



**ACTUARIAL SOCIETY**  
OF SOUTH AFRICA

## **EXAMINATION**

2 May 2023 (am)

### **Subject A211 — Financial Mathematics Intermediate Technical**

*Time allowed: Two hours and fifteen minutes (which includes 15 minutes of reading time)*

*Scan and upload time: Twenty minutes (at the end of the examination)*

*Total marks: 100*

#### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Once you have entered the ASSA Exam Platform, ensure that you have accessed the **Video Room** Invigilation link with both your camera and microphone switched on **before you attempt the examination**.*
2. *Your computer must be placed, and camera angled, so that your head and shoulders as well as your writing area on your desk is visible to the invigilator. Readjust your camera if you bump or move your computer by accident.*
3. *Ensure that you have your exam permit handy. It reflects your candidate number to input **as part of the two hours 15 minutes examination and not before the start of the examination**. Write your candidate number at the top of each page during the examination time only. Do not use your name or member number anywhere on your answer script.*
4. *The cell phone to be used to scan your final answer script must be switched **OFF** during the two hours and 15 minutes examination time. Place the cell phone at the top of your examination pad / writing pages in view of the invigilator.*
5. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may start answering the paper whenever you are ready.*
6. *The question paper is only available on the ASSA Exam Platform as a PDF download and may not be printed.*
7. *You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.*
8. *Attempt all questions, beginning your answer to each question on a new page.*
9. *Write in black or dark blue pen.*

10. *Show calculations where appropriate. You may use blank paper to make notes. This paper must not be scanned as part of your answer script.*
11. *You may not access any file from your computer, use any other computer program (e.g. Email, MS Word or Excel), or open any other browser during the examination.*
12. *You may not use any other material (e.g. a Formulae and Tables book) during the examination. Any such information that may be required will be provided to you within the examination.*
13. *Mark allocations are shown in brackets.*
14. *Assume that months are all equal length, unless otherwise stated.*
15. *At the end of the two hours and 15 minutes examination time, you must stop writing and start scanning and uploading your script. **You may NOT continue to write or review your script during this time.***
16. *Scan ALL your answer pages to PDF so that your candidate number is clear at the top of each page.*
17. *Save your PDF scanned file using your candidate number as file name. Do not use your name or member number anywhere in your answer script nor as file name.*
18. *Transfer your scanned script file to your computer and upload into the ASSA Exam Platform.*
19. *Click on the **Upload Answers** link below the examination paper link. Ensure you click on **Finish** below the upload box and again on **Finish All and Submit**, before the 20-minute upload time is up. (After submission the number of files successfully submitted will be reflected.)*

**Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.**

***END OF INSTRUCTIONS***

## QUESTION 1

List seven factors to consider when assessing the suitability of a model for a particular purpose.

[Total 7]

## QUESTION 2

Person A has taken out a five-year interest-only loan from Bank X. Interest on the loan is guaranteed and payable monthly in arrears at a rate of 6.7% per annum, convertible monthly.

- i. Describe briefly, the certainty/uncertainty of the cashflows on the interest-only loan, from the perspective of Bank X.

[1]

A portion of the interest paid by Person A is used by Bank X to pay the monthly premiums on a five-year term assurance contract, on the life of Person A. The term assurance contract covers the principal amount of the loan.

- ii. Explain how the presence of the term assurance contract affects the certainty/uncertainty of the cashflows of the interest-only loan, from the perspective of Bank X.

[3]

[Total 4]

## QUESTION 3

The annual force of interest,  $\delta(t)$ , is a function of time ( $t$ , where  $t$  is measured in years) and is given by:

$$\delta(t) = 0.002t^3, \quad t \geq 0$$

- i. Calculate the equivalent constant annual effective interest rate applicable between time two and four.

[3]

- ii. Calculate the equivalent constant annual simple discount rate applicable between time one and two.

[4]

- iii. Calculate the combined present value of

- an annuity paying R1,000 per annum in advance for a period of three years, with the first payment made at time zero and
- a lump sum payment of R3,000 made at time four.

[6]

[Total 13]

**PLEASE TURN OVER**

#### QUESTION 4

The notation  $a_{\overline{n}|}^{(p)}$  denotes the present value at time 0 of a level annuity payable  $p$ thly in arrears at the rate of 1 per unit time, over the time interval  $[0, n]$ . For this annuity the payments are made at time  $\frac{1}{p}, \frac{2}{p}, \frac{3}{p}, \dots, n$  and the amount of each payment is  $\frac{1}{p}$ , with  $n$  a positive integer.

- i. Derive, from first principles,  $a_{\overline{n}|}^{(p)} = \frac{1-v^n}{i^{(p)}}$ . Define all notation not defined.

[4]

An investor wants to invest a lump sum of X into a fund today and withdraw an amount of R50,000 every four years over a period of 60 years. After 60 years the fund should be completely drawn down. The first withdrawal will take place four years from today.

The fund will pay compound interest of 7.5% per annum effective.

- ii. Calculate X, using the formula derived in (i).

[6]

[Total 10]

#### QUESTION 5

The following information about zero coupon bonds and a selection of forward rates are given:

$P_n$ = current price of zero coupon bond with term $n$ years	$n$ = term of bond (years)
95	1
80	3
65	5
45	8

$f_{1,1}$	6%
$F_{4,1}$	8.2%
$F_{7,1}$	9.4%

An investor enters into an agreement today to invest

- R50, a year from now and
- R50, four years from now.

Calculate the combined accumulated value of these investments eight years from now.

[Total 7]

**PLEASE TURN OVER**

## QUESTION 6

- i. Describe, in words, what is meant by the phrase "discounted mean term" of a series of cashflows.

[2]

An insurance company has sold a pension product to an individual. Under the arrangement, the insurance company will pay the individual an immediate annuity of R850 per half-year in arrears for 11 years.

- ii. Calculate the discounted mean term of the annuity at an interest rate of 3.5% per annum effective.

[6]

The insurance company holds a fixed-interest bond to meet the liabilities under the pension contract. The present value of the asset is equal to the present value of the liabilities.

The discounted mean term of the fixed-interest bond is 5.91 years.

The insurance company expects a small decrease in interest rates all terms.

- iii. State, with reasons, whether the insurance company will make a profit or a loss with this expected change in the interest rate.

[3]

[Total 11]

## QUESTION 7

A student will take out a 15-year bank loan of R150,000 on 1 January 2024.

The loan is repayable by quarterly instalments in arrears. The instalments for the first five years are X per quarter, for the next five years it is 2X per quarter, and for the last five years it is 3X per quarter. The bank charges a rate of interest of 16% per annum convertible monthly.

- i. Calculate the size of the 42<sup>nd</sup> quarterly payment.

[7]

- ii. Calculate the capital portion of the 42<sup>nd</sup> quarterly payment.

[5]

[Total 12]

**PLEASE TURN OVER**

## QUESTION 8

A fixed-interest bond, redeemable at 96% of par in ten years, pays half-yearly coupons of 9.5% per annum in arrears. The fixed-interest bond has just been issued at a price to give an investor, who pays 25% income tax and 40% capital gains tax, a net redemption yield of 7.25% per annum effective.

- i. Calculate the price per R100 nominal of the fixed interest bond. [7]
  - ii. Calculate the net running yield on the fixed interest bond. [3]
- [Total 10]

## QUESTION 9

An individual is considering investing a sum of R2,514,650 in Project A.

Project A requires the investment in properties that are to be let out to tenants. The details are:

- The individual expects to receive continuous income from rents at an annual rate of R99,000 a year for a period of four years, after an initial two-year period where no income will be received.
  - Rents are expected to increase thereafter at the start of each year at a rate of 1% per annum, with the first increase immediately following the four-year period.
  - From the start of the project, it incurs running costs payable annually in advance. The first payment is R13,000 per annum, and payments will increase by a constant amount of R1,000 per annum, thereafter.
  - At the end of 20 years, the individual expects to sell the properties for R3m, after which there will be no more revenue or costs.
- i. Calculate the net present value of Project A at an interest rate of 3.4% per annum compounded continuously. [17]

An alternative to Project A is Project B that involves the investment of R2,514,650 in an investment fund. The fund is expected to pay an income of R195,000 per annum annually in advance and return the whole invested sum at the end of 20 years. The investor will borrow the investment of R2,514,650 from a bank at a rate of 3.4% per annum compounded continuously. Income from Project B will immediately be used as instalments to repay the loan. Once the loan has been repaid money can be invested at 4.5% per annum effective.

- ii. Calculate the discounted payback period of project B at the borrowing rate. [6]
  - iii. Explain which of the projects has the highest internal rate of return. No calculations are required. [3]
- [Total 26]

[GRAND TOTAL 100]

**END OF EXAMINATION**