EXAMINATION

10 May 2022

Subject A211 — Financial Mathematics

Time allowed:

Two hours and fifteen minutes – examination time

INSTRUCTIONS TO THE CANDIDATE

1. *Ensure that you have your candidate number handy to input **as part of the two hours 15 minutes examination**. Write your candidate number at the top of each page. (DO NOT WRITE YOUR NAME OR MEMBER NUMBER.)*
2. *You are strongly encouraged to use the first 15 minutes as reading time only, however, you may commence answering the paper whenever you are ready. You then have two hours to complete the paper.*
3. *Questions are only available on the ASSA Exam Platform and may not be printed or copied outside of the ASSA Exam Platform.*
4. *You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.*
5. *Attempt all questions, beginning your answer to each question on a new page and numbering your answers clearly.*
6. *Write in black or dark blue pen.*
7. *You should show calculations where this is appropriate.*
8. *You may not use any other computer program (e.g. Email, MS Word or Excel) or files, nor open any other browser during the examination.*
9. *You MAY NOT make use of a Formulae and Tables book during the examination. Any such information that may be required will be provided to you within the examination.*
10. *Mark allocations are shown in brackets.*
11. *You may use additional scrap paper to make notes where this is appropriate. This paper MUST NOT BE HANDED IN as part of your answer script.*

12. *Assume that months are all equal length, unless otherwise stated.*
13. *At the end of the examination time, you must stop writing and you will hand in your script to the invigilator.*
14. *An option to opt out of the examination will become available one hour after the official examination start time. If you select the Opt-Out option, you agree and understand that your entire script/answers will be deleted and cannot be retrieved at a later stage and that your script or part thereof will not be put forward for marking.*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

A government has just signed an agreement with a private consortium of investors to construct a dual toll road (paid freeway), with 15 toll gates to replace an existing road. The private consortium will be responsible for constructing, financing and operating the toll road for 20 years. At the end of that period, the toll road reverts to the government with no additional payments.

Describe briefly the cashflows involved in this project and indicate whether the amount or timing (or both) is fixed or uncertain, from the viewpoint of the private consortium of investors.

Set your answer out in a table format.

[Total 6]

QUESTION 2

On 8 June 2020, just as scientists were battling to study the COVID-19 virus, an article appeared on one of the local news websites stating:

"The South African government relies on models from well-established modelers that suggest we will have at least 40,000 deaths and about 1.2 million symptomatic cases by September 2020. On the other hand, a group of non-affiliated modelers, called PANDA, assert confidently that South Africa will not experience this level of Covid-19-related deaths and that it is unlikely we will exceed 10,000 deaths as a result of this pandemic. Whom then to believe?"

According to the South African government data, reported actual number of deaths due to COVID-19 in South Africa by 20 September 2020 was 15,953.

Explain the discrepancies with reference to the limitations of modelling.

[Total 7]

QUESTION 3

A customer borrows R40,000 under a consumer credit loan. Repayments are calculated to give a flat rate of 8.2%. Instalments are paid monthly in arrears for five years.

Calculate the Annual Percentage Rate of charge (APR) paid by the customer.

[Total 7]

PLEASE TURN OVER

QUESTION 4

A fixed interest bond bears coupons of 8% per annum payable quarterly in arrears and is redeemable at 98% of the par value, on any coupon date between 1 April 2027 and 1 April 2034, inclusive, at the option of the borrower. Coupons are paid on 1 April, 1 July, 1 October and 1 January of each year.

The investor is liable to pay tax at a rate of 25% on the coupon payments and aimed to earn a minimum net yield of 7.1% per annum effective on the bond.

Calculate the price at which the investor purchased R100 nominal of the bond, on 1 June 2021.

[Total 8]

QUESTION 5

i. Let *d* be the annual effective discount rate and $d^{(p)}$ be the annual nominal discount rate compounded p^{th}*-ly* per annum.

Starting with a series representing the present value of payments of $\frac{d^{(p)}}{p}$ payable p^{th}*-ly* in advance, derive $\left(1-\frac{d^{(p)}}{p}\right)^{p}=1-d$. Show all your steps. [4]

ii. A sum of R1,500 is accumulated at a rate of discount of 6.5% per annum convertible quarterly for nine months, then at a nominal rate of interest of 7.5% per annum convertible every four months for 24 months and finally at a constant force of interest of 5.25% per annum, thereafter.

Calculate the accumulated amount of the investment after four years.

[4]

[Total 8]

PLEASE TURN OVER

QUESTION 6

A small investor expects to receive 50 annual payments starting at R10,500 at the end of the first year, increasing by R500 each year thereafter. There is some uncertainty whether these payments will be received.

Due to the uncertainty, the investor decides to increase the basic risk discount rate of 5% per annum effective. The basic risk discount rate will be converted to an annual force of interest and a constant 0,512% will be added to the annual force of interest.

i. Calculate the present value of the payments.

[5]

Instead of increasing the basic risk discount rate the investor wishes to apply a probability which would express the constant addition to the risk discount rate as an annual probability of non-payment.

ii. Calculate the corresponding annual probability for non-payment.

[2]

[Total 7]

QUESTION 7

The force of interest $\delta(t)$, at time *t* is given by

$$\delta(t) = \begin{cases} 0.05 & for\ 0 < t \le 10 \\ 0.003t + 0.0002t^2 & for\ 10 < \text{t} \end{cases}$$

A continuous payment stream is paid at the rate of $e^{-0.003t}$ per unit time between time *t = 1* and time *t = 8*. Calculate, showing all steps, the future value of this payment stream at time *t = 25*.

[Total 11]

PLEASE TURN OVER

QUESTION 8

An investor borrows R1,200,000 at an effective interest rate of 7.25% per annum and invests the money in a project that produces an annuity income of R140,000 per annum payable half-yearly in arrears for 25 years. The income is first used to repay the loan.

Once the loan is repaid, the investor can earn interest at an effective rate of 5.5% per annum on money invested from the annuity payments.

i. Determine the discounted payback period for this investment.

[4]

ii. Determine the accumulated profit the investor will have made at the end of the term of the annuity income.

[6]

[Total 10]

QUESTION 9

On 1 May 2021 the government of a country issued an index-linked bond of term 15 years redeemed at par. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is 3.5%. Interest and capital repayments are indexed by reference to the value of a retail price index with a time lag of eight months. The retail price index value on September 2020 was 203.5 and on March 2021 was 206.

It is assumed that the retail price index increases at an effective rate of 7% per annum after March 2021.

i. Calculate the value of each of the first, second and last coupon per R100 nominal. Clearly state the date on which each of these coupons will be paid.

[6]

ii. Calculate the real value of each of the first, second and last coupon per R100 nominal.

[6]

A tax-exempt purchaser of the bond, at the issue date, aims to secure a real yield of 4% per annum convertible half-yearly.

iii. Write down, and simplify, the equation of value which will accurately calculate the price which should be offered for this bond.

The equation must contain an annuity factor (with $n > 1$) and a discount factor.

[4]

[Total 16]

PLEASE TURN OVER

QUESTION 10

An insurance company has a portfolio of annuity contracts under which it expects to pay R1,000,000 at the end of each of the next 20 years, followed by R500,000 at the end of each of the following 20 years.

i. Calculate the duration (discounted mean term) of the insurance company's liabilities at a rate of interest of 6% per annum effective.

[9]

The insurance company can only invest their funds into a government bond which pays a coupon of 10% per annum in arrears and is redeemed at par in 15 years' time. The yield to maturity of this government bond is 6% per annum effective and a coupon payment has just been made.

ii. Calculate the duration (discounted mean term) of this government bond at a rate of interest of 6% per annum effective.

[5]

iii. Explain why the insurance company cannot immunize its liabilities by purchasing this government bond, only.

[2]

iv. Explain, with reason(s), under which circumstances the insurance company would make a loss if there was a small change in interest rates. No further calculations must be done.

[4]

[Total 20]

PLEASE TURN OVER

Compound Interest

6%									
		n	$(1+i)^n$	v^n	$s_{\overline{n}\|}$	$a_{\overline{n}\|}$	$(Ia)_{\overline{n}\|}$	$(Da)_{\overline{n}\|}$	n
i	0.060 000	1	1.060 00	0.943 40	1.000 0	0.943 4	0.943 4	0.943 4	1
$i^{(2)}$	0.059 126	2	1.123 60	0.890 00	2.060 0	1.833 4	2.723 4	2.776 8	2
$i^{(4)}$	0.058 695	3	1.191 02	0.839 62	3.183 6	2.673 0	5.242 2	5.449 8	3
$i^{(12)}$	0.058 411	4	1.262 48	0.792 09	4.374 6	3.465 1	8.410 6	8.914 9	4
		5	1.338 23	0.747 26	5.637 1	4.212 4	12.146 9	13.127 3	5
δ	0.058 269	6	1.418 52	0.704 96	6.975 3	4.917 3	16.376 7	18.044 6	6
		7	1.503 63	0.665 06	8.393 8	5.582 4	21.032 1	23.627 0	7
		8	1.593 85	0.627 41	9.897 5	6.209 8	26.051 4	29.836 8	8
$(1+i)^{1/2}$	1.029 563	9	1.689 48	0.591 90	11.491 3	6.801 7	31.378 5	36.638 5	9
$(1+i)^{1/4}$	1.014 674	10	1.790 85	0.558 39	13.180 8	7.360 1	36.962 4	43.998 5	10
$(1+i)^{1/12}$	1.004 868	11	1.898 30	0.526 79	14.971 6	7.886 9	42.757 1	51.885 4	11
		12	2.012 20	0.496 97	16.869 9	8.383 8	48.720 7	60.269 3	12
		13	2.132 93	0.468 84	18.882 1	8.852 7	54.815 6	69.122 0	13
v	0.943 396	14	2.260 90	0.442 30	21.015 1	9.295 0	61.007 8	78.416 9	14
$v^{1/2}$	0.971 286	15	2.396 56	0.417 27	23.276 0	9.712 2	67.266 8	88.129 2	15
$v^{1/4}$	0.985 538	16	2.540 35	0.393 65	25.672 5	10.105 9	73.565 1	98.235 1	16
$v^{1/12}$	0.995 156	17	2.692 77	0.371 36	28.212 9	10.477 3	79.878 3	108.712 3	17
		18	2.854 34	0.350 34	30.905 7	10.827 6	86.184 5	119.539 9	18
		19	3.025 60	0.330 51	33.760 0	11.158 1	92.464 3	130.698 1	19
d	0.056 604	20	3.207 14	0.311 80	36.785 6	11.469 9	98.700 4	142.168 0	20
$d^{(2)}$	0.057 428	21	3.399 56	0.294 16	39.992 7	11.764 1	104.877 6	153.932 1	21
$d^{(4)}$	0.057 847	22	3.603 54	0.277 51	43.392 3	12.041 6	110.982 7	165.973 6	22
$d^{(12)}$	0.058 128	23	3.819 75	0.261 80	46.995 8	12.303 4	117.004 1	178.277 0	23
		24	4.048 93	0.246 98	50.815 6	12.550 4	122.931 6	190.827 4	24
		25	4.291 87	0.233 00	54.864 5	12.783 4	128.756 5	203.610 7	25
$i/i^{(2)}$	1.014 782	26	4.549 38	0.219 81	59.156 4	13.003 2	134.471 6	216.613 9	26
$i/i^{(4)}$	1.022 227	27	4.822 35	0.207 37	63.705 8	13.210 5	140.070 5	229.824 4	27
$i/i^{(12)}$	1.027 211	28	5.111 69	0.195 63	68.528 1	13.406 2	145.548 2	243.230 6	28
		29	5.418 39	0.184 56	73.639 8	13.590 7	150.900 3	256.821 3	29
i/δ	1.029 709	30	5.743 49	0.174 11	79.058 2	13.764 8	156.123 6	270.586 1	30
$i/d^{(2)}$	1.044 782	31	6.088 10	0.164 25	84.801 7	13.929 1	161.215 5	284.515 2	31
$i/d^{(4)}$	1.037 227	32	6.453 39	0.154 96	90.889 8	14.084 0	166.174 2	298.599 3	32
$i/d^{(12)}$	1.032 211	33	6.840 59	0.146 19	97.343 2	14.230 2	170.998 3	312.829 5	33
		34	7.251 03	0.137 91	104.183 8	14.368 1	175.687 3	327.197 6	34
		35	7.686 09	0.130 11	111.434 8	14.498 2	180.241 0	341.695 9	35
		36	8.147 25	0.122 74	119.120 9	14.621 0	184.659 6	356.316 9	36
		37	8.636 09	0.115 79	127.268 1	14.736 8	188.944 0	371.053 7	37
		38	9.154 25	0.109 24	135.904 2	14.846 0	193.095 1	385.899 7	38
		39	9.703 51	0.103 06	145.058 5	14.949 1	197.114 2	400.848 8	39
		40	10.285 72	0.097 22	154.762 0	15.046 3	201.003 1	415.895 1	40
		41	10.902 86	0.091 72	165.047 7	15.138 0	204.763 6	431.033 1	41
		42	11.557 03	0.086 53	175.950 5	15.224 5	208.397 8	446.257 6	42
		43	12.250 45	0.081 63	187.507 6	15.306 2	211.907 8	461.563 8	43
		44	12.985 48	0.077 01	199.758 0	15.383 2	215.296 2	476.947 0	44
		45	13.764 61	0.072 65	212.743 5	15.455 8	218.565 5	492.402 8	45
		46	14.590 49	0.068 54	226.508 1	15.524 4	221.718 2	507.927 2	46
		47	15.465 92	0.064 66	241.098 6	15.589 0	224.757 2	523.516 2	47
		48	16.393 87	0.061 00	256.564 5	15.650 0	227.685 1	539.166 2	48
		49	17.377 50	0.057 55	272.958 4	15.707 6	230.504 8	554.873 8	49
		50	18.420 15	0.054 29	290.335 9	15.761 9	233.219 2	570.635 7	50
		60	32.987 69	0.030 31	533.128 2	16.161 4	255.204 2	730.642 9	60
		70	59.075 93	0.016 93	967.932 2	16.384 5	269.711 7	893.590 9	70
		80	105.795 99	0.009 45	1 746.599 9	16.509 1	279.058 4	1 058.181 2	80
		90	189.464 51	0.005 28	3 141.075 2	16.578 7	284.973 3	1 223.688 3	90
		100	339.302 08	0.002 95	5 638.368 1	16.617 5	288.664 6	1 389.707 6	100

58

[GRAND TOTAL 100]

END OF EXAMINATION

