 12{,}000 \times (14.759354326) + 1{,}000 \times (138.415277994)$$
$$= R2{,}830{,}177.52990389$$

$$NPV = R2{,}830{,}203.24 - R2{,}830{,}177.24$$
$$= R25.71$$

ii.

$NPV_{DPP} \geq 0$

$NPV = -2{,}514{,}650 + 195{,}000 \times \ddot{a}_{\overline{n|}} \geq 0$

$$\ddot{a}_{\overline{n|}} = \frac{(1 - \exp(-0.034n))}{0.033428} \geq \frac{2{,}514{,}650}{195{,}000}$$

$n \geq 16.58$

$DPP = 16$

iii.
Project B has the highest internal rate of return.
Project A has an NPV close to 0 at the borrowing rate, indicating that the internal rate of return is close to 3.4%.
The discounted payback period of project B is at time 16, which is three years before the last income payment.

Part (i) was answered well with many marks available for the correct discounting of the cashflows.

A common error was to set the 3.4% equal to the effective interest rate per annum instead of the continuous interest rate per annum.

In part (ii) candidates should remember that with discrete cashflows the discounted payback period (DPP) should be an integer value and when setting up the equation to solve the DPP " $\geq$ *" should be used.*

Also, $n \geq 16.58$ *implies the DPP is at the 17th payment. As this is in reference to an annuity due (where the first payment occurs at t=0) this will occur at time 16.*

