# SPECIMEN EXAMINATION PAPER

## MEMORANDUM

April 2019

## Subject A211 — Financial Mathematics

**QUESTION 1**

$$1500*(1+0.05*(2.5)*2)*\left(1-\frac{0.16}{4}\right)^{-n}=2495.18$$

n = 7 $\Rightarrow$ 21 months

**QUESTION 2**

Payments from t=2 to t=6, accumulated to t=6:

$$\int_2^6 e^{-0.02t}*e^{\int_t^6 0.06ds}dt=\int_2^6 e^{-0.02t}*e^{0.06(6)}*e^{-0.06t}dt$$

$$=e^{0.36}\int_2^6 e^{-0.08t}dt=e^{0.36}\left[\frac{e^{-0.08t}}{-0.08}\right]_2^6=4.181029018$$

Accumulate to t=12

$$4.181029018*\exp\int_6^{12}(0.05+0.0002t^2)dt=4.181029018*\exp\left[0.05t+\frac{0.0002t^3}{3}\right]_6^{12}$$

$$=6.24$$

**QUESTION 3**

$$21.5=1.1\left(\frac{92}{96}\right)v^{\frac{2}{12}}+\left(\frac{103}{93}\right)\left(\frac{92}{104}\right)v^{1\frac{2}{12}}+32\left(\frac{92}{108}\right)v^{1\frac{4}{12}}$$

Sub $r$=27.8% into RHS of equation and show that LHS $\approx$ RHS

**QUESTION 4**

i. $y_1=f_{0,1}=0.04$

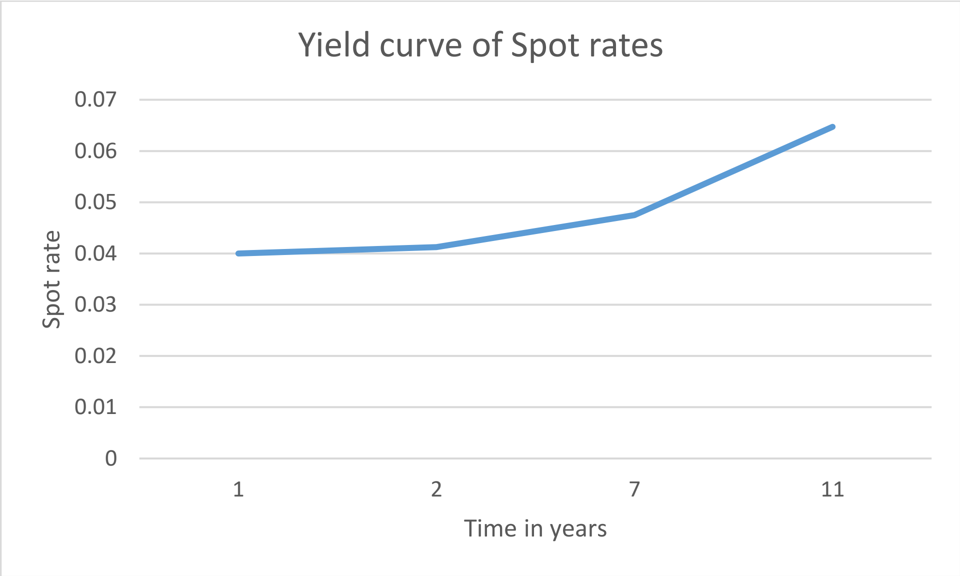
$$(1+y_1)*(1+f_{1,1})=(1+y_2)^2\Rightarrow y_2=0.041249259$$

$$(1+y_2)^2*(1+f_{2,5})^5=(1+y_7)^7\Rightarrow y_7=0.047492308$$

$$(1+y_2)^2*(1+f_{2,9})^9=(1+y_{11})^{11}\Rightarrow y_{11}=0.064714176$$

| | | |
|---|---|---|
| $y_1$ | 1 | 0.04 |
| $y_2$ | 2 | 0.041249259 |
| $y_7$ | 3 | 0.047492308 |
| $y_{11}$ | 4 | 0.064714176 |

ii.




iii. Expectations theory.

The yield curve is upward sloping. This may be due to an expectation of a rise in future short-term interest rates. This expectation pushes up the demand for and thus price of short-term securities, resulting in lower yield for short-term securities.

Liquidity preference theory.

The upward sloping yield curve may be due to the risk-premium required by investors to hold longer dated securities. This pushes the yield of long-term securities upward.

**QUESTION 5**

A model is described as stochastic if it allows for the random variation in at least one input variable.

- Often the output from a stochastic model is in the form of many simulated possible outcomes of a process, so distributions can be studied.
- Sometimes stochastic models have analytical/closed form solutions, such that simulation is not required, but they are still stochastic as they allow for factors to be random variables.
- If a stochastic model is sufficiently tractable, it may be possible to derive the results one wishes by analytical methods.

A deterministic model can be thought of as a special case of a stochastic model where only a single outcome from the underlying random processes is considered.

- The results for a deterministic model can often be obtained by direct calculation, but sometimes it is necessary to use numerical approximations, either to integrate functions or to solve differential equations.

**QUESTION 6**

i. Advantages

- Complex systems cannot be properly described by mathematical models. Simulation modelling is a way to study the operation of a complex system in reduced time.
- The act of building the model creates insights into the problem
- Different future strategies or actions can be compared to see what best suit the requirements of the user
- Can control the experimental conditions
- More cost effective compared to implementing experimental solutions in practice

Disadvantages

- Model development requires investment of time and expertise
- In a stochastic model, for any given set of inputs each run gives only estimates of a model's outputs. So to study the outputs for any given set of inputs, several independent runs of the model are needed.
- As a general rule, models are more useful for comparing the results of input variations than for optimising outputs
- Models can look impressive when run on a computer so that there is a danger that one gets lulled into a false sense of confidence.
- Models rely heavily on the data input.
- There is a danger of using a model as a "black box" from which it is assumed that all results are valid
- It is not possible to include all future events in a model.
- It may be difficult to interpret some of the outputs of the model.

ii. Communication should take into account

- Knowledge of the target audience
  - Language & detail provided should be appropriate
- Viewpoint of the target audience
- Client should accept the model as valid & useful
- Limitations of use are appreciated

**QUESTION 7**

i. $3000a_{\overline{20|}} - 25(Ia)_{\overline{20|}} = 28\ 499.87 \quad @7.5\%$

ii. Loan outstanding @ t=7: $2825a_{\overline{13|}} - 25(Ia)_{\overline{13|}} = 21\ 736.17 \quad @7.5\%$

Interest content: $21\ 736.17 * 0,075 = 1\ 630.21$

Capital content: $2\ 800\ - 1\ 630.21 = 1\ 169.79$

**QUESTION 8**

$i^{(2)} = 0.054263858 \qquad \frac{0.0625}{1.02}(1-1.02) = 0.049019608$

$i^{(2)} > g(1-t_1) \Rightarrow$ Capital gain, redeem as late as possible, n =$10\frac{3}{4}$

P= $v_i^{0.25}(0.8)(6.25)\ddot{a}^{(2)}_{\overline{11|}i^{(2)}} + 102v_i^{10.75} - 0.2\left[102 - P\right]v_i^{10.75}$

P=98.5384684 thus P=98.54

**QUESTION 9**

Let x be the redemption amount for the 5 year bond and y be the redemption amount for the 20 year bond. Then

$$PV(L) = 60000a_{\overline{9|}} + 750000v^{10} = 772\,176 \ @\ i = 7\%$$
$$PV(A) = xv^5 + yv^{20} \quad @i = 7\%$$
$$DMT(L) = \frac{60000(Ia)_{\overline{9|}} + 10*750000v^{10}}{60000a_{\overline{9|}} + 750000v^{10}} = 7.24182 \ @\ \text{i=7\%}$$

$$\frac{DMT(L)}{(1+i)} = vol_L = 6.768056$$
$$PV'_A = -5x(1+i)^{-6} - 20y(1+i)^{-21} \ @i = 7\%$$
$$\Rightarrow Vol_A = \frac{5xv^6 + 20yv^{21}}{xv^5 + yv^{20}} = 6.76806$$

$$xv^5 + yv^{20} = 772176$$
$$5xv^6 + 20yv^{21} = 6.76806*772176$$

Solving, give $xv^5$ = 656 770.48 and $yv^{20}$ = 115 405.52
$x$ = 468 268.27 and $y$ = 29 822.98

**QUESTION 10**

i. $$(1-\frac{d^{(p)}}{p})^p = v = (1+\frac{i^{(m)}}{m})^{-m} \Rightarrow (1-\frac{d^{(p)}}{p})^p = (1+\frac{i^{(m)}}{m})^{-m}$$

$$d^{(p)} = p\left\{1-\left(1+\frac{i^{(m)}}{m}\right)^{-\frac{m}{p}}\right\}$$

ii. $$d^{(m)} = m\left\{1-\left(1+\frac{i^{(m)}}{m}\right)^{-\frac{m}{m}}\right\} \Rightarrow \frac{d^{(m)}}{m} = 1-\left(1+\frac{i^{(m)}}{m}\right)^{-1}$$

$$\frac{d^{(m)}}{m} = 1-\left(\frac{1}{1+\frac{i^{(m)}}{m}}\right) = \frac{\frac{i^{(m)}}{m}}{1+\frac{i^{(m)}}{m}} \Rightarrow d^{(m)} = \frac{mi^{(m)}}{m+i^{(m)}}$$

$$\frac{1}{d^{(m)}} - \frac{1}{i^{(m)}} = \left[\frac{mi^{(m)}}{m+i^{(m)}}\right]^{-1} - \frac{1}{i^{(m)}} = \frac{m+i^{(m)}}{mi^{(m)}} - \frac{1}{i^{(m)}} = \frac{m+i^{(m)}-m}{mi^{(m)}} = \frac{i^{(m)}}{mi^{(m)}} = \frac{1}{m}$$

**QUESTION 11**

i. <u>PV of outgo (work in ‘000 000)</u>

$$1.5\bar{a}_{\overline{3}|} + 0.3v^3\ddot{a}^{(4)}_{\overline{12}|} + v^3(1 + 1.05v + 1.05^2v^2 + \ldots + 1.05^{11}v^{11}) \quad @9\%$$

$$\frac{1.05}{1.09} = \frac{1}{1+j} \Rightarrow j = 3.809523\%$$

$$= 1.5\bar{a}_{\overline{3}|} + 0.3v^3\ddot{a}^{(4)}_{\overline{12}|} + v^3\ddot{a}_{\overline{12}|j\%} = 13.3234$$

<u>PV of income(work in ‘000 000)</u>

$$\bar{a}_{\overline{3}|}v^3 + 1.9\bar{a}_{\overline{3}|}v^6 + 2.5\bar{a}_{\overline{6}|}v^9 + 8v^{15} \quad @9\% = 12.6252$$

NPV(@9%) = 12.6252 – 13.3234 = -0.69823 < 0

$\Rightarrow$ IRR < 9%

ii. Calculate NPV for only 12 years @7% p.a

<u>PV of outgo(work in ‘000 000)</u>:

$$1.5\bar{a}_{\overline{3}|} + 0.3v^3\ddot{a}^{(4)}_{\overline{12}|} + v^3(1 + 1.05v + 1.05^2v^2 + \ldots + 1.05^8v^8) \quad @7\%$$

$$\frac{1.05}{1.07} = \frac{1}{1+j} \Rightarrow j = 1.904761\%$$

$$= 1.5\bar{a}_{\overline{3}|} + 0.3v^3\ddot{a}^{(4)}_{\overline{12}|} + v^3\ddot{a}_{\overline{9}|j\%} = 12.5581$$

<u>PV of income(work in ‘000 000)</u>

$$\bar{a}_{\overline{3}|}v^3 + 1.9\bar{a}_{\overline{3}|}v^6 + 2.5\bar{a}_{\overline{3}|}v^9 \quad @7\% = 9.34598$$

NPV(@9%) = 9.34598 – 12.5581 = -3.21212 < 0

$\Rightarrow$ DPP > 12