Actuarial Society of South Africa

EXAMINERS REPORT

MAY 2022

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

Many candidates can also increase their probability of passing this exam by practising good exam technique, which is often missing in its most basic form.

Question 1

Timing	Timing Unknown/Know	Size	Size Unknown/Know	+/-	Type
Initial	Known	Very large	Known (Unknown if reason given)	Negative	outlay for the construction of the toll road.
Regular in the 20 years	Unknown	Smaller than initial	Unknown	Negative	maintenance of the road (e.g., resurfacing, toll gates maintenance
Everyday	Known		Unknown	Positive	Toll fees, number of vehicles, increases to fees unknown

[Total 6]

This question was often poorly answered with candidates ignoring the facts given in the question and not providing all the information asked for.

Many candidates did not follow the instructions to tabulate their answers and subsequently missed important points due to lack of a structured answer.

Question 2

- Model development requires considerable investment of time and expertise.
  Both sets of modelers were faced with a pandemic which had just surfaced and so had no considerable time to develop proper models.
- A model produces one set of outputs for each asset of inputs.
  If one/both organizations used a relatively small number of independent runs of a stochastic model the mean output could have been affected by randomness.
- Models rely heavily on data input. For COVID-19 the data sources may have been unreliable. Public health institutions and private health institutions are known to produce data on health and care with differences in quality and type.
- If a model has not passed the tests of validity and verification, the impressive output of the model is a poor substitute for its ability to imitate the corresponding real-world system being modelled. In both cases, verification of the models was difficult because COVID-19 is a new virus with little or no test data to verify the model. This may explain the wide difference between the model predictions and the observed data.
- It is not possible to allow for all future possibilities. One model may have allowed for a possibility that the other excluded, leading to the discrepancy.

[Total 7]

Questions on this part of the work continues to be answered poorly.

Candidates must tailor their answers to the specific situation given in the question.

Common errors were

- *Listing all limitations of modelling without applying it to the specific situations.*
- *Listing issues with models on COVID without linking it to the limitations of modelling.*

Question 3

Let X be the monthly repayment.

Then the flat rate: $\dfrac{5 \times 12 \times X - 40,000}{5 \times 40,000} = 0.082$

$\Rightarrow X = 940$

Let i be the APR (annual percentage rate of return), so that

$$12 \times X a_{\overline{5}|i}^{(12)} = 40,000 \ \Rightarrow a_{\overline{5}|i}^{(12)} = \frac{40,000}{11,280} = 3.5461$$

To calculate APR by linear interpolation, we note that APR $\approx$ 2xFlat rate = 0.164

Therefore, our equation of value above is

$$\frac{1-(1+i)^{-5}}{12\left((1+i)^{\frac{1}{12}}-1\right)} = 3.5461$$

Let $i_1 = 0.15$ be the first approximation. Substituting on LHS we get 3.5768 > 3.5461

Let $i_2 = 0.16$ be the second approximation. Substituting on LHS we get 3.50798 < 3.5461

Thus $i_3 = i_1 + \left(\dfrac{3.5768 - 3.5461}{3.5768 - 3.50798}\right) \times (0.15 - 0.16)$ = 0.154461

Therefore, APR is 0.154 = 15.4% (*Rounded to nearer 0.1%)*

[Total 7]

The question was answered well in general.

The most common error was the incorrect rounding of the APR.

Also note, that for any interest rate which cannot be solved algebraically, the linear interpolation approach must be shown in full to obtain full credit. Solving the interest rate on the calculator does not equate to solving it algebraically.

Question 4

The effective yield is $i = 7.1\% \Rightarrow i^{(4)} = 0.069184$

Also $\dfrac{8\times(1-0.25)}{98} = 0.061224$. Since $i^{(4)} > (1-t_1)\dfrac{D}{R}$, there is capital gain

The investor pays a price assuming redemption is at the latest possible date which is 1 April 2034.

Therefore, per R100 nominal we have

$$P = (1.071)^{2/12} \times \left[8(1-0.25)a_{\overline{13}|7.1\%}^{(4)} + 98v_{7.1\%}^{13}\right]$$

$$a_{\overline{13}|7.1\%}^{(4)} = 8.528588720$$

$$P = (1.071)^{2/12} \times \left[91.347186297\right] = 92.397471440$$

$\Rightarrow$ Price = R92.39 per R100 nominal

[Total 8]

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The question was answered well in general.

Candidates must remember to always show the capital gains tax test in full.

The most common errors were

- *Calculating the term of the bond incorrectly.*
- *Making an incorrect adjustment to the equation of value to calculate the price of the bond on the purchase date.*
- *Deducting capital gain tax although no capital gain tax was payable.*

Question 5

i. Assuming p -payments of $\frac{d^{(p)}}{p}$ is payable, in advance at each period of length $\frac{1}{p}$ in a year, then $\frac{d^{(p)}}{p}+\frac{d^{(p)}}{p}v^{\frac{1}{p}}+\frac{d^{(p)}}{p}v^{\frac{2}{p}}+\ldots+\frac{d^{(p)}}{p}v^{1-\frac{1}{p}}=d$

$$\Rightarrow \frac{d^{(p)}}{p}\sum_{j=0}^{p-1}v^{\frac{j}{p}}=d$$

$$\Rightarrow \frac{d^{(p)}}{p}\left(\frac{1-v}{1-v^{\frac{1}{p}}}\right)=d$$

$$\Rightarrow \frac{d^{(p)}}{p}\left(\frac{d}{1-v^{\frac{1}{p}}}\right)=d$$

$$\Rightarrow \frac{d^{(p)}}{p}=1-v^{\frac{1}{p}}$$

$$\Rightarrow v=\left(1-\frac{d^{(p)}}{p}\right)^{p}$$

$$\Rightarrow 1-d=\left(1-\frac{d^{(p)}}{p}\right)^{p}$$

[4]

ii. Accumulated value = $1,500\left(1-\frac{0.065}{4}\right)^{-12/4}\times\left(1+\frac{0.075}{3}\right)^{24/4}\times e^{0.0525\times1.25}$

$$=1,500\left(1-\frac{0.065}{4}\right)^{-3}\times\left(1+\frac{0.075}{3}\right)^{6}\times e^{0.0525\times1.25}$$

= 1,500 ×1.050378355×1.159693418×1.067826207 = 1,951.105668555

[4]

[Total 8]

Question (i) was a theory question from the study material and was very poorly answered. An instruction was given in the question as to how to start the solution, yet this was often ignored.

Question (ii) was answered well, however some candidates added the discount rate instead of subtracting it in the first accumulation factor or forgot that to accumulate with an effective discount rate the exponent of the accumulation factor must be negative.

Question 6

i. $\delta = \ln(1.05) = 0.0487902$ is the force of interest and $\mu = 0.00512$ is the addition to the force of interest.

$\delta' = 0.0487902 + 0.00521 = 0.053910164$

$\exp(\delta') = 1.055389786 = 1 + i' \Rightarrow i' = 0.055389786$

Therefore EPV = $10{,}000a_{\overline{50|}} + 500(Ia)_{\overline{50|}}$

$a_{\overline{50|}} = 16.835088239$ and $(Ia)_{\overline{50|}} = \dfrac{\ddot{a}_{50} - 50v^{50}}{i} = 259.834491699$

$= 168{,}350.882200395 + 129{,}917.245849402 = 298{,}268.128049797$

[5]

ii. $v(\delta') = \exp(-0.053910164) = 0.947517224$

$v(i = 5\%) = \exp(-0.0487902) = 0.952380952$

$0.947517224 = 0.952380952 \times \text{Probability}$

$\text{Probability} = 0.994893085$

[2]
[Total 7]

Although question 6 was a type of question not necessarily seen before, the steps for part (i) was clearly set out in the question.

Many candidates took the "basic risk discount rate" (as stated in the question) to mean effective annual discount rate (=d). The "basic risk discount rate" is the rate of interest (=i) that should be used in a project taking account of the risk that project possess to the investors.

The most common errors in part (i) were

- *Setting the "basic risk discount rate"= d.*
- *Not converting the 5% interest rate to a force of interest before adding the 0.512%.*

Part (ii) was the worst answered question in the paper with many candidates not making an attempt. The idea behind part (ii) is to convert the additional force of interest (0.512%), which forms part of the discount factor in (i), into a probability.

Question 7

$$\text{Future value} = \left(\int_1^8 \exp(-0.003t)\times\exp\left(\int_t^8 0.05ds\right)dt\right)\times\exp\left(\int_8^{10} 0.05ds\right)$$

$$\times\exp\left(\int_{10}^{25}(0.003s+0.0002s^2)ds\right)$$

Part 1

$$\left(\int_1^8 \exp\left(-0.003t\right)\times\exp\left(0.05t\Big|_t^8\right)dt\right)=\int_1^8 \exp\left(-0.003t\right)\times\exp\left(0.4-0.05t\right)dt$$

$$=\exp\left(0.4\right)\times\int_1^8 \exp\left(-0.053t\right)dt=\exp\left(0.4\right)\times\left[\frac{\exp(-0.053t)}{-0.053}\right]_1^8$$

$$=\frac{\exp(0.4)}{-0.053}\times\left[0.654424-0.94838\right]=8.274170106$$

Part 2

$$\exp\left(\int_8^{10}0.05ds\right)=\exp\left(0.05t\Big|_8^{10}\right)=\exp(0.1)=1.105171$$

Part 3

$$\exp\left(\int_{10}^{25}(0.003s+0.0002s^2)ds\right) = \exp\left(\frac{0.003}{2}s^2+\frac{0.0002}{3}s^3\Big|_{10}^{25}\right)$$

$$=\exp\left(\frac{0.003}{2}(25^2-10^2)+\frac{0.0002}{3}(25^3-10^3)\right) = \exp(1.7625)=5.826986667$$

Total

$8.274170106\times 1.105171\times 5.826986667 = R53.28413868$

[Total 11]

This question was answered well.

Question 8

i. Let t be such that $140,000a^{(2)}_{\overline{t}|7.25\%} = 1,200,000$

$\Rightarrow a^{(2)}_{\overline{t}|7.25\%} = 8.57143$

$\Rightarrow v^t = 0.410148 \quad \Rightarrow t = 13.47339474$

Therefore, discounted payback period (DPP) = 13.5 years since payments are half-yearly

[4]

ii. The accumulated in value at the DPP is

$\left(140,000a^{(2)}_{\overline{13.5}|7.25\%} - 1,200,000\right)\times 1.0725^{13.5} = 1,424.016067817\times 1.0725^{13.5} = 3,663.350318$

Thus, the investor's profit = $3,663.350318\times 1.055^{11.5} \quad + 140,000s^{(2)}_{\overline{11.5}|5.5\%}$

$= 6,780.812521820 + 2,195,533.713716470 = 2,202,314.526238290$

[6]

[Total 10]

Part (i) was answered well. Although many candidates did not round the discounted payback period to the next half-year (which was the payment period of the income).

In part (ii) many candidates assumed that the accumulated profit would be zero at the discounted payback period. This only holds when the income is payable continuously, which was not the case.

The accumulated profit must be calculated at the discounted payback period using the interest rate of the loan. After the discounted payback period the interest rate which the investor can earn, becomes applicable.

Question 9

i. First Coupon

*Coupon (**November 2021**)* = $\frac{3.5}{2}\times\frac{RPI_{\text{March 2021}}}{RPI_{\text{Sept 2020}}}=\frac{3.5}{2}\times\frac{206}{203.5}=1.771498771$

Second Coupon

*Coupon (**May 2022**)* =

$$\frac{3.5}{2}\times\frac{RPI_{Sept\ 2021}}{RPI_{Sept\ 2020}}=\frac{3.5}{2}\times\frac{206\times(1.07)^{\frac{1}{2}}}{203.5}=\frac{3.5}{2}\times\frac{213.0881}{203.5}=1.832452578$$

Where $RPI_{Sept\ 2021}=RPI_{\text{March 2021}}\times(1.07)^{\frac{6}{12}}=206\times(1.07)^{\frac{1}{2}}$

OR

$$Coupon(Nov\ 2021)\times(1.07)^{\frac{1}{2}}=1.771498771\times(1.07)^{\frac{1}{2}}=1.832452578$$

Last Coupon

*Coupon (**May 2036**)* = $\frac{3.5}{2}\times\frac{RPI_{\text{March 2035}}}{RPI_{\text{Sept 2020}}}=\frac{3.5}{2}\times\frac{206\times(1.07)^{14.5}}{203.5}=4.725041551$

Where $RPI_{\text{March 2035}}=RPI_{\text{March 2021}}\times(1.07)^{14\frac{1}{2}}=206\times(1.07)^{14\frac{1}{2}}$

OR

$$Coupon(Nov\ 2021)\times(1.07)^{14.5}=1.771498771\times(1.07)^{14.5}=4.725041551$$

[6]

ii. Taking inflation out:

First Coupon

$$\textit{Coupon } (\textbf{\textit{November 2021}}) = \frac{3.5}{2} \times \frac{206}{203.5} \times \frac{RPI_{May\ 2021}}{RPI_{Nov\ 2021}}$$

$$= \frac{3.5}{2} \times \frac{206}{203.5} \times \frac{206(1.07)^{\frac{2}{12}}}{206(1.07)^{\frac{8}{12}}} = 1.771498771 \times (1.07)^{-\frac{6}{12}} = 1.712572503$$

Where $RPI_{May\ 2021} = RPI_{\text{March 2021}} \times (1.07)^{\frac{2}{12}} = 206 \times (1.07)^{\frac{2}{12}}$ and

$RPI_{\text{Nov 2021}} = RPI_{\text{March 2021}} \times (1.07)^{\frac{8}{12}} = 206 \times (1.07)^{\frac{8}{12}}$

Second Coupon

$$\textit{Coupon } (\textbf{\textit{May 2022}}) = \frac{3.5}{2} \times \frac{206 \times (1.07)^{\frac{1}{2}}}{203.5} \times \frac{RPI_{May\ 2021}}{RPI_{\text{May 2022}}}$$

$$= \frac{3.5}{2} \times \frac{206 \times (1.07)^{\frac{1}{2}}}{203.5} \times \frac{206(1.07)^{\frac{2}{12}}}{206(1.07)^{\frac{14}{12}}} = 1.771498771 \times (1.07)^{-\frac{6}{12}} = 1.712572503$$

Where $RPI_{May\ 2022} = RPI_{\text{March 2021}} \times (1.07)^{\frac{8}{12}} = 206 \times (1.07)^{\frac{8}{12}}$

Last Coupon

$\textit{Coupon } (\textbf{\textit{May 2036}}) =$

$$= \frac{3.5}{2} \times \frac{206 \times (1.07)^{14.5}}{203.5} \times \frac{206(1.07)^{\frac{2}{12}}}{206(1.07)^{15\frac{2}{12}}} = 1.771498771 \times (1.07)^{-\frac{6}{12}} = 1.712572503$$

[6]

iii. $$\textit{Redemption} = 100 \times \frac{206 \times (1.07)^{14.5}}{203.5} \times \frac{206(1.07)^{\frac{2}{12}}}{206(1.07)^{15\frac{2}{12}}} = 101.2285012 \times (1.07)^{-\frac{6}{12}}$$

$$\text{Price} = 1.712572503\left[v_{2\%}^{1} + v_{2\%}^{2} + \ldots + v_{2\%}^{30}\right] + 101.2285012 \times (1.07)^{-\frac{6}{12}} \times v_{2\%}^{30}$$

$$= 1.712572503 a_{\overline{30}|2\%} + 101.2285012 \times (1.07)^{-\frac{6}{12}} \times v_{2\%}^{30}$$

$$= (1.712572503) \times 2 a_{\overline{15}|4\%}^{(2)} + 101.2285012 \times (1.07)^{-\frac{6}{12}} \times v_{2\%}^{30}$$

[4]

[Total 16]

Question 9 was answered poorly.

Although the question is on a part of the work that many candidates find difficult, the breakdown of the question into parts should have made the question easier.

The most common errors were

- *Using 3.5% as the half-yearly coupon.*
- *Allowing for the time lagging incorrectly, which lead to incorrect indexes being used when allowing for inflation.*
- *Using incorrect indexes when removing inflation.*

Question 10

i. Working in millions:

We have $PV_L(6\%) = a_{\overline{20|}} + 0.5v^{20}a_{\overline{20|}}$

$= a_{\overline{20|}}\left(1+0.5v^{20}\right)$

$\Rightarrow PV_L(6\%) = 13.258109045045100$

Now numerator of DMT_L (discounted mean term) (6%) is

$1v^1 + 2v^2 + \ldots + 20v^{20} + 0.5\left(21v^{21} + 22v^{22} + \ldots + 40v^{40}\right)$

$= (Ia)_{\overline{20|}} + 0.5v^{20}\left(21v^1 + 22v^2 + \ldots + 40v^{20}\right)$

$= (Ia)_{\overline{20|}} + 0.5v^{20}\left(20a_{\overline{20|}} + (Ia)_{\overline{20|}}\right)$

$= (Ia)_{\overline{20|}} \times \left(1+0.5v^{20}\right) + 10a_{\overline{20|}}v^{20}$

We have $a_{\overline{20|}} = 11.4699$ @ 6% and $(Ia)_{\overline{20|}} = \dfrac{\ddot{a}_{20} - 20v^{20}}{i} = 98.700365899$

Therefore, numerator of DMT_L @ 6%

$= 98.700365899291600 + 51.151\ 376845989000 = 149.851742745281000$

$\Rightarrow DMT_L = \dfrac{149.852}{13.2581} = 11.3026$

[9]

ii. <u>Per R100 nominal</u>

$$PV_A(6\%) = 10a_{\overline{15|}} + 100v^{15} = 138.848995951$$

$$\Rightarrow \text{ numerator of } DMT_A \ (6\%) = 10(Ia)_{\overline{15|}} + 15 \times 100v^{15}$$

$$\text{We have } (Ia)_{\overline{15|}} = \frac{\ddot{a}_{15} - 15v^{15}}{i} = 67.266800266\,.$$

numerator of DMT_A (6%)

$$= 672.668002662 + 625.897591103 = 1{,}298.565593765$$

$$\text{Therefore } DMT_A = \frac{1{,}298.565593765}{138.848995951} = 9.352358545$$

[5]

iii. Since $DMT_L \neq DMT_A$ at $i = 6\%$, then one of Reddington's conditions of immunization is violated and so the insurance company cannot immunize with the portfolio of assets and liabilities as given.

[2]

iv. We have $DMT_L > DMT_A$ a portfolio with a high DMT is more susceptible to small changes in interest rates. Therefore, if there is a small <u>decrease</u> in interest rates from 6%, then percentage <u>increase</u> in present value for liabilities is more than the percentage increase in present value for assets. Which results in negative net present value (6%).

[4]
[Total 13]

Part (i) and (ii) were answered well.

The common mistake was in the numerator of the discounted mean term in part (i) where the level annuity term was often missing.

In part (iii) candidates often referred to convexity as a reason why immunization could not occur, ignoring the numerical values calculated in (i) and (ii).

In part (iv) many candidates ignored the instruction in the question (which asked for the circumstance under which a loss would be made) by re-writing the answer to part (iii) referring to the fact that the portfolio was not immunized.

