

# A TYPOLOGY OF MODELS USED IN ACTUARIAL SCIENCE

by RJ Thomson

## ABSTRACT

This paper proposes a categorisation of the models used in actuarial science. It illustrates the application of that categorisation by using it to classify numerous such models. It is suggested that this categorisation, together with the illustrative classification, may be used as a typology for the classification of other such models, and that this typology may be found useful as a candidate exemplar, or as a basis for further refinement, in the discourse of actuarial science.

## KEYWORDS

Actuarial models; categorisation; classification; typology

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## 1. INTRODUCTION

1.1 The centrality of models to actuarial science is well documented in the literature of the discipline. Jewell (1980), for example, takes a wide-ranging ramble through the models used in life and general insurance, and gives an extensive bibliography of the subject. Since that report, numerous further articles have been written relating to the development and application of models in actuarial science. The purpose of this paper is not to discuss this feature, which could well constitute the subject of a broader inquiry. The scope of this paper is confined to the categorisation of the models used in actuarial science.

1.2 For the purposes of this paper, a ‘model’ is defined as “a representation of a real system in the form of a homomorphic system that satisfies or instantiates [a] theory” about the real system (Thomson, 2004). It may be noted here that, under this definition, no model is theory-free.

1.3 The purpose of categorising these models is to improve the discourse of the discipline. It will enable actuaries to discuss the general principles that should apply to each category of model. Instead of discussing those principles repeatedly whenever a model is developed, actuaries will be able to refer to the generic discussion for the category of model concerned, and focus instead on the application of that discussion to the particular model being developed, and on issues peculiar to that model.

1.4 The models used in actuarial science are not necessarily different by nature from those used in other disciplines. In fact, by virtue of its inter-disciplinary nature, actuarial science borrows freely from the methods and models of other disciplines. Jewell (op. cit.) suggests that it would benefit by further cross-fertilisation. While it would be of interest to consider the relevance to other disciplines (for example, industrial engineering) of a categorisation of the models used in actuarial science, that is beyond the scope of this paper.

1.5 In this paper, the ‘categorisation’ of models means the process of defining categories of models. The ‘classification’ of models means the process of allocating particular models to the categories defined. A ‘typology’ means a system of categorisation, specified by definition and illustrated by the classification of a specified set of models, that may be used for the classification of any model that may be used in the discipline. ‘Exemplars’ are ‘concrete problem solutions, accepted by the group as... paradigmatic’—typically the text-book examples given to students (Kuhn, 1974).

1.6 Section 2 reviews the literature on the subject. Section 3 suggests a categorisation of the models used in actuarial science. Section 4 suggests a classification of a specified set of such models. Section 5 concludes.

## 2. LITERATURE REVIEW

### 2.1 BOOTH ET AL. (1999)

Booth et al. (1999: 593–598) give an overview of the categories of models used in actuarial science. They suggest that models may be categorised:

- by the nature of the problem to which they are to be applied;
- by the field of actuarial practice in which they are to be applied; or
- by the nature of the models themselves.

#### 2.1.1 CATEGORISING BY THE NATURE OF THE PROBLEM

##### 2.1.1.1 Categorising by the nature of the problem, they list:

“models for valuing investments, selecting portfolios, pricing insurance contracts, estimating reserves, valuing portfolios, controlling pension scheme finances, and so on.”

2.1.1.2 While the nature of the problem is a meaningful criterion for categorisation, it is not clear here how that nature itself is to be categorised. In what sense, for example, is ‘valuing investments’ different by nature from ‘valuing portfolios’?

#### 2.1.2 CATEGORISING BY THE FIELD OF ACTUARIAL PRACTICE

##### 2.1.2.1 Categorising by the field of actuarial practice, they list:

“asset management, life insurance, general insurance and pension funds”;

i.e. the major traditional fields, to which healthcare should be added.

2.1.2.2 To the extent that the nature of problems corresponds to particular fields of actuarial practice, the above categorisations are similar. To the extent that they do not, classification by the nature of the problem is more meaningful as it avoids duplication of categories across fields of practice.

2.1.2.3 Also, a progressive evolution of modelling in actuarial science requires, as Jewell (op. cit.: S129) expresses it:

“a variety of novel methods and models, often transposed from other fields... to be presented and examined *on their own merits*, rather than subject to a tradition-oriented screening process” (emphasis in the original).

A categorisation by field of application would be counter-productive, as it would serve only to fortify the barriers between the paradigms of the respective fields.

2.1.2.4 While the above criteria may be helpful in identifying the category of model needed for a particular application, they are not helpful in classifying a particular model. For example, the capital-asset pricing model may be applied to all the problems listed, and in each of the fields of actuarial practice listed.

### 2.1.3 CATEGORISING BY THE NATURE OF THE MODEL

The categorisation of Booth et al. (op. cit.) by the nature of the model itself follows that of Derman (1996), which is discussed below.

## 2.2 DERMAN (1996)

2.2.1 Derman (op. cit.) identifies the following categories:

- fundamental models;
- phenomenological models; and
- statistical models.

2.2.2 He presents this categorisation without justification or authority. His definitions relate explicitly to models used in financial economics. He apparently intends the categories to be mutually exclusive and complementary. But even in that context, it would be difficult to determine from his definitions which category certain models would fall into. In the wider context of actuarial science this difficulty would be insuperable.

## 2.3 BELL ET AL. (1998)

2.3.1 Bell et al. (1998) define a ‘scientific model’ as “an abstract and simplified representation of a given phenomenon.” They then define a ‘mathematical model’ as a “scientific model in which the representation is expressed in mathematical terms” and a ‘stochastic model’ as “a mathematical model in which the representation is expressed in terms of probabilities.” They define a ‘dynamic model’ as a “stochastic model that incorporates a systematic process for revising the model in response to observed results.” They define a ‘deterministic model’ as “a simplification of a stochastic model in which the proportion of occurrences of a given event estimated by the stochastic model is assumed to occur with probability one.” Finally they define an ‘actuarial model’ as a stochastic model of ‘actuarial risks’, “based on assumptions about the probabilities that

will apply to the actuarial risk variables in the future, including assumptions about the future environment.” For the purposes of this definition, an ‘actuarial risk’ is “a phenomenon that has economic consequences and is subject to uncertainty with respect to... occurrence, timing [or] severity.”

2.3.2 This categorisation is useful so far as its definitions accord with what would naturally be understood by the terms defined. It may be argued that the definition of a deterministic model as a particular type of stochastic model is meaningful in that it recognises the artificial treatment of uncertainty in the former. On the other hand, it may be argued that the concept of a deterministic model is more fundamental in that the range of a probability measure is deterministic; the concept of a deterministic value must therefore be understood before a probability measure can be defined.

2.3.3 The definition of a ‘dynamic model’ does not allow for the dynamic modelling of decision-making: it merely relates to the revision of models from time to time. Such revisions do not change the nature of a model unless the ‘systematic process for revising the model’ is part of the model itself. It therefore appears to cover both models in which decision-making processes are assumed and those in which they are not.

2.3.4 The scope of this paper is confined to ‘mathematical models’ in the sense defined by Bell et al. (op. cit.). It is not, however, confined to ‘actuarial models’ in the sense defined by Bell et al. (op. cit.). That classification turns on:

- whether they model ‘actuarial risks’;
- whether they are ‘based on assumptions about the probabilities that will apply to the actuarial risk variables in the future’; and
- whether those assumptions include ‘assumptions about the future environment’.

2.3.5 The question whether a phenomenon is an ‘actuarial risk’ turns on:

- whether it has ‘economic consequences’; and
- whether it is ‘subject to uncertainty with respect to... occurrence, timing [or] severity’.

(¶2.3.1)

2.3.6 Not all models used in actuarial science are models of ‘actuarial risks’ in this sense, nor are they necessarily ‘based on assumptions about... probabilities’. For example, a yield curve, which clearly falls within the definition of a mathematical or symbolic model, does not fall within the definition of an ‘actuarial model’. Also, while a model such as the graduation of mortality rates is a mathematical model based on assumptions about probabilities, it does not necessarily model actuarial risks. In itself it does not model a phenomenon with economic consequences. Only if such a model is used in a model of contracts based on life contingencies does it become part of an ‘actuarial model’ as defined by Bell et al. (op. cit.). It is clear that many of the models typically used in actuarial practice do not necessarily constitute ‘actuarial models’ as defined above.

2.3.7 It has been suggested to the author that a (parametric) graduation of mortality rates does not fall within the definition of a ‘model’; that the graduation process is merely a process of determining parameters and the graduated rates are merely a function of the parameter estimates. In order to decide the matter, the definition in ¶1.2 may be invoked. Here the real system is a population of lives experiencing mortality. The theory is that their mortality may be represented by a smooth function of age (and

possibly duration since entry). The model is homomorphic to the real system: the real system produces actual mortality rates and the homomorphic system produces expected mortality rates. In this way the model represents the population's mortality in a manner that satisfies the theory. On the basis of this argument, such a graduation does fall within the definition of a 'model'. In Bell et al.'s (op. cit.) nomenclature, a mortality graduation falls within the definition of a 'mathematical model'.

## 2.4 PEMBERTON (1999)

2.4.1 Following Feldblum (1995), Pemberton (1999: 148) distinguishes a 'scenario model' as:

"... the development of a small number of pictures of possible futures which are defined by a qualitative account of how the world would be."

2.4.2 Pemberton (op. cit.: 154–155) categorises the models used by actuaries as:

- 'assumption-based models' (typically a 'rule of thumb');
- 'fact-based models' (like deterministic models for the valuation of life offices and pension funds); and
- 'extended fact-based models' (i.e. fact-based models modified to allow for 'the changing nature of the causal influence of the situation').

2.4.3 He acknowledges that, while this categorisation accommodates deterministic models, "scenario and stochastic models are generally more difficult to classify."

2.4.4 'Assumption-based models' are outside of the scope of this paper. It is questionable whether a 'rule of thumb' should be dignified as a 'model'. In view of the subjectivity of models, references to 'fact-based models' have also been avoided.

## 2.5 ACKOFF & SASIENI (1968)

2.5.1 The models used in actuarial practice are similar in certain respects to those used in operations research, and consideration of the categorisation of models in that discipline is instructive. While most of the standard texts of operations research discuss the importance of modelling in that discipline, they do not generally formalise a typology of the models used. An exception is Ackoff & Sasieni (1968). The 'mathematical model' of Bell et al. (op. cit.) corresponds to what Ackoff & Sasieni (op. cit.: 61) define as a 'symbolic model', which is typified by the use of 'letters, numbers and other types of symbols to represent variables and the relationships between them'. They 'take the form of mathematical relationships... that reflect the structure of that which they represent'. (The other primary types are 'iconic' and 'analogue' models, which fall outside the scope of the models considered in this paper.)

2.5.2 At a secondary level they distinguish between 'explanatory' and 'descriptive' models: the former contain 'controlled variables', whereas the latter do not. The designation of the former as 'explanatory' is justified on the basis that, in order for control to be achieved, a causal relationship must exist between the control variable and the objective variable.

2.5.3 The question whether a model is ‘explanatory’ or ‘descriptive’ turns on whether it contains controlled variables. Clearly, for this purpose, a ‘controlled variable’ is a variable controlled by a decision-maker whose decisions are not part of the model itself. A stochastic asset–liability model used for portfolio selection is explanatory, the controlled variables being the proportions invested in the respective asset categories. But if such a model is integrated into a model that optimises the portfolio-selection problem, then the values of those variables are outputs from the models, not inputs to them. Thus, from the point of view of such integrated models, the variables concerned are not externally controlled and these models are therefore not explanatory. Many models used in actuarial practice, from graduations to stochastic investment models, are clearly descriptive.

2.5.4 It is also questionable whether ‘explanatory’ is the most appropriate description of the models so named. In analysing the purposes to which models may be applied, it is useful to distinguish between explanation, prediction and control, and models may be developed for any of those purposes. A descriptive model such as a stochastic investment model may, however, be used either for the explanation of history or for the prediction of its future course. The description of a model used for control purposes as ‘explanatory’ is therefore misleading.

## 2.6 KNOX ET AL. (UNPUBLISHED)

2.6.1 Knox et al. (unpublished: 5) also discuss the variety of types or forms of models developed by actuaries:

“... the models developed by actuaries include:

- individual-based models, which equate the present values of the uncertain future inflows and outgoes and thereby permit the calculation of individual premiums and individual reserves after the commencement of a policy;
- collective-based models, where individuals are grouped so that it is the aggregate cash flows [that] are considered in setting contribution rates or benefits;
- asset–liability management models, which represent the interactions between the two sides of the balance sheet; and
- financial models, which attempt to represent some aspects of economic reality.”

2.6.2 While this list is not intended to be exhaustive (a life table, for example, cannot be included), it is also unclear how the categories suggested are supposed to relate to each other. For example, the valuation of life-office liabilities might be made on data aggregated by age; it would seem unnecessary to distinguish such a valuation from one in which the liabilities were calculated for each policy before aggregation.

## 2.7 CONCLUSION

2.7.1 From the above discussion it is clear that the categorisation, in the actuarial literature, of models used in actuarial practice leaves much to be desired.

2.7.2 Working from the definition in ¶1.2, it would be logical to recognise the homomorphism between the model and the real system in any categorisation of models.

Between the real system and the model there is a process, which explains how the model is developed from information and theory about the real system and from the judgement or subjective input of the agents involved in the development of the system. In the model, as in the real system, there is input and output. If the model may be represented by a function in closed form, the input is an element or subset of the domain of the function and the output is an element or subset of its range. If, for example, the model may be represented by a regression equation, the input comprises the independent variables and the output the dependent variable. If the model is an algorithm, the input comprises the variables that differ from one application of the algorithm to the next and the output is the value of the variable or variables produced by the algorithm. The nature of a model may be categorised either by the process by which it is developed or by the process of application (i.e. how the output is derived from the input). Both the process of development and the process of application may be described by the nature of each of their constituent components.

2.7.3 This means that the mathematical formulation of a model is not in itself a criterion for the classification of the model. From a mathematical point of view, a model may be expressed as a function, a set of functions, or an algorithm. The mathematical expressions of two different models may be identical in form, but if the nature of any of the constituents of the process of development or application is different, they will be classified into different categories. An example of this is given in ¶3.2.3 below. In this paper, the specification of the mathematical forms of models is therefore avoided.

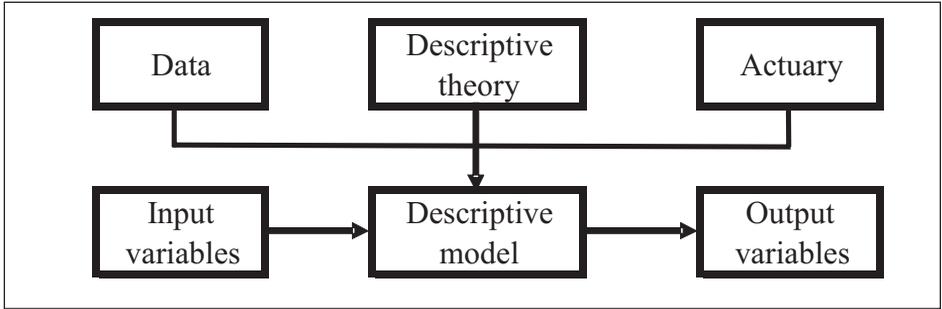
### 3. CATEGORISATION OF MODELS

#### 3.1 DESCRIPTIVE MODELS

3.1.1 A distinction needs to be drawn between a descriptive model and a predictive model. A stochastic investment model may be parameterised as a descriptive model in that it describes the historical relationships between the variables modelled, but (subject to reparameterisation) it may be intended to be used as a predictive model. Such a model is a general model in that it describes or predicts the market rather than a particular person, institution or fund.

3.1.2 The contributions of data, descriptive theory and the actuary's judgement to the specification of a descriptive model may be represented as shown in Figure 1. In this figure, the process of development of the model is shown vertically (from sources of information to the formulation of the model) and the process of application of the model is shown horizontally (from input to output). Here, for example, 'actuary' represents the actuary as a source of information being input into the model. This includes actuarial judgement with regard to levels of significance, accuracy, smoothness, parsimony, complexity etc. These considerations will be affected by the nature of the problem to be addressed by the actuary and the purpose to which the model is to be applied. A stochastic investment model clearly conforms to this scheme.

Figure 1. A descriptive model

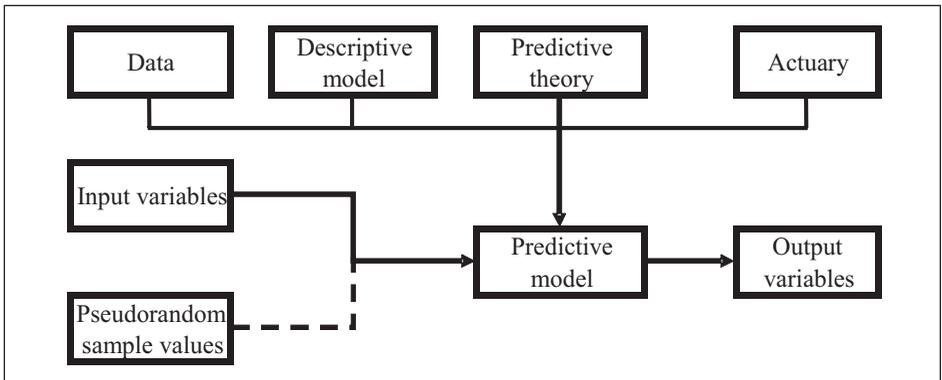


### 3.2 PREDICTIVE MODELS

3.2.1 The future will not necessarily be the same as the past. In its application as a predictive model, some theory may therefore be needed in order to reparameterise the model for predictive purposes. This may, for example, involve the relationship between the current yield curve and expected future interest rates, or between the beta values implied by the model for various asset categories and the expected future returns on those categories. Further data are also needed, both for the revision of the parameters to reflect predicted conditions and for the current values of the variables modelled. This is shown in Figure 2.

3.2.2 Again, the process of development is shown vertically and the process of application horizontally. Here the ‘input variables’ are deterministic. They may (as, for example in an ARIMA model) be recent values of the variables being modelled. They may also be values (such as the current yield curve) that may be used in determining the parameters of the predictive model. Besides input variables, if (as in this case) the predictive model is stochastic, its input will include pseudorandom sample values. If the

Figure 2. A predictive model



output variable or its distribution can be expressed in terms of the input variables in closed form or by methods of approximation other than simulation, or if the model is deterministic, the simulation of sample values would be unnecessary; hence the dotted line. The use of simulation is given special prominence here because, so far as the modelled sources of uncertainty are concerned, it demonstrates with particular clarity the homomorphism of the model to the real system. It also demonstrates how frequentist notions of probability may be used to represent uncertainty.

3.2.3 It may be noted that, as indicated in ¶2.7.3, a predictive model may have a mathematically identical form to a descriptive model. For example, consider the single-index model (Elton & Gruber, 1995: 131):

$$R_i = \alpha_i + \beta_i R_m + e_i;$$

where:

$R_i$  is the return on the  $i$ th share;

$R_m$  is the return on a share-market index; and

$e_i$  is a random variable with mean 0.

This may be used as a descriptive model, in which case  $R_i$  and  $R_m$  are the values of input variables selected from the data specified in the development of the model and  $\alpha_i, \beta_i$  and  $e_i$  are output variables. Alternatively, it may be used as a predictive model to predict a distribution for (or the expected value of)  $R_i$ , in which case  $\alpha_i$  and  $\beta_i$  are parameters of the model (derived in part from the descriptive model, but possibly adjusted on the basis of other information, theory and the actuary's subjective judgement) and  $e_i$  is a pseudo-random variable (whose distribution may be similarly derived), and  $R_m$  is either an input variable or a pseudorandom variable.

### 3.3 FUND MODELS

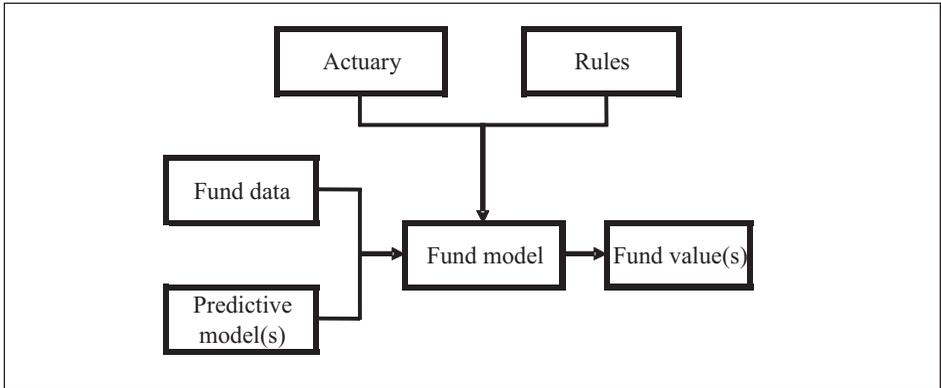
3.3.1 A model of a particular fund (e.g. a retirement fund or life fund) is based on the rules governing the fund or the contracts entered into by it. It specifies a fund value as a function of:

- current fund data, i.e. data relating specifically to the decision-maker's assets and liabilities (including the values of any control variables such as the asset allocation supposed to be selected);
- the returns on the various asset categories in future years; and
- variables such as inflation rates and the rates of mortality and other contingencies that may be expected to affect the future cash flows in respect of the liabilities, administrative expenses and tax.

The first item corresponds to the data that would be required for a valuation of the fund. The latter two correspond to the assumptions that would have to be made. The fund value may, for example, be the surplus at a specified time horizon (or at the current date), but this depends on the criteria to be used for the purposes of decision-making.

3.3.2 The fund model is illustrated in Figure 3. (For simplicity, the sources of information in the predictive models have been omitted. Note that a fund model is not a particular type of predictive model; it obtains its input in part from the output of a predictive model.)

Figure 3. A fund model



3.3.3 For the purposes of definition of a fund model, a ‘fund’ does not necessarily mean an entire institution. It may include the financial system relating to an individual contract, a set of contracts or an individual agent.

3.3.4 It has been suggested to the author that a fund model may be used descriptively. An example of this might be an analysis of surplus, where a fund model is used to determine what would have happened if some or all of the assumptions used in the previous valuation had been realised. For the purpose of this paper, it is more consistent to treat such models as predictive, as, despite the fact that an historical period is being considered, it is nevertheless being considered on the basis of assumptions—i.e. as if it were a future period, subject to prediction. The point of a descriptive model is that it may be used as a basis for a predictive model. While an analysis of surplus may be used to identify important departures from the assumptions made at the last valuation, it cannot of itself form the basis for a predictive model.

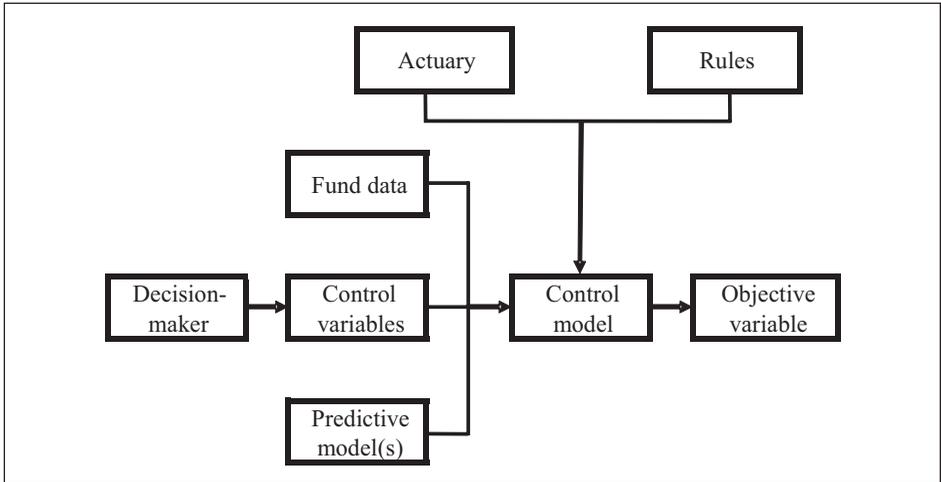
### 3.4 CONTROL MODELS

3.4.1 A control model is a fund model in which certain items of the fund data (e.g. the asset allocation) are treated as control variables and an objective variable (e.g. the contribution rate required at the time horizon) is defined in terms of the fund value. A control model is illustrated in Figure 4.

3.4.2 The purpose of a control model is to show the effects of alternative values of the control variables. The control variables are those over which the decision-maker has control in the real system. The model shows how the objective variable (or its distribution) is affected by particular choices of the control variables. It corresponds to the ‘explanatory model’ of Ackoff & Sasieni (1968). However, reference has been made to a ‘control model’ rather than an ‘explanatory model’. The input variables of a descriptive or predictive model may well have explanatory value, even though they may not necessarily be controllable.

3.4.3 If the predictive model is stochastic, the objective variable is a random

Figure 4. A control model



variable, and the output is a pseudorandom sample of its distribution. In other words, the output of the control model is deterministic or stochastic according to the output from the predictive model from which the control model derives its input.

### 3.5 NORMATIVE MODELS

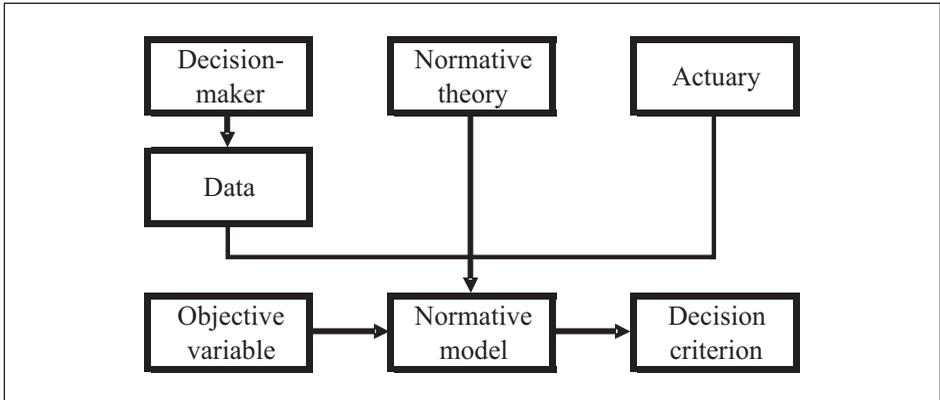
3.5.1 The distinction drawn in Thomson (2003a) between descriptive and normative decision theory is important. In actuarial practice, a descriptive theory may help to clarify understanding of a phenomenon, and it may serve as the basis for the development of a model for use in prediction or control. For example, a good descriptive theory of decision-making by prospective policyholders may help the actuary to price products so as to maximise profits. But for the purpose of making recommendations to a decision-maker (for example, to a prospective policyholder or a price-taking investor), a normative theory is required. For further discussion of this point, the reader is referred to Thomson (op. cit.: 656–7).

3.5.2 The sources of information in a normative model are shown in Figure 5. The normative theory may, for example, be expected utility theory, in which case the normative model is the decision-maker's utility function. The process differs from that of a descriptive or predictive model in that the data (e.g., in the case of expected utility theory, elicited values of the decision-maker's utility function) come direct from the decision-maker. No data external to the decision-maker are required for the specification of the structure of the model.

### 3.6 DECISION-MAKING MODELS

3.6.1 The application of normative and control models to the formulation of actuarial advice is shown in Figure 6. Since the decision-making process is part of the

Figure 5. A normative model



model, and since the output is normative advice for the purposes of decision-making, this model is referred to as a ‘decision-making model’.

3.6.2 Reference is made in Figure 6 to an ‘idealised decision-maker’. This follows the theory proposed by Thomson (2003a). The recommendation made by the actuary is not necessarily what the actuary would decide if she/he were the decision-maker, because the actuary’s risk tolerance is not necessarily the same as the decision-maker’s. Nor is it what the decision-maker would have decided, because the decision-maker would not have the skills necessary to develop and apply a descriptive model and a normative theory for that purpose. As explained in ¶¶2.3.18–19 of that paper, the decision recommended (i.e. the advice) is the decision that an idealised decision-maker would have made. The resulting decision is not necessarily prudent—that would depend on the risk-tolerance of the decision-maker—but, as argued in section 2.3 of that paper, it is rational.

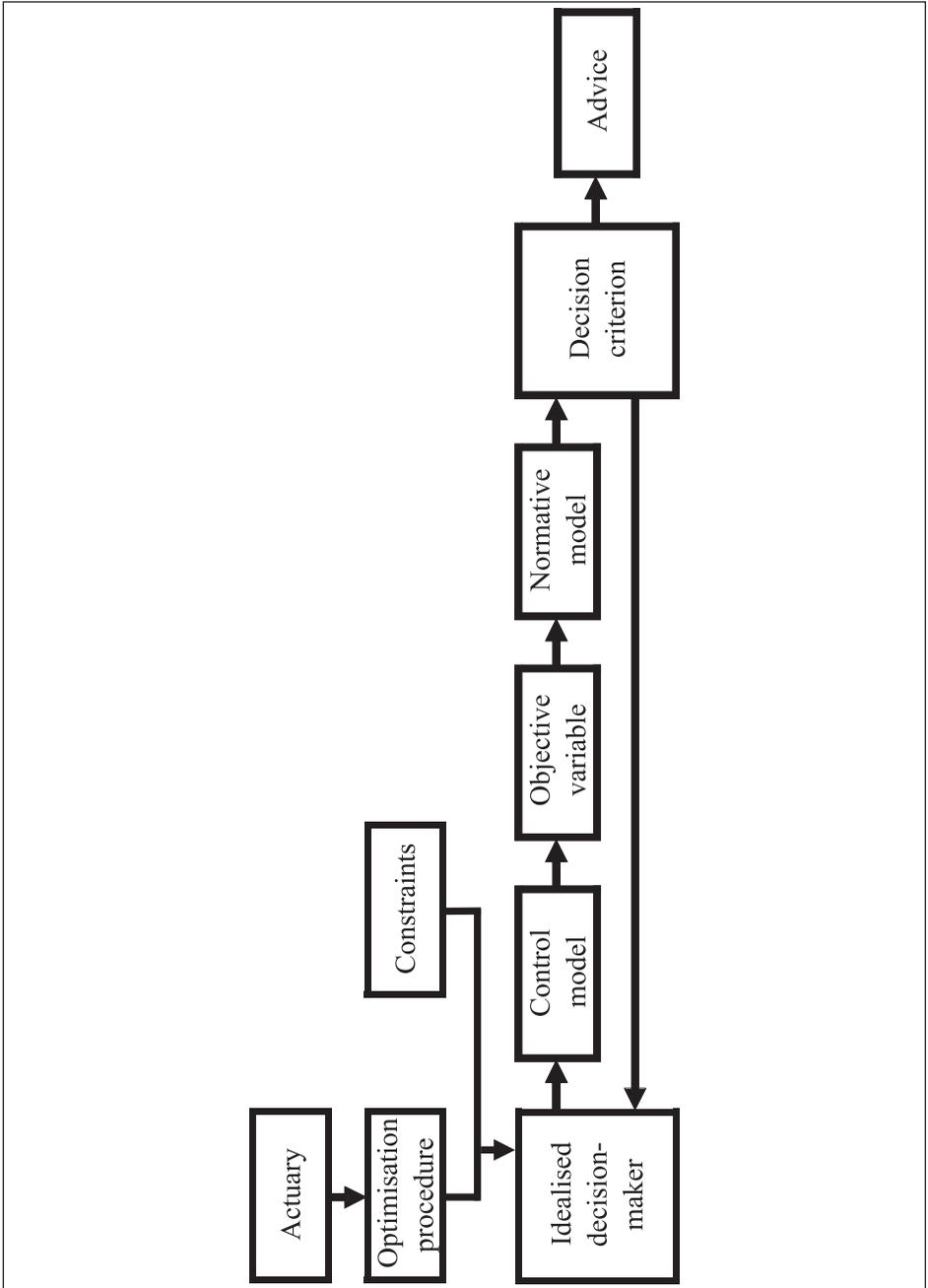
3.6.3 The backward arrow returning to the idealised decision-maker indicates that, as in Thomson (2003b), the descriptive model, and therefore the control model, may be stochastic, requiring numerous simulations to define the objective function, and the optimisation procedure may be dynamic, allowing for repeated decision-making up to the time horizon. In practice the decision procedure may involve the satisfaction of decision criteria rather than the optimisation of such criteria.

### 3.7 SUMMARY

In the above discussion, models used in actuarial practice have been categorised as:

- descriptive models;
- predictive models;
- fund models;
- control models;
- normative models; and
- decision-making models.

Figure 6. A decision-making model



### 3.7.1 DESCRIPTIVE MODELS

A descriptive model is one that describes historical relationships between the variables modelled.

### 3.7.2 PREDICTIVE MODELS

A predictive model is one that predicts future relationships between the variables modelled.

### 3.7.3 FUND MODELS

A fund model is one that expresses fund values in terms of fund data, the output variables of predictive models, and the rules of the fund. A ‘fund’ comprises the financial system relating to an individual contract, a set of contracts, an individual agent, or a financial institution. ‘Rules’ may be legal provisions or reasonable expectations.

### 3.7.4 CONTROL MODELS

A control model is a fund model that expresses an objective variable in terms of predictive models and control variables specified by the decision-maker.

### 3.7.5 NORMATIVE MODELS

A normative model is one that expresses the preferences of a decision-maker between financially risky prospects in terms of data relating to the attitude of the decision-maker to wealth and risk.

### 3.7.6 DECISION-MAKING MODELS

A decision-making model is one that determines the advice to be given by the actuary to the decision-maker corresponding to the decision, optimised subject to certain constraints, that would be made by an idealised decision-maker, whose preferences are described by a particular normative model and whose subjective probabilities of future outcomes are described by a particular control model.

## 3.8 TIME

Most of the models used in actuarial practice involve functions of time (or equivalently age). Others either contemplate a specified time horizon (as in mean–variance decision-making models and in normative models) or a variable that implicitly varies over time (such as the valuation rate of interest in an immunisation model). In each type of model, time may be measured in discrete intervals or as a continuous variable. In the continuous-time case a time horizon may tend to zero. As discussed in Thomson (2003c), continuous-time models, though they may be more elegant, are not necessarily better representations of reality than discrete-time models. Because of the lack of closed-form solutions, continuous-time models may in many cases require numerical approximation. On the other hand, it may be preferable to use a continuous-time model if (as in the modelling of mortality rates) it simplifies the use of the model. In some circumstances (as in Thomson, 2003b) continuous-time models may be used to model processes within

discrete time intervals. In this paper, the distinction between discrete and continuous time has not been used to distinguish categories of models. These features may be seen, instead, as characteristics of models of the various categories.

### 3.9 GENERAL AND SPECIFIC MODELS

3.9.1 A descriptive or predictive model may be general or specific. Here a 'general' model is one that expresses relationships between variables relating to the market, which are outside of the control of the decision-maker. A 'specific model' is one that expresses relationships between variables at least some of which relate to or are within the control of the decision-maker.

3.9.2 It may be argued that actuarial science is more concerned with the practice of decision-making than with the pursuit of truth for its own sake. This means that predictive models are of more interest than descriptive models and specific models are of more interest than general models. However, a general descriptive model may have explanatory value that may be useful to the actuary in the development of specific predictive models. Descriptive and general models serve as points of departure for predictive and specific models. A predictive model is more convincing both to the actuary and to the client if it is explicable in relation to a descriptive model. The advantage of a general model is that it can be applied to different decision-makers' situations without necessitating the reconsideration of the model in each situation.

3.9.3 A normative model is by definition specific; it must reflect the decision-maker's preferences. A fund model is also specific; it requires data specific to the fund and a specification of the rules of the fund, and it may include a specific predictive model. A control model and a decision-making model are therefore also specific.

3.9.4 As in the matter of time, the distinction between general and specific models has not been used to distinguish categories of models. Again, these features may be seen, instead, as characteristics of models of the various categories.

### 3.10 THE RELATIONSHIP BETWEEN MODELS AND SYSTEMS

It may be observed that, just as a model is defined in relation to a real system, so, for example, control models and decision-making models may be defined in relation to real control systems and real decision-making systems. In practice, predictive models are used in control systems, whether those systems are formally (i.e. mathematically) modelled or not. Similarly, control models and systems are used in decision-making systems, whether those systems are formally modelled or not. In an informal control system (i.e. a control system that is not formally modelled), a decision-maker is subjectively weighing alternatives in the light of information about her/his general environment and specific situation. Similarly, in an informal decision-making system she/he is subjectively selecting the best (or a satisfactory) alternative.



- Decision-making models:
- asset-selection models based on mean–variance portfolio theory or expected-utility theory
- the capital-asset pricing model
- option pricing models
- liability pricing models

## 4.2 DISCUSSION

4.2.1 Most of the above classification follows clearly from the definitions. It may be helpful to discuss possible exceptions.

4.2.2 The capital-asset pricing model is included as a decision-making model because it treats hypothetical market participants as idealised decision-makers with homogeneous expectations. Here it is assumed that the model is used for the purposes of pricing. The rules in the control model are trivial: the investor receives the return on the assets without the intermediation of a financial institution. Constraints may or may not apply. But there is a normative model (viz. mean–variance optimisation), which is iteratively applied through the process of achieving equilibrium. And the model produces advice with regard to pricing and asset allocation. Because the model has a solution in closed form, pseudorandom sampling is not required.

4.2.3 An option-pricing model similarly involves hypothetical market participants as idealised decision-makers who, in this case, eliminate arbitrage against a hedge portfolio. Once again, it is assumed that this model is used for the purposes of pricing. Here the rules describe the option pay-out in terms of the price of the underlying security.

4.2.4 The classification of models for the pricing of general contingent claims or liabilities using risk-neutral methods or deflators may be similarly justified.

4.2.5 The models referred to above may, however, be used for descriptive purposes. For example, the option-pricing model may be used to determine the implied volatility from actual prices. In that context they may be categorised as descriptive models. Similar observations may apply to other models.

## 5. CONCLUSION

It has been shown in this paper that the categorisation suggested by the author may be applied quite generally to the classification of models used in actuarial practice. While this categorisation, together with the associated classification of models, cannot be claimed to be unique, it does constitute a tentative typology of such models, which may be found useful as a candidate exemplar, or as a basis for further refinement, in the discourse of actuarial science.

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